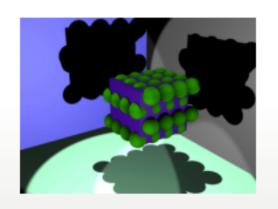


AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY



# MODELLING POROSITY OF HIGH SURFACE AREA NANOPOWDERS OF THE GALLIUM NITRIDE GAN SEMICONDUCTOR

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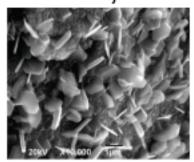
#### Gallium nitride GaN

- two crystallographic forms at RT:
  - wurzite (hexagonal): a = 3.168 Å and c = 5.178 Å
  - zinc blende (cubic): a = 4.51 Å
- thermal stability up to 1000 °C
- high chemical stability
- strong piezoelectric effect
- wide bandgap semiconductor: 3.4 eV (hexagonal)
- alloys with InN and AlN; alloy bandgap range: 1.8-6.2 eV
- GaN bandgap is a function of particle size for R < 11 nm</li>



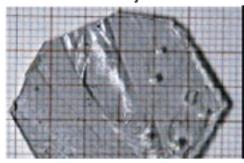
#### Materials forms of GaN

#### Microcrystals



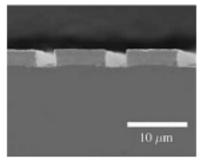
L. Lei, D. He; Cryst. Growth Des., 2009, 9, 1263.

#### Bulk crystals



T. Fekeda, D. Ehre httaet, J. Cryst. Growth, 2007, 305, 304. ▲

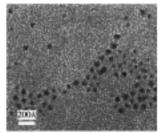
#### Thin films



H. K. Cho et al.; Su perlattices and Microstruct, 2004, 36, 385.

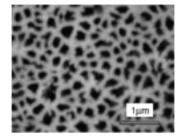
#### Gallium nitride GaN

#### Quantum dots



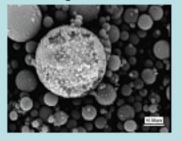
 I. MickSetal., Appl. Phys. Lett., 1999, 75, 4.

#### Porous surface



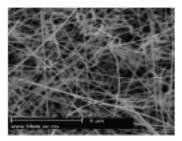
F. K. Yam, Z. Hassan, S. S. Ng; Thin Solid Films, 2007, 515, 3469.

#### Nanopowders



J. F. Janik etal.; J. Phys. Chem. Solids, 2004, 65, 639.

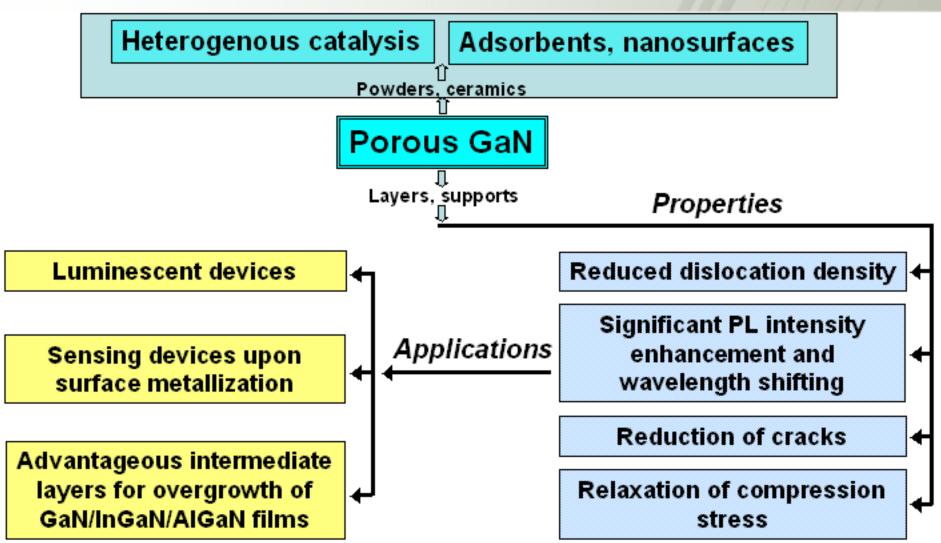
#### Nanotubes Nanofibers



W. Luetal.; J. Cryst. Growth, 2007, 307, 1.

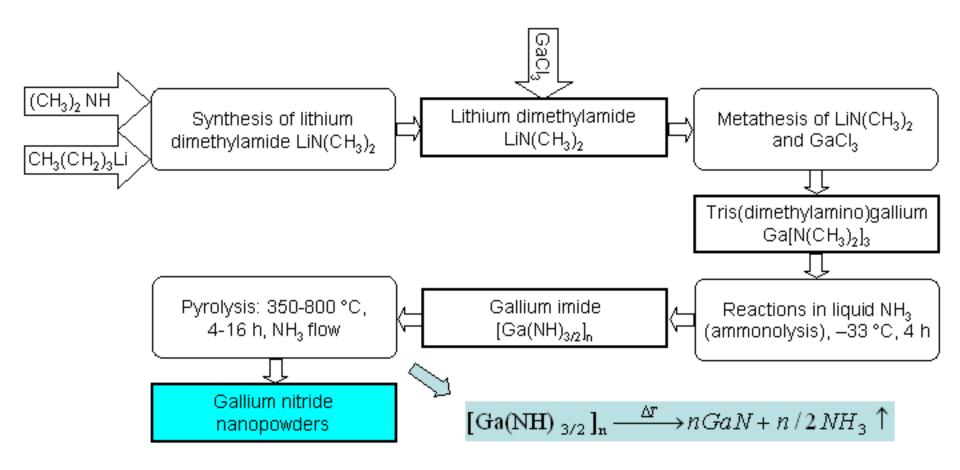


#### Porous GaN





## This work – anaerobic synthesis of GaN nanopowders

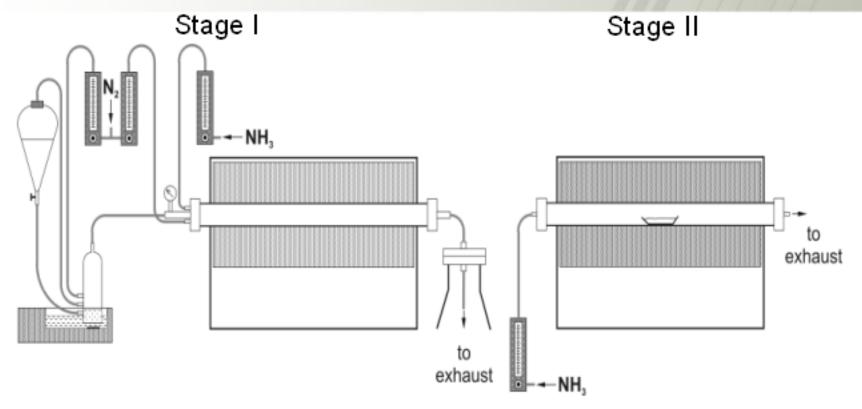


[1] J. F. Janik, R. L. Wells; Chem. Mater. 1996, 8, 2708.

[2] J. F. Janik, R. L. Wells, J. L. Coffer, J. V. St. John, W. T. Pennington, G. L. Schimek; Chem. Mater. 1998, 10, 1613.



## This work – aerosol synthesis of GaN nanopowders



$$2Ga(NO_3)_3 \xrightarrow{\phantom{-}1050\,^{\circ}\mathcal{C}\phantom{+}} Ga_2O_3 + "3N_2O_5" \uparrow \cdots Ga_2O_3 + 2NH_3 \xrightarrow{\phantom{-}950-975\,^{\circ}\mathcal{C}\phantom{+}} 2GaN + 3H_2O \uparrow C + 2NH_3 \xrightarrow{\phantom{-}$$

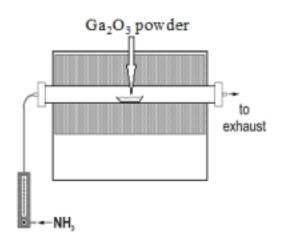
E. A. Pruss, G. L. Wood, W. J. Kroenke, R. T. Paine; Chem. Mater. 2000, 12, 19.

