Preserving Diversity in Evolution Strategy for Shape Design of Rotating Elastic Disc

Roman Dębski, Rafał Dreżewski, Marek Kisiel-Dorohinicki
Department of Computer Science
AGH University of Science and Technology, Kraków, Poland

drezew@agh.edu.pl

Abstract

In the paper the optimization of rotating variable-thickness annular elastic discs based on a simulation model by means of (μ, λ)-evolution strategies is discussed. Additional mutation operator is introduced as a means to preserve diversity in the population. The considerations are illustrated by a review of obtained experimental results.

Keywords: evolution strategies, preserving diversity, optimal shape design

1 Introduction

For very complex engineering problems even approximate mathematical models often prove so difficult that their analysis becomes a non-trivial task for most traditional methods. In such cases simulation experiments may be useful to verify the correctness of the proposed solutions, which may be further improved (optimized) by the human designer. However the globally optimal solution may be hardly found this way. It seems that this task may be successfully handled by some (meta)heuristic computational techniques such as *evolutionary algorithms*. This term covers a wide range of search and optimization methods, based on analogies to phenomena of natural evolution. Particularly evolution strategies are distinguished by real-valued representation (thus they are most often used for continuous optimization problems), Gaussian mutation as main variation operator, and deterministic selection scheme [4, 7].

The paper discusses the optimization of rotating variable-thickness annular elastic discs based on the proposed simulation model by means of evolution strategies. The goal of the design is to find such a shape of the disc that would ensure maximal elastic carrying capacity. While in many papers the shape is modeled with the use of some fixed class of functions (e.g. hyperbolic, n-th order polynomial), in the model presented the profile of the disc is defined by spline curves. To preserve diversity in the population additional mutation operator is introduced. This additional mutation range and rate decrease during the course of the algorithm, allowing for more intensive exploration of the search space at the very beginning and then for more and more accurate approximation of the solution.

The paper is organized as follows. Classical evolution strategies and additional mutation operator are described in section 2. Section 3 presents the optimization problem: the design of rotating variable-thickness annular elastic disc, proposed representation of the solutions, and the model used to evaluate their quality (fitness). Selected experimental results with the proposed (μ, λ)-evolution strategy conclude the work.

2 Evolution strategies

Evolution strategies (ES) were developed by Rechenberg and Schwefel in the 1960s at the Technical University of Berlin. The first applications were aimed at hydrodynamical problems like shape optimization of a bended pipe and drag minimization of a joint plate [2]. ES is a special instance of an evolutionary algorithm characterized by real-valued vector representation, Gaussian mutation as main variation operator, self-adaptation of mutation rate, and deterministic selection mechanisms.

2.1 Classical approach

Algorithmic framework of contemporary evolution strategies may be described with the use of following notation introduced by Schwefel [6]:

- $(\mu + \lambda)$-ES generates λ offspring from μ parents and selects the μ best individuals from $\mu + \lambda$ (parents and offspring) individuals $(1 \leq \mu \leq \lambda)$,
- (μ, λ)-ES denotes an ES that each time step generates λ offspring from μ parents and selects the μ best individuals only from λ (offspring) individuals $(1 \leq \mu < \lambda)$.

The individuals in a population consist of the objective variables vector $\boldsymbol{x}$ and a vector of *strategy parameters* $\boldsymbol{\sigma}$, where σ_i denotes the standard deviation used when applying a zero-mean Gaussian mutation to the i-th component in parent vector. These parameters are incorporated into the representation of individual in order to obtain evolutionary *self-adaptation* of an ES [7, 1]. The mutation operator changes strategy parameters according to:

$$\sigma_i' = \sigma_i \exp(\tau_0 N(0,1) + \tau N_i(0,1)) \tag{1}$$

and the objective variables (a simplified case of uncorrelated mutations):

$$x_i' = x_i + N(0, \sigma_i') \tag{2}$$

where the constant $\tau \propto \frac{1}{\sqrt{2\sqrt{n}}}$, $\tau_0 \propto \frac{1}{\sqrt{2n}}$, $N(0,1)$ is a standard Gaussian random variable sampled once for all n dimensions and $N_i(0,1)$ is a standard Gaussian random variable sampled for each of the n dimensions.

