Slope stability analysis with numerical and limit equilibrium methods

Marek Cała and Jerzy Flisiak
Department of Geomechanics, Civil Engineering & Geotechnics, University of Mining & Metallurgy
Al. Mickiewicza 30, 30-059 Kraków
e-mail: cala@agh.edu.pl

Abstract

This paper deals with the slope stability engineering. The stability of slopes is traditionally estimated using 2D limit equilibrium methods (LEM). However these methods have several disadvantages and may neglect some important factors. Due to the rapid development of computing efficiency, several numerical methods are gaining increasing popularity in slope stability engineering. The most popular numerical method of slope stability estimation is shear strength reduction technique (SSR). After performing several calculations it turned out that for a simple slopes factor of safety (FS) obtained from numerical methods is usually the same as FS obtained from LEM. However for complex geometry slopes considerable differences between FS values may be expected. Application of SSR for complex geology slopes is usually restricted to the weakest "link" estimation – part of slope with the lowest FS. Finite Difference Method code *FLAC* gives the opportunity to analyze several slip surfaces by using modified SSR technique. The method is based on reducing shear properties of soils after identification of first slip surface. The modified shear strength reduction technique (MSSR) allows performing complete estimation of stability for any type of slope.

Keywords: slope stability, shear strength reduction

1. Introduction

The stability of slopes is traditionally estimated using 2D limit equilibrium methods (LEM). However these methods have several disadvantages and may neglect some important factors. Due to the rapid development of computing efficiency, several numerical methods are gaining increasing popularity in slope stability engineering. Finite Element Method (FEM) and Finite Difference Method (FDM) are very often used for that purpose. The factor of safety (FS) for slope may be computed by reducing shear strength of rock or soil in stages until the slope fails. This method is called shear strength reduction technique (SSR).

FLAC code [1] is often applied for estimating FS for rock slopes [2, 3] or even foliated rock slopes [4]. It may be also applied in evaluating stability of soft rock slope perforated by underground openings [5]. *FLAC* is also widely used for analyzing stability of soil slopes [6, 7, 8, 9, 10]. Sometimes *FLAC* is even used for slope stability engineering in combination with other methods. Reference [11] shows examples of the application of *FLAC* combined with LEM. A lot of examples of applications of numerical methods to slope stability analysis may be found in several conference proceedings [12, 13].

2. Stability of simple geometry slopes

Many authors have pointed out several advantages of SSR over the limit equilibrium methods. But usually they checked the effectiveness of SSR on rather small models of simple geometry. In this study, the accuracy of the SSR was investigated through comparisons with limit analysis solutions. FS estimated by SSR was compared with FS obtained from Fellenius, Bishop, Morgenstern-Price and Janbu.

To check the influence of elastic properties (Young's modulus = E, Poison's ratio = ν) more than 100 numerical simulations for the homogeneous and isotropic slopes were performed. The values of E were changed from 25 MPa to 1000 MPa and ν from 0.1 to 0.4. All the slopes in this paper were simulated with *FLAC* in plane strain conditions using small-strain mode.

It was found that although the elastic properties have a significant influence on the computed deformations prior to failure, they negligibly influenced FS. The difference in FS was below 1 %. This confirms conclusions from reference [14] which even recommends application of nominal values of E = 100 MPa and ν = 0.3 for slope stability analysis with SSR. After preliminary calculations it seems that this statement is also valid for heterogeneous slopes.

To compare SSR and LEM more than 200 numerical simulations for the isotropic and homogeneous slopes were carried out. Embankments were simulated with slope angles (α) ranging from 18.43° (1:3) to 63.43° (2:1). The height of the embankment was changed from 15 m to 35 m. The soil was given values of angle of internal friction (ϕ) ranging from 10° to 30° and cohesion (c) from 25 kPa to 75 kPa. Figure 1 shows FS calculated with SSR are within few percent of the FS obtained from LEM.

The next task was to compare FS from SSR and LEM for a slope consisting of two different geological units. The soil below the embankment (foundation layer) was given friction angle $\phi = 10°$ and cohesion $c = 0$. The stability of the embankment of height 25 m, friction angle $\phi = 20°$ and cohesion c = 30 kPa for several sloping angles was analyzed.

Figure 2 shows FS calculated with SSR are within a few percent of the FS obtained from LEM for sloping angles from 18.43° to about 41°. For sloping angles from 41° to 64.43° FS calculated with SSR are even 20% lower than FS from LEM.

This may be explained by the fact that slip surfaces obtained from SSR are localized deeper than slip surfaces from LEM. That may suggest that the complex geology of the slope has a significant influence on the value of FS predicted by LEM and SSR.

