Combinatorial spectra of graphs

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In the article [1] we have created a generalization of the Hamiltonian spectrum of a graph G called the H-Hamiltonian spectrum of the graph G denoted by $\mathscr{H}_H(G)$. Not only does this generalization give us the opportunity to talk, for example, about the isomorphism of graphs and the regularity of graphs in the language of these spectra, but there are several relations between $\mathscr{H}_H(G)$ and $\mathscr{H}_{H'}(G)$ for related H and H', for example for $H' = \overline{H}$. And so this brings some basic calculus to this area.

This approach has led us to one more generalization, which we call the combinatorial spectrum. This spectrum is now for R-weighted graphs, more precisely for sets of R-weighted graphs and we denote it $\mathscr{H} * \mathscr{G}$. This will play role of multiplication, we also get some addition $+$. When we denote $\mathscr{P}(Graphs)$ the set of all sets of graphs we will get that $(\mathscr{P}(Graphs), *, +)$ is a R-module, semigroup (or comutative monoid) and something close to ring. The most important thing is that most of the basic concepts of graph theory, such as maximum pairing, vertex and edge connectivity and coloring, Ramsey numbers, isomorphisms and regularity, can be expressed in the language of these combinatorial spectra. Together with the already mentioned calculus, it gives hope that all these concepts could be linked, studied together and transfer the results from one to the other.

References

[1] Dzúrik, M. (2021). An upper bound of a generalized upper hamiltonian number of a graph. *Archivum Mathematicum*, (5), 299?311. https://doi.org/10.5817/am2021-5-299