If the number of parents $\mu > 1$, the objective variables and internal strategy parameters can be recombined with usual recombination operators, for example *intermediate recombination* [3], which acts on two parents $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ and creates an offspring $\boldsymbol{x'}$ as the weighted average:

$$x_i' = \alpha x_{1i} + (1-\alpha) x_{2i} \tag{3}$$

where $\alpha \in \langle 0, 1 \rangle$ and $i = 1, \ldots, n$. The same may be applied to standard deviations:

$$\sigma_i' = \alpha \sigma_{1i} + (1-\alpha) \sigma_{2i} \tag{4}$$

It is not necessary to apply the same recombination operator for objective variables and standard deviations. For example one can use *discrete recombination* for standard deviations and *intermediate recombination* for objective variables.

2.2 Preserving diversity

An evolutionary algorithm works properly (in terms of searching for a global solution) if the population consists of individuals different enough, i.e. the so-called diversity in the population is preserved. Yet many algorithms tend to prematurely loose this useful diversity, and as a result, there is possibility that population gets stuck in some local extremum instead of searching for a global one. To avoid premature convergence in classical ES the mechanism of self-adaptation, as described above, was proposed. Yet this mechanism proves often not sufficient for very complex multi-modal problems.

In order to prevent the premature convergence of ES, an additional operator may be introduced. The operator is applied every time when the average of standard deviation, calculated as a Gaussian norm:

$$\sigma_{avg}(t) = \frac{1}{\mu}\sum_{i=1}^{\mu}\sqrt{\sum_{j=1}^{n}\left[\sigma_{ij}\left(t\right)\right]^2} \tag{5}$$

becomes less than an a priori set minimum value: $\sigma_{avg} < k_\sigma$. This additional variation was applied to a limited number of individuals depending on the generation t:

$$L(t) = \mu\left(1 - \frac{t}{T}\right)^2 \tag{6}$$

The additional mutation range in generation t is defined via its standard deviation:

$$\sigma_{ij}(t) = k_\sigma\left(1 - \frac{t}{T}\right)^2 \quad i = 1, \ldots, \mu, \quad j = 1, \ldots, N \tag{7}$$

where: T – total (maximum) number of generations,

σ_{ij} – standard deviations associated with the j-th gene of the i-th individual.

3 The optimization problem

The most important assumptions about the physical model of the discussed design problem are as follows [5]:

1. We consider an annular elastic disc of variable thickness $h = h(r)$ rotating with constant angular velocity ω and subject to uniform traction p_b at the outer radius b. The disc is clamped at the inner radius a.
2. The classical theory of thin discs with small gradient dh/dr is assumed and hence the stresses τ_{rz} and σ_z are neglected.
3. The material is linear-elastic with Young's modulus E, Poisson's ratio ν and subject to the Huber-Mises-Hencky (H-M-H) yield condition.
4. The small-strain theory is adopted.

3.1 Basic equations

The condition of internal equilibrium in polar coordinates may be expressed as follows:

$$\frac{1}{h}\frac{d}{dr}\left(h\sigma_r\right) + \frac{\sigma_r - \sigma_\theta}{r} + \rho_m\omega^2 r = 0 \tag{8}$$