[2] J. F. Janik, M. Drygas, S. Stelmakh, E. Grzanka, B. Palosz, R. T. Paine; phys. stat. sol. a 2006, 203, 1301.



## This work – synthesis of GaN nanopowders from $Ga_2O_3$

#### Reaction of bulk gallium oxide powder with ammonia



$$Ga_2O_3 + 2NH_3 \xrightarrow{950\,^{o}C} 2GaN + 3H_2O \uparrow$$



## High surface area GaN powders – to date

"Adsorption characteristics of powders of nanometric gallium nitride and aluminium nitride" - L. Czepirski, J. F. Janik, E. Komorowska-Czepirska, R. L. Wells; Adsorpt. Sci. Technol. 2002, 20 (8), 723.

Max. BET surface area: 224 m<sup>2</sup>/g

"Template assisted design of microporous gallium nitride materials" – G. Chaplais, K. Schlichte, O. Stark, R. A. Fischer, S. Kaskel; *Chem. Comm.* **2003**, 730.

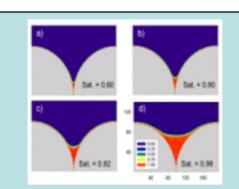
Max. BET surface area: 320 m²/g

"Porosity control in pre-ceramic molecular precursor-derived GaN based materials" – G. Chaplais, S. Kaskel; J. Mater. Chem. 2004, 14, 1017.

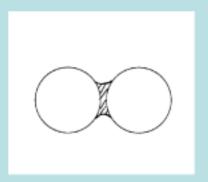
Max. BET surface area: 331 m<sup>2</sup>/g



## Capillary condensation *vs.* pore size and spherical particle morphology

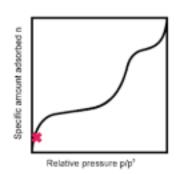


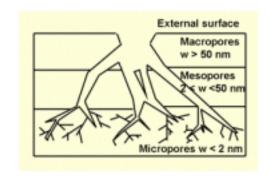
Liquid meniscus formation between spherical particles Seonmin Kim, Univ. of Maryland, USA



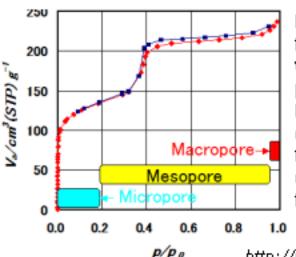
Bridging between two spheres due to capillary condensation







Dr. Nishith Verma, Kanpur, India



If a capillary's radius increases sharply, then capillary condensation (adsorption) will cease until an equilibrium vapor pressure is reached which satisfies the larger pore radius. However, during evaporation (desorption), liquid will remain filled to the larger pore radius until an equilibrium vapor pressure that satisfies the smaller pore radius is reached.

http://www.nippon-bel.co.jp/tech/seminar12\_e.html

- [1] D. Dollimore, D., G. R. Heal; J. Colloid Interf. Sci. 1973, 42(2), 233.
- [2] D. C. Howard, R. Wilson; J. Colloid Interf. Sci. 1976, 57(2), 276.



### Langmuir theory

#### Assumptions of Langmuir theory



http://upload.wikimedia.org/wikipedia/commons/0/0b/Langmuir\_izoterma.png

- Fixed number of adsorption sites are available on the surface.
- All the vacant sites are of equal size and shape on the surface.
- Each site can hold maximum of one gaseous molecule and a constant amount of heat energy is released during this process.
- Dynamic equilibrium exists between adsorbed gaseous molecules and the free gaseous molecules.

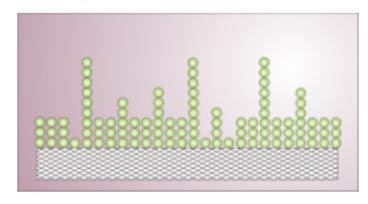
$$A(g) + B(s) \stackrel{\text{adsorption}}{\longleftarrow} AB$$

Adsorption is monolayer.



### Brunauer, Emmett, Teller (BET) theory

#### Assumptions of BET theory



http://upload.wikimedia.org/wikipedia/commons/8/83/BET\_Multilayer\_Adsorption.jpg

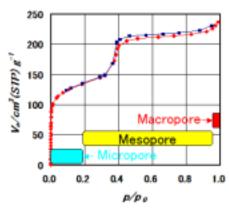
The concept of the theory is an extension of the Langmuir theory, which is a theory for monolayer molecular adsorption, to multilayer adsorption with the following hypotheses.

- Gas molecules physically adsorb on the surface with random distribution of sites covered by one, two, three, etc. molecules.
- There is no interaction between each adsorption layer.
- The Langmuir theory can be applied to each layer.



## Barrett, Joyner, Halenda (BJH) theory





.. The relative pressure p/p₀ at which decondensation (from capillary phase to multilayer adsorbed phase) occurs in a cylindrical pore is given by the Kelvin law.

$$kTln(p/p_0) = -2 \sigma v_1/(R-t)$$



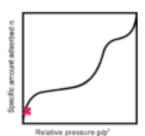
 $p_0$  - saturation vapor pressure,  $\sigma$  - surface tension,  $v_1$  - liquid molecular volume,

R - pore radius, t - multilayer thickness

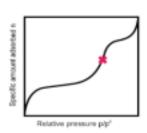


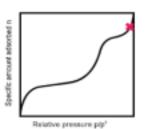
 The curvature of the solid surface has no influence on t. Thus, t can be experimentally determined using non porous adsorbents.