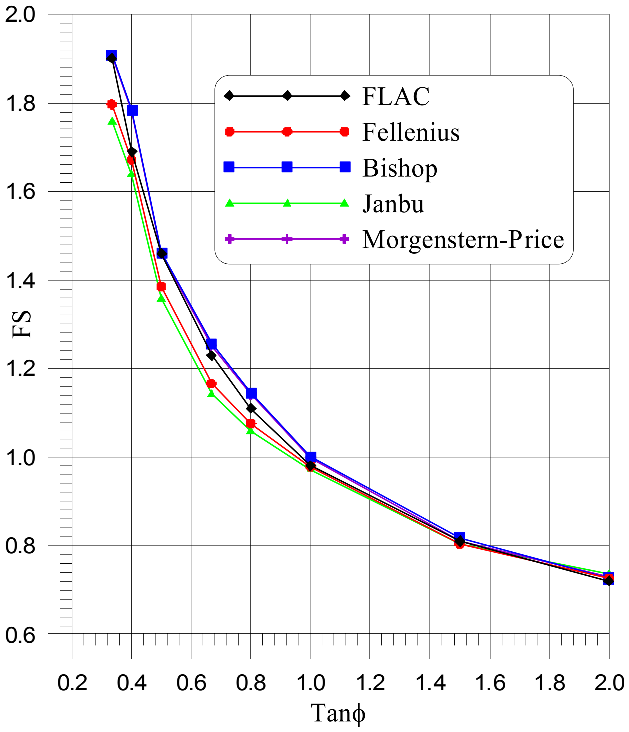

Figure 1. FS for embankment of height 25 m, friction angle $\phi = 20^{o}$, cohesion c = 30 kPa for several sloping angles with SSR and LEM.

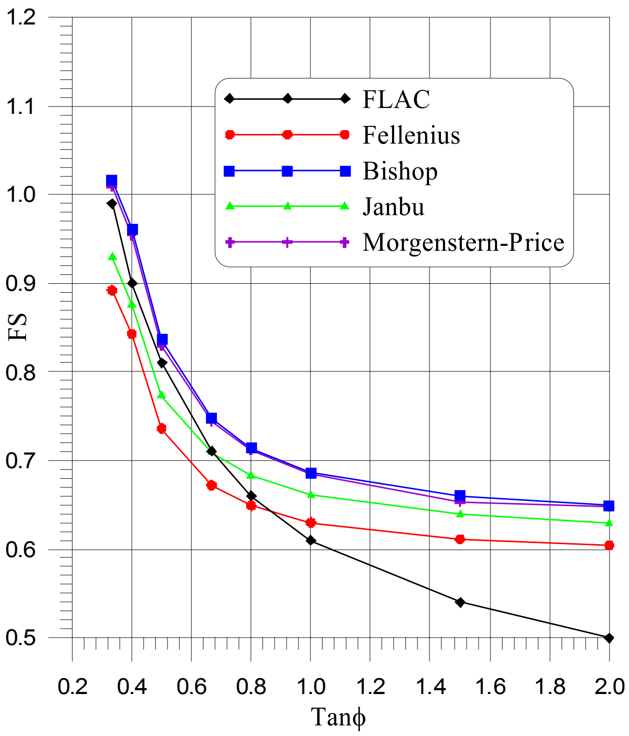

Figure 2. FS for embankment (height 25 m) consisting of two geological units for several sloping angles with SSR and LEM.

3. Slope with weak stratum

For to investigate the influence of weak stratum on FS some 350 models were analyzed. The thickness of weak stratum was changed from 1.0 to 10.0 m. The weak layer was localized from 0 to 50 m from the top of the slope (fig.3).

It was assumed that embankment is 25 m high and has a slope angle equal 45°. It consists of two different geological units. The soil was given friction angle $\phi = 30°$ and cohesion c = 75 kPa. The weak, thin layer had friction angle equal $\phi = 10°$ and cohesion c = 25 kPa. Both soils had unit weight $\gamma = 20$ kN/m^3. The thickness "g" of horizontal weak layer was changed from 1.0 m to 10.0 m and its distance "h" from top of the slope changed from 0 to 50 m.

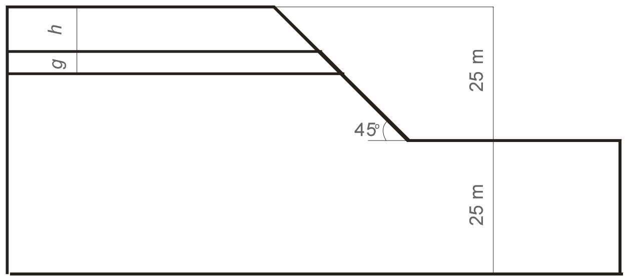

Figure 3. Slope with weak stratum.