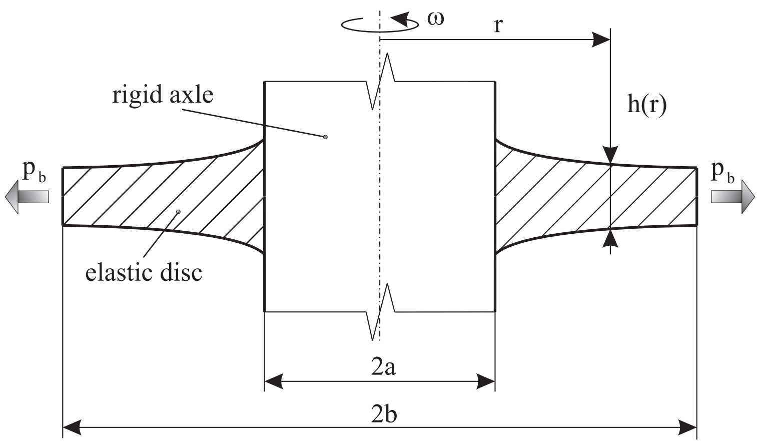


Figure 1: Annular disc under consideration

where ρ_m stands for mass density and h for the disc thickness. Constitutive equations (described by the Hooke's law) take the following form:

$$\sigma_r = \frac{E}{1-\nu^2}\left(\frac{du}{dr} + \nu\frac{u}{r}\right), \quad \sigma_\theta = \frac{E}{1-\nu^2}\left(\frac{u}{r} + \nu\frac{du}{dr}\right) \tag{9}$$

and combine radial displacement u with radial σ_r and circumferential σ_θ stresses.

Taking advantage of constitutive equations (9) after simple calculations the condition of internal equilibrium has the form:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\left(1 + \frac{r}{h}\frac{dh}{dr}\right)\frac{du}{dr} - \frac{1}{r^2}\left(1 - \nu\frac{r}{h}\frac{dh}{dr}\right)u = -\rho_m\omega^2\frac{1-\nu^2}{E}r \tag{10}$$

Boundary conditions:

$$u(a) = 0, \quad \frac{E}{1-\nu^2}\left[\frac{du(b)}{dr} + \nu\frac{u(b)}{r}\right] = p_b \tag{11}$$

allow to find a numerical solution depending on external loadings (angular velocity ω and uniform traction p_b). The stress intensity calculated according to Huber-Mises-Hencky hypothesis:

$$\sigma_i{}^2 = \sigma_r{}^2 + \sigma_\theta{}^2 - \sigma_r\sigma_\theta \tag{12}$$

takes its maximal value at the boundary of the disc (usually at the inner radius) or at a certain point inside the disc. It obviously depends on the shape of the disc. When the maximum value of the stress intensity reaches the value of yield stress σ_0:

$$|\sigma_i(r_0)|_{max} = \sigma_0 \tag{13}$$

the elastic carrying capacity is exhausted. Therefore (13) allows to find the external loadings value (a combination of angular velocity ω and uniform traction p_b), which we call the elastic carrying capacity of the disc.

3.2 Shape representation

As it was mentioned above, the profile of the disc is represented by the 3rd order spline built on equidistant nodes (with δ being the distance between nodes). Coefficients of the spline may be found from the set of linear equations:

$$\begin{pmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 4 & 1 & \dots & 0 \\ 0 & 1 & 4 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & 1 & 4 & 1 \\ 0 & \dots & 0 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} M_0 \\ M_1 \\ M_2 \\ \vdots \\ M_{n-1} \\ M_n \end{pmatrix} = \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_{n-1} \\ d_n \end{pmatrix} \tag{14}$$

where:

$$d_j = \frac{6}{\delta^2}\left(y_{j-1} - 2y_j + y_{j+1}\right), \quad j = 1, \dots, n-1, \tag{15}$$

$$d_0 = \frac{6}{\delta^2}\left(y_1 - y_0\right), \quad d_n = \frac{6}{\delta^2}\left(y_n - y_{n-1}\right). \tag{16}$$

After some calculations one may find the value of interpolating function as well as its derivative at any point from the range $r \in \langle a, b\rangle$ (or, after introducing dimensionless radius $x = r/a, x \in \langle a, b/a\rangle$):

$$\begin{aligned} s(x) = & \frac{1}{6\delta}\left[M_{j-1}\left(x_j - x\right)^3 + M_j\left(x - x_{j-1}\right)^3\right] + \\ & \left[\frac{y_j - y_{j-1}}{\delta} - \frac{\delta}{6}\left(M_j - M_{j-1}\right)\right]\left(x - x_{j-1}\right) + \\ & y_{j-1} - M_{j-1}\frac{\delta^2}{6} \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{ds(x)}{dx} = & \frac{1}{2\delta}\left[-M_{j-1}\left(x_j - x\right)^2 + M_j\left(x - x_{j-1}\right)^2\right] + \\ & \frac{y_j - y_{j-1}}{\delta} - \frac{\delta}{6}\left(M_j - M_{j-1}\right) \end{aligned} \tag{18}$$

where $x \in \langle x_{j-1}, x_j\rangle$ and $j = 1, \dots, n$.