$$kTln(p/p_0) = U(t)$$









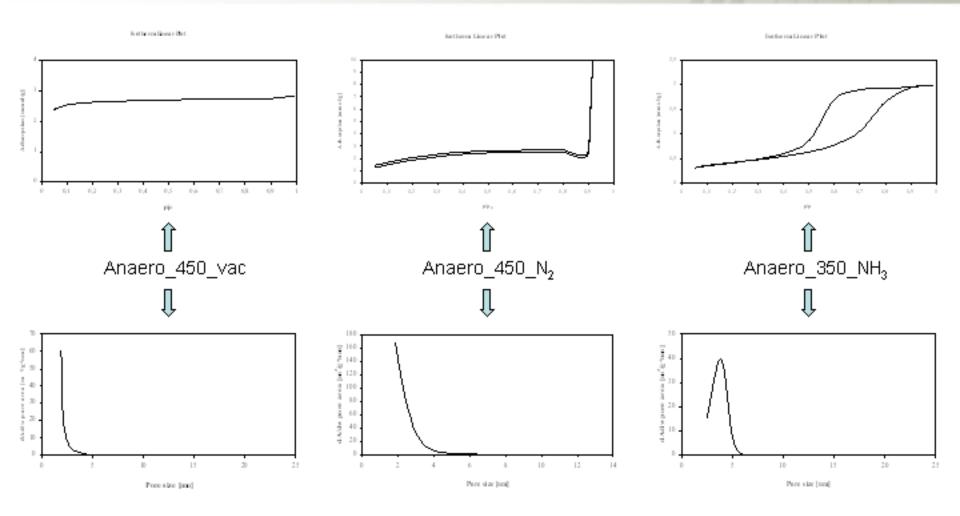


## Samples and experimental results

No.	Sample	D <sub>GaN</sub> (XRD) [nm]	<b>d</b> <sub>He</sub> [g/cm <sup>3</sup> ]	<b>S</b> BET [m²/g]	<b>S</b> <sub>Lang</sub> [m²/g]	<b>S</b> <sub>meso</sub> (BJH/des) [m²/g]	<b>V</b> <sub>meso</sub> (BJH/des) [cm³/g]	Av. pore size (BJH/des) [nm]
1	Anaer 450 vac	1.0	3.6	179	265	30.2	0.021	2.7
2	Anaer 450 N <sub>2</sub>	1.2	3.8	165	262	145	0.089	2.4
3	Anaer_350_NH <sub>3</sub>	1.6	4.0	134	216	96	0.070	3.7
4	Anaer_450_NH <sub>3</sub>	1.8	4.5	222	357	216	0.138	2.6
5	Anaer_600_NH <sub>3</sub>	3.0	5.2	103	162	133	0.154	4.6
6	Anaer 700 NH₃	6.0	5.2	58	91	76	0.146	6.9
7	Anaer 800 NH₃	8.1	5.0	42	66	66	0.143	17.8
8	Aero D/M 950	17.0	6.0	23.5	37.0	38.0	0.181	19.0
9	Aero_DMF_950	18.0	6.0	24.0	37.5	42.0	0.213	20.3
10	Aero_MeOH_950	18.0	6.1	20.8	32.1	22.8	0.086	15.1
11	Aero_H <sub>2</sub> O_975	30.0	6.0	6.3	9.7	5.2	0.015	11.5
12	Ga <sub>2</sub> O₃ 950	▼39.0	6.1	8.0	12.2	6.7	0.019	11.6

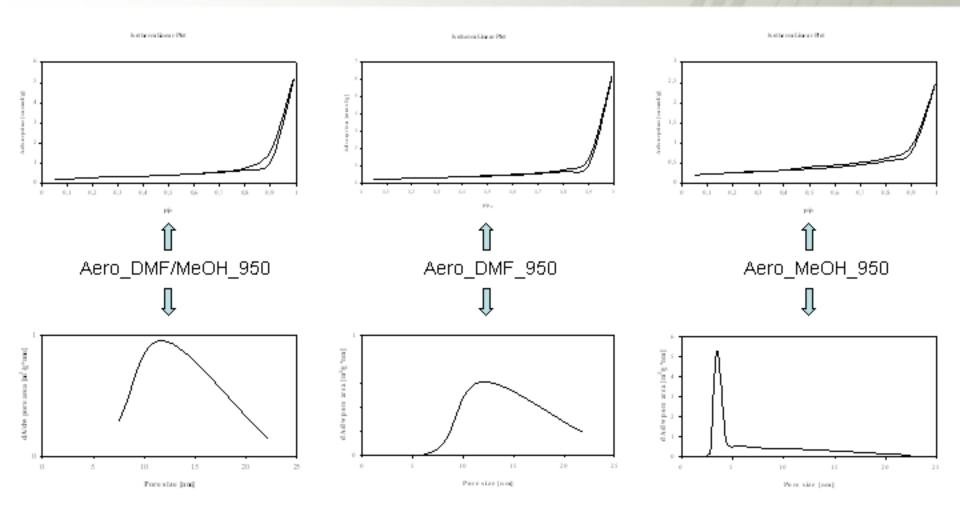


## Adsorption isotherms (N<sub>2</sub>, 77.5 K) and mesopore distribution (BJH) - 1



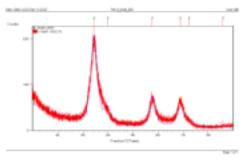


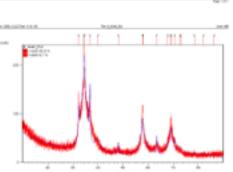
## Adsorption isotherms (N<sub>2</sub>, 77.5 K) and mesopore distribution (BJH) - 2

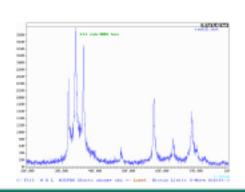


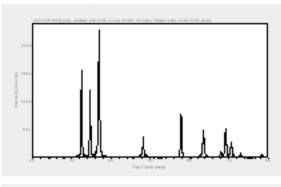


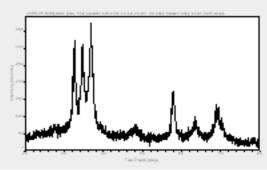
### XRD size determination aspect

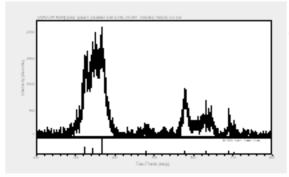












Scherrer's equation:

#### Av. crystallite size = $K \cdot \lambda / B \cdot \cos \Theta$

K (crystallite shape factor) = 0.9 (usually)

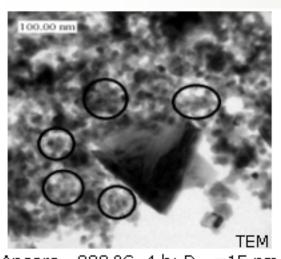
- $\lambda$  (X-ray wavelength) = 1.540598 Å (Cu K<sub>a</sub>)
- B (line broadening at half the maximum) = sq. root (B<sub>obs.</sub><sup>2</sup> B<sub>std.</sub><sup>2</sup>)
- Θ (Bragg angle)
- possible superimposed peaks for the hexagonal and cubic polytypes
- potentially defected hexagonal polytype
- vaquely addressed problem of lattice strain contribution to line broadening in nanograins
- equivocal interpretation of X-ray diffraction outcome for small nanocrystallites due to significant contribution of surface layers



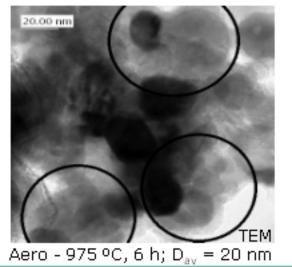
Increased uncertainty in average size determinations for D < ca. 10 nm



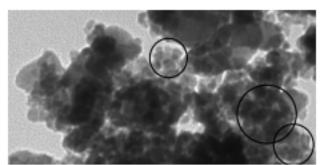
### TEM pictures of GaN powders

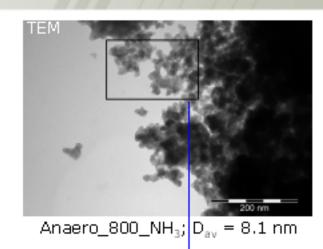


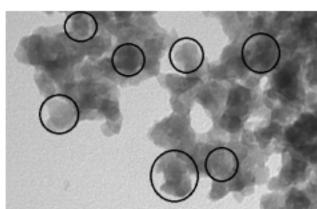
Anaero - 900 °C, 4 h; D<sub>av</sub> =15 nm



Anaero\_600\_NH<sub>3</sub>; D<sub>av</sub> = 3.0 nm



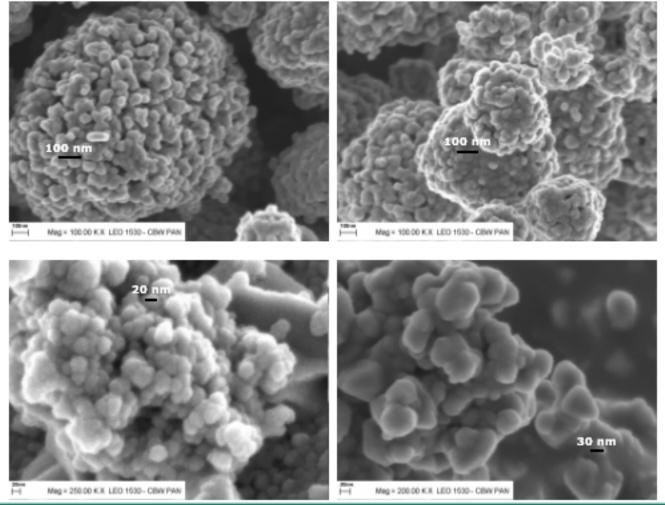






### SEM pictures of GaN powders

#### Aerosol-assisted synthesis, 950-975 °C, 6 h



IVC18/ICN+T2010/ICSS-14/VASSCAA-5, Beijing, China, 2010



## "Porosity" model ramifications phenomenological approach

Can pore system and pore surface area for gallium nitride GaN nanopowders be adequately modelled by interparticle spaces and surface area, respectively, of closely packed equiradial spherical particles as the simplest model case?