Figure 4 shows the FS values for a 1.0 m thick weak layer and fig. 5 for a 5.0 thick one. The decrease of FS is quite small if the thin weak layer is located close to top of the slope. Increasing the weak layer thickness produces considerable decrease of FS. The differences in FS values are significant especially in case of small thickness (1 m – 3 m) of weak stratum.

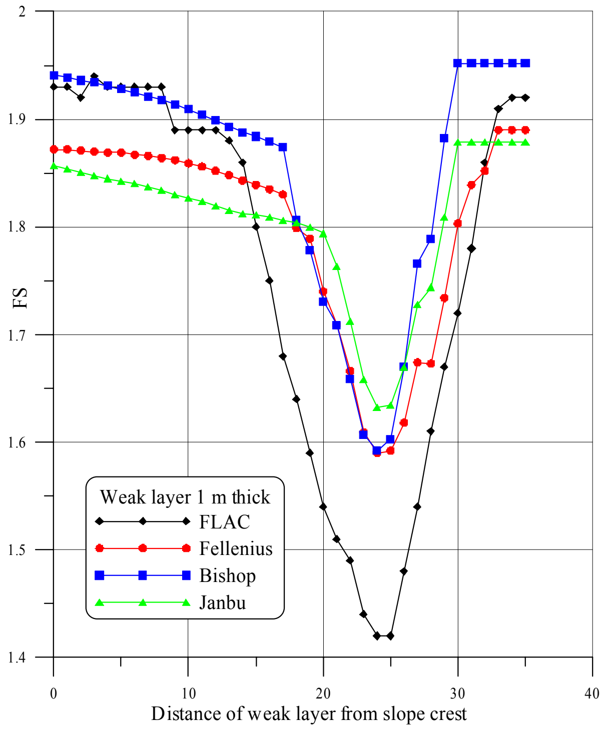

Figure 4. FS values for a 1.0 m thick weak layer.

Increase of weak layer thickness (irrespectively of its localization) reduces differences between FS values from LEM and SSR. Especially FS values estimated with Bishop's method are within 8 % of the FS obtained from SSR.

For the thickness of the weak layer less or equal 5 m SSR produces lower FS values than any of the LEM method.

For the weak layer 5 m thick Bishop's method produces FS = 1.114 and SSR shows FS = 1.07. Further increase of weak layer thickness (7.5 m and 10 m) produces lowest FS values from Bishop's method (FS = 0.926 and FS = 0.811 respectively). SSR technique shows respectively FS = 0.95 and FS = 0.87 in this case.

It seems that application of Bishop's method produces the most reliable results among LEM. These results are simultaneously closest to the FS values obtained from SSR. Application of Fellenius's method produces not reliable FS values in case of weak layer localization below slope toe. It shows the influence of weak layer on FS values even if the roof of the stratum lays 15 m below the slope toe.

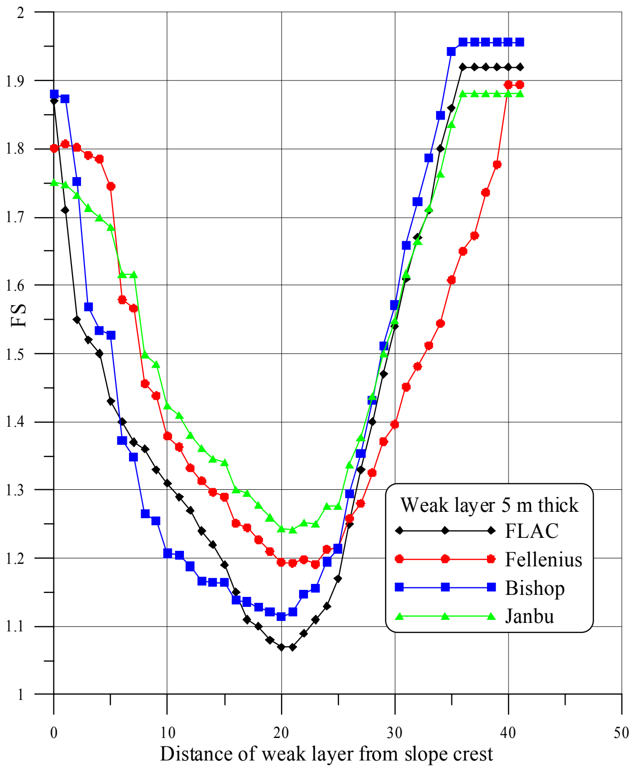

Figure 5. FS values for a 5.0 m thick weak layer.

It must be also pointed that failure surfaces identified by SSR technique are sometimes considerably different than surfaces identified by LEM (fig.6).

Figure 6 shows the situation when FS resulted from SSR is considerably lower and unit volume of failed slope is significantly higher than estimated from LEM.