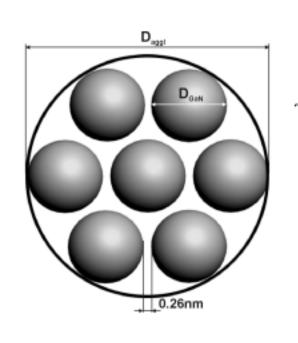


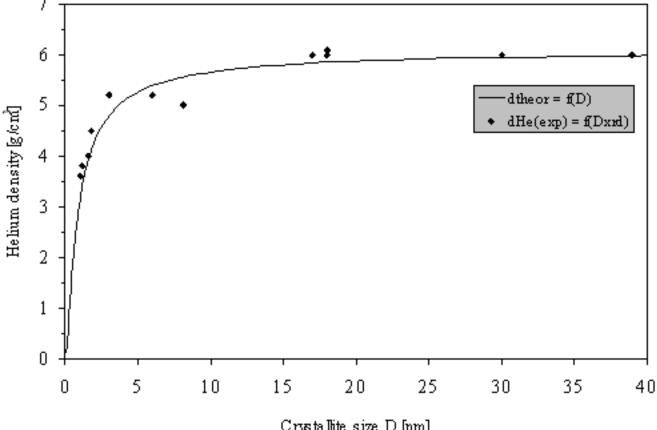
Model validation by testing correlation between the experimental standard "porosity" parameters, i.e., BET, Langmuir, and BJH-derived specific surface areas and the experimental "particle size" parameter, i.e., average crystallite diameter from XRD determinations with respect to predictions of the model.



### Helium density aspect

$$d_{aggl} [g/cm^{3}] = \frac{m_{aggl}}{V_{aggl}} = 6.1 \cdot \frac{n \cdot \frac{4}{3} \pi (\frac{D_{GaN}}{2})^{3}}{n \cdot \frac{4}{3} \pi (\frac{D_{GaN} + D_{He}}{2})^{3}} = 6.1 \cdot \frac{D_{GaN}^{-3}}{(D_{GaN} + D_{He})^{3}} = 6.1 \cdot \frac{D_{GaN}^{-3}}{(D_{GaN} + 0.26)^{3}}$$

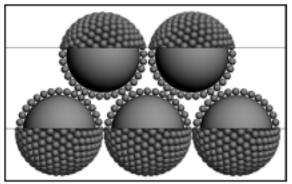


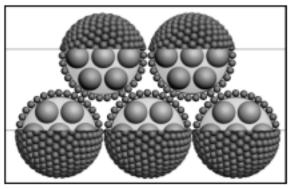


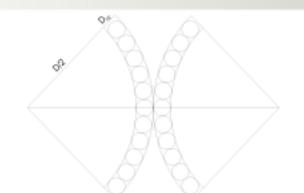
Crystallite size D [nm]

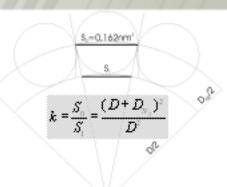


## Model 1 – double adsorbate layer particle bridging









$$n/cm^{3} = \frac{0.7405}{\frac{4}{3}\pi(\frac{D+2D_{N_{2}}}{2})^{3} \cdot 10^{-21}}; \text{ n - number of particles per 1 cm}^{3}$$

$$S_p[m^2] = 4\pi(\frac{D}{2})^2 \cdot 10^{-18}$$
,  $S_p$  - particle surface area

$$S\left[m^{2}/cm^{3}\right] = n \cdot S_{\mu} = \frac{0.7405}{\frac{4}{3}\pi(\frac{D+2D_{\mu_{2}}}{2})^{3} \cdot 10^{-21}} \cdot 4\pi(\frac{D}{2})^{2} \cdot 10^{-15} = \frac{4443 \cdot D^{2}}{(D+2D_{\mu_{2}})^{3}}$$

$$S[m^2/g] = \frac{4443 \cdot D^2}{(D+2D_{N_1})^3} \cdot \frac{(D+D_{H_2})^3}{6.1 \cdot D^3} = \frac{728.36 \cdot D^2}{(D+2D_{N_1})^3} \cdot \frac{(D+D_{H_2})^3}{D^3}$$

D - particle (crystallite/agglomerate) diameter [nm] D. = 0.260 nm - helium atom kinetic diameter

D. = 0.455 nm - nitrogen molecule diameter

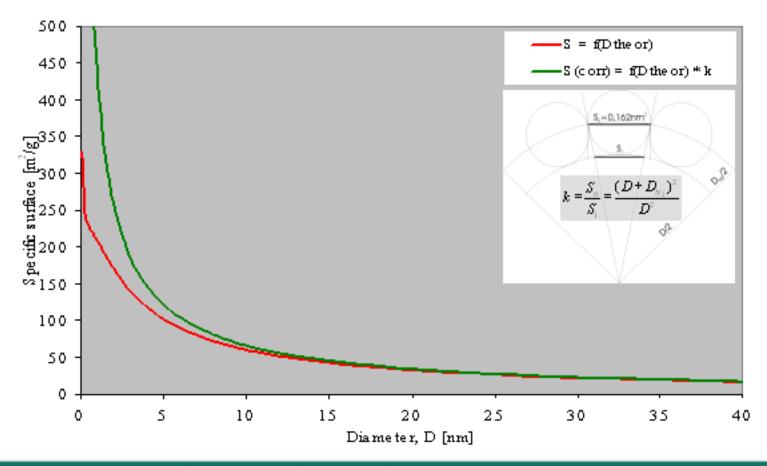
$$S_{corr}[m^2/g] = S[m^2/g] \cdot k$$

$$S_{corr}[m^2/g] = S[m^2/g] \cdot k = \frac{728.36 \cdot (D + D_{N_c})^2}{(D + 2D_{N_c})^3} \cdot \frac{(D + D_{H_c})^3}{D^3}$$



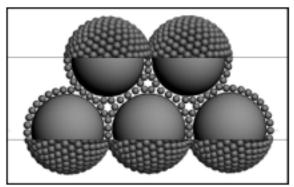
## Model 1 – double adsorbate layer particle bridging

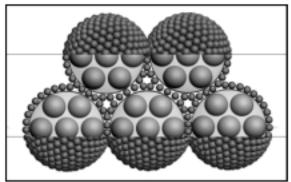
Model 1 - theoretical relationships between the specific surface and sphere diameter

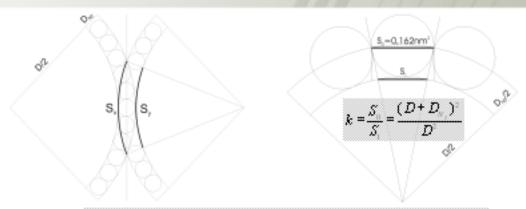




## Model 2 – single adsorbate layer particle bridging







D - particle (crystallite/agglomerate) diameter [nm]

D. = 0.260 nm - helium kinetic diameter

D<sub>v2</sub> = 0.455 nm - nitrogen molecule diameter

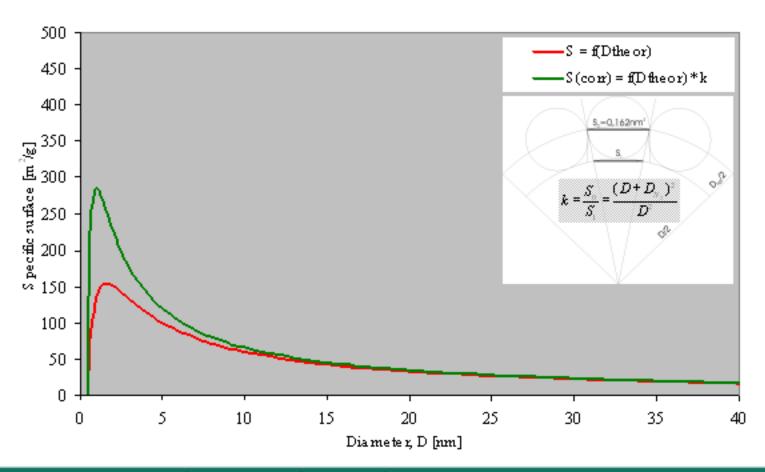
$$S[m^2/g] = (D^2 - \frac{3 \cdot D^2 \cdot D_{N_2}}{D + 2D_{N_2}}) \cdot \frac{728.36}{(D + D_{N_2})^3} \cdot \frac{(D + D_{H_2})^3}{D^3}$$

$$S_{corr}[m^2/g] = S[m^2/g] \cdot k = (1 - \frac{3 \cdot D_{N_2}}{D + 2D_{N_2}}) \cdot \frac{728.36}{(D + D_{N_2})} \cdot \frac{(D + D_{H_2})^3}{D^3}$$



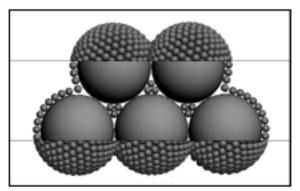
## Model 2 – single adsorbate layer particle bridging

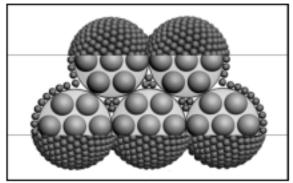
Model 2 - theoretical relationships between the specific surface and sphere diameter





## Model 3 – no adsorbate layer particle bridging







D - particle (crystallite/agglomerate) diameter [nm]

D. = 0.260 nm - helium kinetic diameter

D<sub>NZ</sub> = 0.455 nm - nitrogen molecule diameter

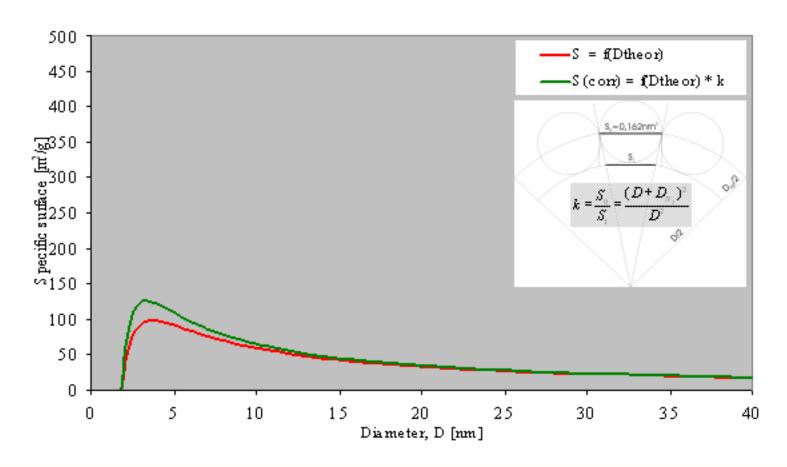
$$S[m^2/g] = \left(1 - \frac{6D_{N_2}}{D + 2D_{N_2}}\right) \cdot \frac{728.36}{D} \cdot \frac{(D + D_{H_2})^3}{D^3}$$

$$S_{corr}[m^2/g] = S[m^2/g] \cdot k = \left(1 - \frac{6D_{N_2}}{D + 2D_{N_2}}\right) \cdot \frac{728.36 \cdot (D + D_{N_2})^2}{D^3} \cdot \frac{(D + D_{H_2})^3}{D^3}$$



## Model 3 – no adsorbate layer particle bridging

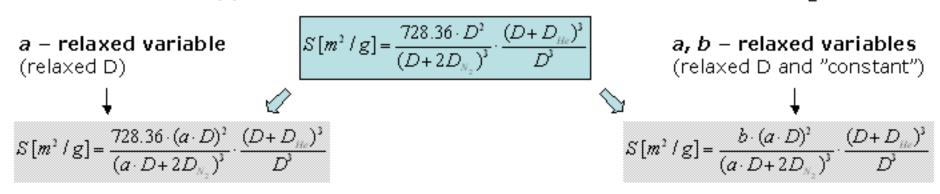
Model 3 - theoretical relationships between the specific surface and sphere diameter



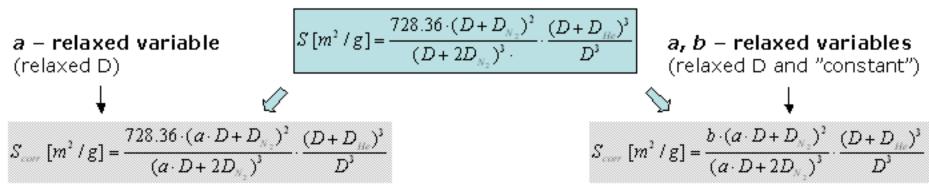


### Model 1 - correlation equations

(i) No correction for the cross-sectional area of N<sub>2</sub>



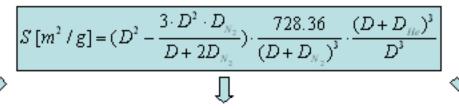
(ii) Corrected for the cross-sectional area of N<sub>2</sub>

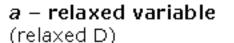




#### Model 2 - correlation equations

(i) No correction for the cross-sectional area of N<sub>2</sub>





a, b - relaxed variablesa, b, c - relaxed variables(relaxed D and one "constant")(relaxed D and two "constants")

$$S[m^{2}/g] = ((a \cdot D)^{2} - \frac{3 \cdot D^{2} \cdot D_{m}}{a \cdot D + 2D_{m}}) \cdot \frac{728.36}{(a \cdot D + D_{m})^{3}} \cdot \frac{(D + D_{m})^{3}}{D^{3}}$$

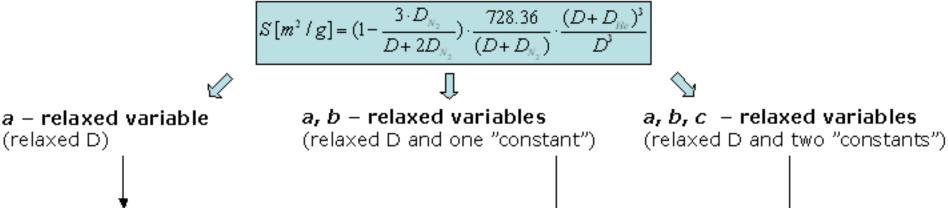
$$S[m^{2}/g] = ((a \cdot D)^{2} - \frac{3 \cdot D^{2} \cdot D}{a \cdot D + 2D}) \cdot \frac{b}{(a \cdot D + D)^{3}} \cdot \frac{(D + D_{in})^{3}}{D^{3}}$$

$$S[m^{2}/g] = ((a \cdot D)^{2} - \frac{c \cdot D^{2} \cdot D_{y_{0}}}{a \cdot D + 2D_{y_{0}}}) \cdot \frac{b}{(a \cdot D + D_{y_{0}})^{3}} \cdot \frac{(D + D_{y_{0}})^{3}}{D^{3}}$$