It seems that SSR technique is much more sensitive than LEM in case of complex geometry cases.

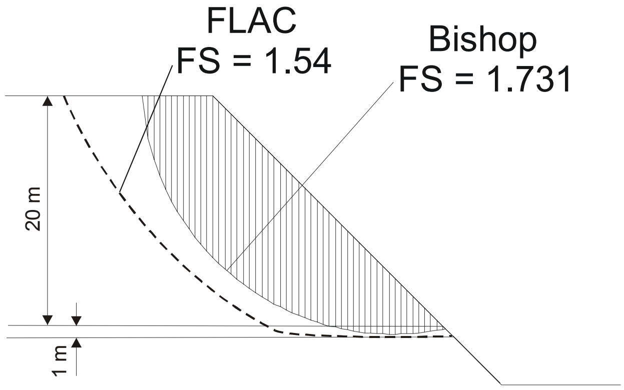

Figure 6. Critical slip surfaces identified by SSR and LEM.

4. Modified shear reduction technique

Application of SSR for complex geology slopes is usually restricted to the weakest "link" estimation – part of the slope with the lowest FS. However the Finite Difference Method code *FLAC* gives the opportunity to analyze several slip surfaces using modified shear strength reduction technique – MSSR [15].

This method is based on reducing shear properties of soils *after* identification of first slip surface (FS_1). It is simply the continuation of classic SSR, but after first instability occurrence. It is possible only using Finite Difference Method. Program *FLAC* uses the explicit, Lagrangian calculation scheme. The full dynamic equations of motion are used, even when modelling systems that are essentially static. This enables *FLAC* to follow physically unstable processes without numerical distress. In fact, *FLAC* is most effective when applied to nonlinear or large-strain problems, or to situations in which physical instability may occur. This may lead to identification of several other slip surfaces. Let's consider benched slope stability (fig.7).

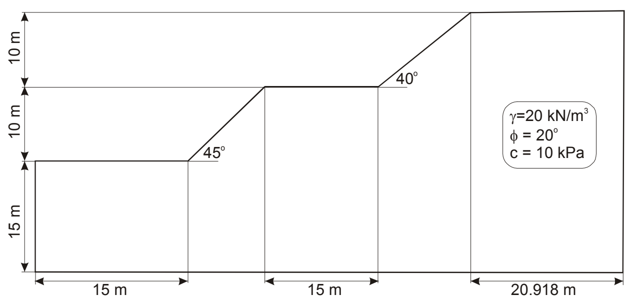

Figure 7. Benched slope problem geometry.

Figure 8 shows the slip surfaces identified in benched slope by MSSR and LEM. Failure of the lower part of the slope was detected first. FS_1=0.90 calculated by SSR is very close to FS = 0.921 given by Bishop's method. And precisely here ends the range of classical SSR technique – especially with application of any Finite Element Method code. However FLAC is created especially for modeling physical instability.

This allows to continue shear strength reduction and to identify another possible slip surfaces. In analysed case, next identified failure surface is located in the upper part of the slope. FS_2=1.00 calculated by MSSR is again very close to FS = 1.008 given by Bishop's method. And finally application of MSSR allowed to evaluate FS for entire slope – FS_3=1.24 is also very close to FS = 1.228 given by Bishop's method.

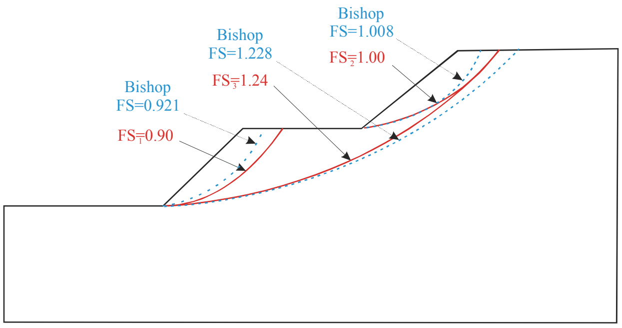

Fig.8. Several slip surfaces identified in benched slope by MSSR and LEM.

It seems that FS calculated with MSSR are within a few percent of the FS obtained from LEM for simple cases. It must be however underlined that effectiveness of MSSR must be verified on "real" cases.

5. Summary

For a simple, homogeneous slope FS calculated with SSR are usually the same as FS obtained from LEM. In the case of a simple geometry slope consisting of two geological units, FS calculated with SSR may be considerably different than FS from LEM.

In the case of complex geometry and geology slopes SSR technique is much more "sensitive" than LEM. Application of SSR with *FLAC* may be recommended for the large-scale slopes of complex geometry.

Another step forward is the modified shear strength reduction technique – MSSR. Such a powerful tool as MSSR with *FLAC* gives the opportunity for the complete stability analysis for any slope.

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