#### Model 2 - correlation equations, cntd.

#### (i) Corrected for the cross-sectional area of N<sub>2</sub>



$$S[m^2/g] = (1 - \frac{3 \cdot D_{w}}{a \cdot D + 2D_{w}}) \cdot \frac{728.36}{a \cdot D + D_{w}} \cdot \frac{(D + D_{w})^3}{D^3}$$

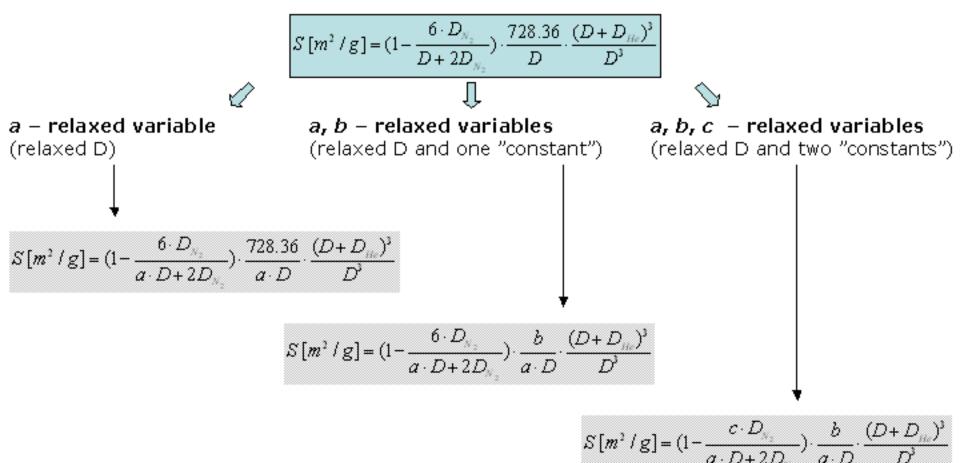
$$S[m^{2}/g] = (1 - \frac{3 \cdot D_{n}}{a \cdot D + 2D_{n}}) \cdot \frac{b}{a \cdot D + D_{n}} \cdot \frac{(D + D_{n})^{3}}{D^{3}}$$

$$S[m^{2}/g] = (1 - \frac{c \cdot D_{m}}{a \cdot D + 2D_{m}}) \cdot \frac{b}{a \cdot D + D_{m}} \cdot \frac{(D + D_{m})^{3}}{D^{3}}$$



#### Model 3 - correlation equations

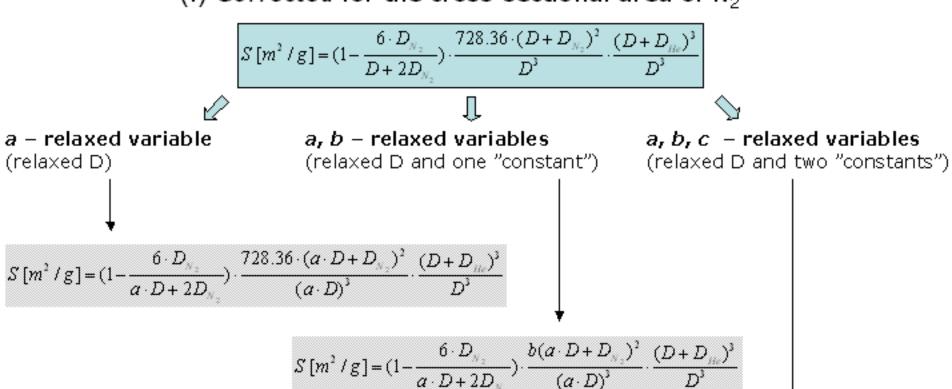
(i) No correction for the cross-sectional area of N<sub>2</sub>





### Model 3 - correlation equations, cntd.

(i) Corrected for the cross-sectional area of N<sub>2</sub>



$$S[m^{2}/g] = (1 - \frac{c \cdot D_{w_{0}}}{a \cdot D + 2D_{w_{0}}}) \cdot \frac{b(a \cdot D + D_{w_{0}})^{2}}{(a \cdot D)^{3}} \cdot \frac{(D + D_{w_{0}})^{3}}{D^{3}}$$

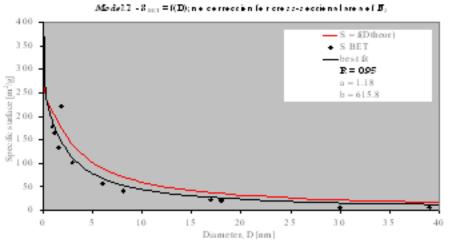


#### Correlation coefficients

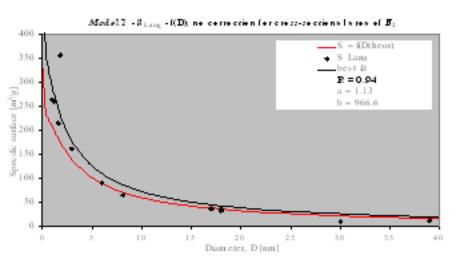
CORRELATION COEFFICIENT R a,b,c - adjusted constant(s) in the fitted model equations													
Model 1 (double adsorbate layer particle bridging)													
Correction for cross-sectional	BET			LA	ANGMU	IR	ВЈН						
area of N2	a	a,b	a,b,c	a	a,b	a,b,c	a	a,b	a,b,c				
Yes	0.84	0.94	nla	0.87	0.92	n/a	0	0.64	n/a				
No	0.91	0.95	nla	0.87	0.94	n/a	0.64	0.76	nla				
Model 2 (single adsorbate laver varticle bridging)													
Correction for cross-sectional	BET			LANGMUIR			ВЈН						
area of No	â	a,b	a,b,c	â	a,b	a,b,c	a	a,b	a,b,c				
Yes	0.65	0.95	0.93	0.95	0.95	0.92	0.55	0.86	0.90				
No	0.95	0.95	0.95	0.80	0.95	0.95	0.82	0.82	0.80				
Model 3 (no adsorbate layer particle bridging)													
Correction for cross-sectional	BET			LANGMUIR			ВЈН						
area of N2	â	a,b	a,b,c	à	a,b	a,b,c	a	a,b	a,b,c				
Yes	0.94	0.95	0.95	0.73	0.95	0.95	0.88	0.89	0.90				
No	0.88	0.95	0.95	0.56	0.95	0.95	0.81	0.90	0.90				

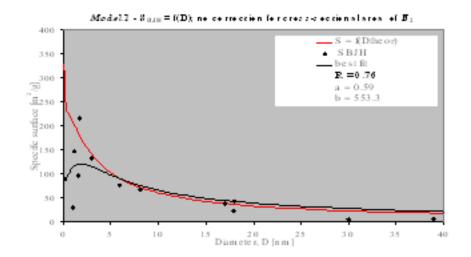


## Model 1 – selected best fits for BET, Langmuir, and BJH data



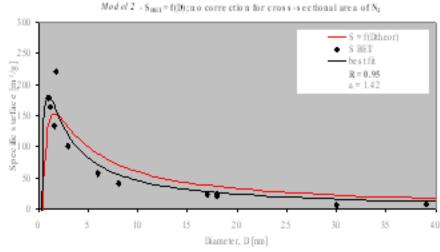
- very good overall fits for the BET and Langmuir data, a rather bad fit for the BJH data
- (ii) distinct characteristics of the  $S_{\rm BJH} = f(D)$ relationship for D < ca. 5 nm - not compatible with Model 1
- (iii) uncertainty in the trend for D < ca. 2-3 nm what are nano-GaN low size limits?

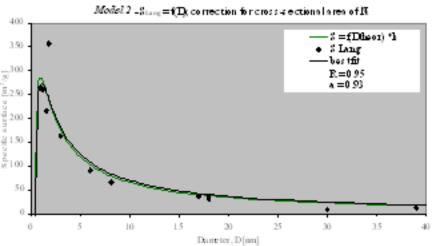




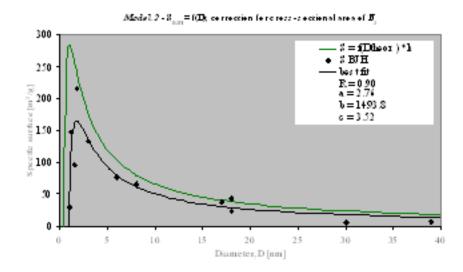


## Model 2 – selected best fits for BET, Langmuir, and BJH data





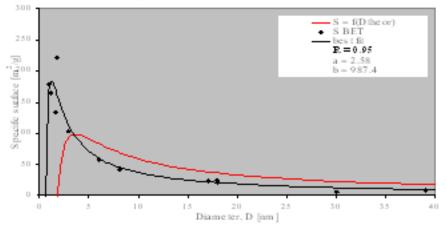
- very good overall fits for the BET and Langmuir data, and a good fit for the BJH data
- (ii) similar relationship characteristics suggested for all data (local maxima in the smallest D range)
- (iii) correction for the cross-sectional area of N<sub>2</sub> beneficial for the BJH data fit
- (iv) small spread between the model curves (red or green) and the best fit curves (black)





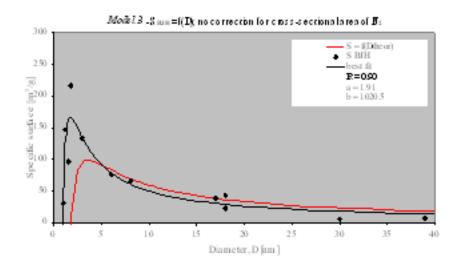
## Model 3 – selected best fits for BET, Langmuir, and BJH data

Model 3 - 3  $_{\rm BBT}$  = f(D); no correction for cross-sectional area of  $B_1$ 



- very good overall fits for the BET and Langmuir data, and a good fit for the BJH data
- (ii) similar relationship characteristics suggested for all data (local maxima in the smallest D range)
- (iii) relatively large spread between the model curves (red) and the best fit curves (black)

Model3 - 8 tm= f (D); no correction for cross-sections in word B: 400 S = f(Dthcor)3.50 S Lang est fit R = 0.95a = 2.50b = 1573.g 1.90 81 00 2 50 25 -5 1.5 20 30 35 Diameter, D [nm]





#### Conclusions

- A pool of twelve GaN nanopowders with crystallite sizes spanning the 1 to 40 nm range (from powder XRD scans) was shown each to consist of tight agglomerates of regularly shaped crystallites forming larger loose aggregates as supported by characteristic size dependent helium densities and SEM/TEM examinations.
- 2. A general model of porosity of such nanopowders was proposed wherein the pare system and pare surface area could be represented by the interparticle spaces and surface area, respectively, of closely packed equiradial spherical agglomerates of particles:
  - (i) three cases of the model were investigated, namely, Model 1 with double adsorbate layer particle bridging, Model 2 – with single adorbate layer particle bridging, and Model 3 – with no adsorbate layer particle bridging; all model equations for S = f(D) were derived with the option to include corrections for the sphere curvature adjusted cross-sectional area of the adsorbate molecule,
  - (ii) within the limits of the smallest particle sizes going from ca. 5 to 1 nm, Model 1 predicted steeply increased surface areas with values reaching a few hundred m²/g while Model 2 and Model 3 suggested local maxima of lower magnitude for the diameters of ca. 1-2 nm and 3-4 nm, respectively.



### Conclusions, cntd.

- 3. The model-based relationships of the surface area as function of sphere diameter, now with some relaxed variables, were statistically fitted for the cases of the BET, Langmuir, and BJH surface areas (from low temperature nitrogen adsorption experiments) being a function of the average crystallite size (from powder XRD scans):
  - the best fits for the BET and Langmuir surface areas vs. D<sub>XRD</sub> yielded for both the correlation coefficients R equal to 0.95 while for the BJH data R reached 0.90, supporting the overall very good relevance of the applied models,
  - (ii) it appeared that the circumstances somewhat between Model 1 and Model 2 are the closest to best describe the experimental data as far as the magnitude of the particular surface area is concerned,
  - (iii) it is our opinion that the observed discrepancies between the model and the experimental data in each case are, at least, partly due to the unknown degree of agglomeration of the GaN crystallites and its likely dependence on D<sub>XRD</sub>; a mismatch between the D<sub>aggl</sub> and D<sub>XRD</sub> causes the displacement of the experimental and model curves.

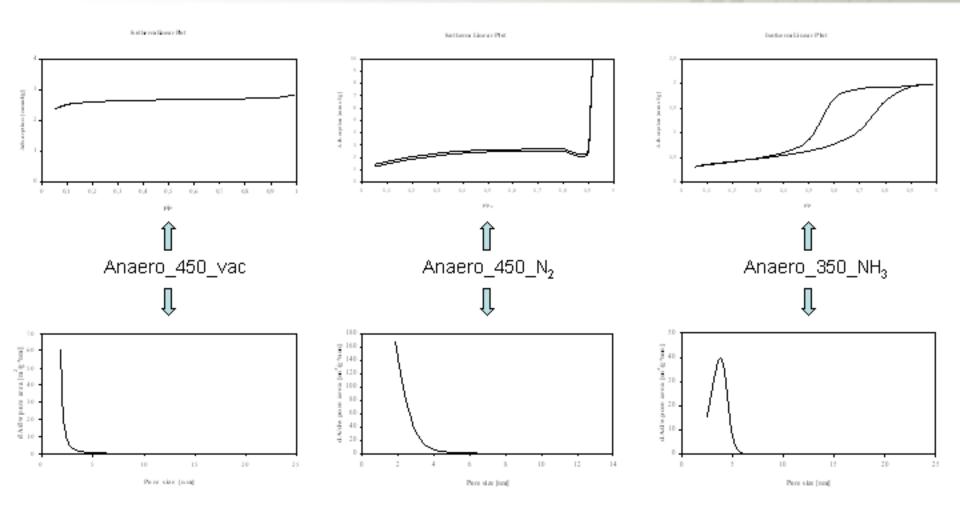
Acknowledgement. Research supported by the Polish Ministry of Science and Higher Education/MNISW, Grant No. N N 507 443534.



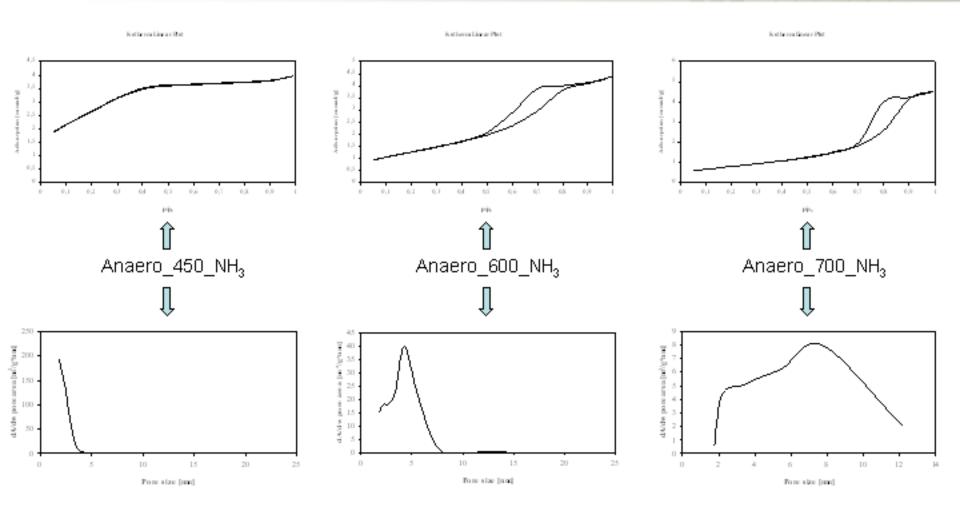
AGH University of Science and Technology Krakow, Poland



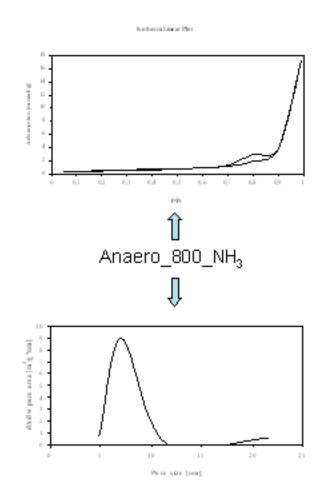




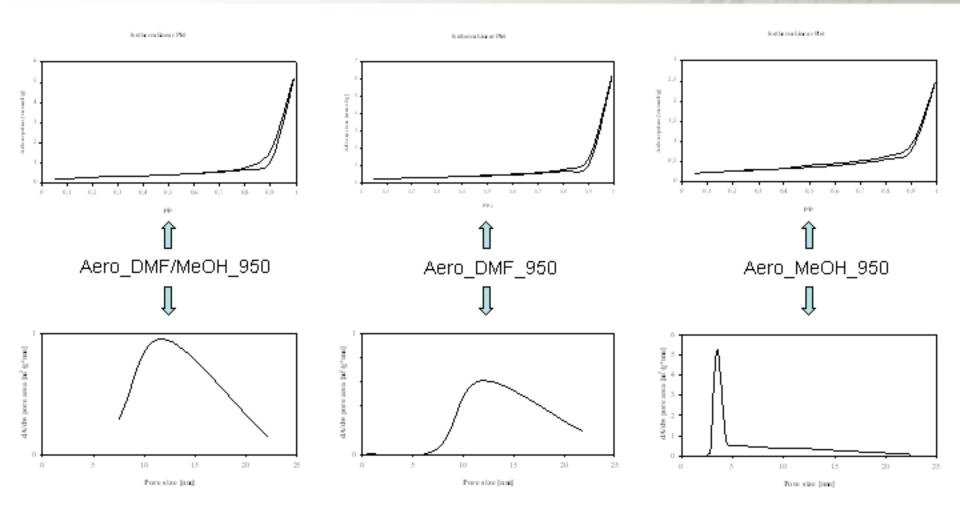




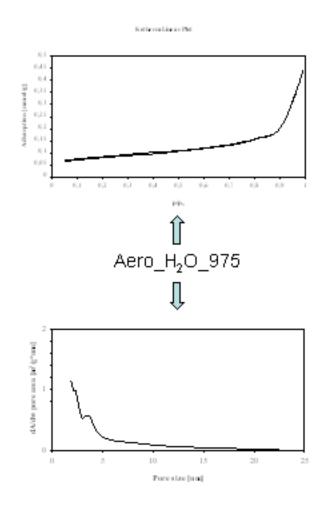


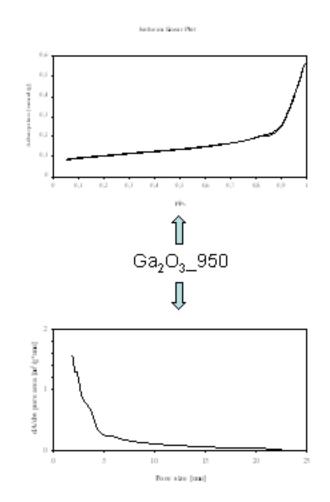














## Appendix Langmuir theory - 1

#### Assumptions of Langmuir theory



http://upload.wikimedia.org/wikipedia/commons/0/0b/Langmuir\_izoterma.png

- Fixed number of adsorption sites are available on the surface.
- All the vacant sites are of equal size and shape on the surface.
- Each site can hold maximum of one gaseous molecule and a constant amount of heat energy is released during this process.
- Dynamic equilibrium exists between adsorbed gaseous molecules and the free gaseous molecules.

$$A(g) + B(s) \stackrel{\text{adsorption}}{\longleftarrow} AB$$

Adsorption is monolayer.



## Appendix Langmuir theory - 2

Langmuir adsorption equation in terms of pressure P:

$$\frac{P}{V} = \frac{P}{V_{mono}} + \frac{1}{KV_{mono}}$$

If we plot a graph between P/V vs. P, we will obtain a straight line with

slope = 
$$\frac{1}{V_{mono}}$$
 and intercept =  $\frac{1}{KV_{mono}}$ 

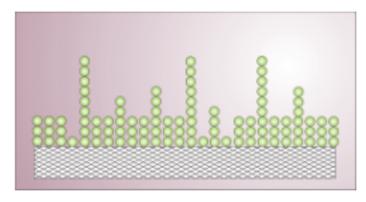
 $V_{\text{mono}}$  - adsorbed volume of gas covering the surface with a monolayer

K – constant; 
$$K = \frac{K_a}{K_d}$$
;  $K_a$  – rate of adsorption,  $K_d$  – rate of desorption



## Appendix Brunauer, Emmett, Teller (BET) theory -1

#### Assumptions of BET theory



http://upload.wikimedia.org/wikipedia/commons/8/83/BET\_Multilayer\_Adsorption.jpg

The concept of the theory is an extension of the Langmuir theory, which is a theory for monolayer molecular adsorption, to multilayer adsorption with the following hypotheses.

- Gas molecules physically adsorb on the surface with random distribution of sites covered by one, two, three, etc. molecules.
- There is no interaction between each adsorption layer.
- The Langmuir theory can be applied to each layer.



## Appendix BET theory - 2

Classical form of the BET equation:

$$a = a_m \cdot \frac{Ch}{(1-h)[1+(C-1)h]}$$

where:

a - the adsorption capacity equilibrium humidity h

h - the relative pressure ( $h = p/p_s$ )

p - the equilibrium partial pressure of the adsorbate vapor

 $p_{\epsilon}$  - the saturated vapor pressure of the adsorbate at absolute temperature T

a - monolayer capacity

C- energy constant:

$$C \approx \exp[-(E_1 - E_L)/RT]$$

E, - heat effect of adsorption in the first layer

E, - latent heat of condensation

R-the gas constant

- the coordinates of the points of monolayer  $h_{m}$  calculated according to the known formula:

$$h_{\infty} = 1/(\sqrt{C} + 1)$$

- the first layer heat of adsorption calculated from the equation:

$$E_1 - E_2 = RT \ln(C)$$

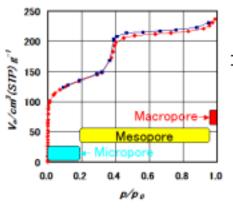
where: -  $E_t$  is the first layer heat of adsorption,  $E_L$  is the heat of condensation, R is the gas constant, and T is the absolute temperature,

- surface area of samples calculated from  $a_m$  values assuming that cross-sectional area of adsorbed nitrogen molecule is 0.162 nm<sup>2</sup>.



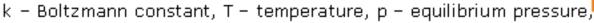
## Appendix Barrett, Joyner, Halenda (BJH) theory





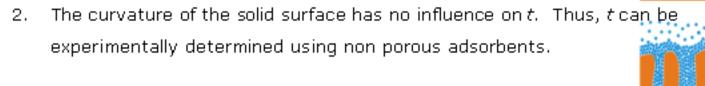
The relative pressure p/p<sub>o</sub> at which decondensation (from capillary phase to multilayer adsorbed phase) occurs in a cylindrical pore is given by the Kelvin law.

$$kTln(p/p_0) = -2 \sigma v_1/(R-t)$$



 $p_o$  - saturation vapor pressure,  $\sigma$  - surface tension,  $v_1$  - liquid molecular volume,

R - pore radius, t - multilayer thickness



$$kTln(p/p_0) = U(t)$$