For numerical calculations we introduce the following dimensionless quantities:

$$\beta = \frac{b}{a}, \quad x = \frac{r}{a}, \quad y = \frac{h}{a}, \quad w = \frac{u}{w}. \tag{19}$$

The above group of parameters is connected only with the geometry of the disc while the next one describe the material of the disc and its external loadings:

$$S = \frac{\sigma_0}{E}, \quad \Omega = \frac{\sqrt{3}\rho_m\omega^2 a^2}{2\sigma_0}, \quad p = \frac{p_b}{\sigma_0}. \tag{20}$$

3.3 The model

After introducing basic equations (the physical model) one may formulate the optimization problem by defining a decision variables vector, a feasible region and an objective function. The decision variables vector

$$\boldsymbol{Y} = (y_1, y_2, \dots, y_n) \in M \subset \mathbf{R}^n \tag{21}$$

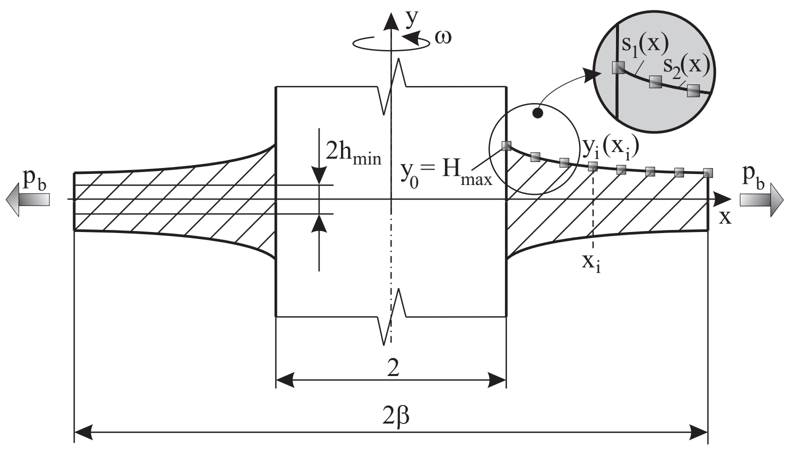


Figure 2: Profile representation and constrains of the shape

represents the shape of the disc in n equidistant points.

The feasible region:

$$M = \{\boldsymbol{Y} \in \mathbf{R}^n \mid k_d \cdot h_{min} \leq y_j \leq k_g \cdot H_{max} \quad \forall j = 1, \dots, n\} \tag{22}$$

assumes that the disc can be neither too thin (not thinner than $k_d \cdot h_{min}$) nor too thick (not thicker than $k_g \cdot H_{max}$). Additionally, the stress intensity must satisfy (13).

Objective function is described by the following formula:

$$\Phi = \left\{ c \left[\frac{1}{\beta - 1} \int_1^\beta \sigma_i(x)\, dx \right] + (1 - c)\sqrt{p^2 + \Omega^2} \right\} \to max \tag{23}$$

where $0 \leq c \leq 1$ makes it possible to set the importance of each of the two criteria taken into account. The first of them (with the multiplier c) is connected with the equalization of the stress intensity and the second one with the external loadings (it is worth noting that if $c = 0$ this criterion becomes a simple maximization of elastic carrying capacity). Such a generalization is very helpful in estimating the limit carrying capacity or decohesive carrying capacity.

4 Experimental results

The system used in simulations was composed of two subsystems: the first one controlled evolutionary algorithm while the second one realized simulation processes. Such architecture may be easy adapted for parallel computation (e.g. based on master-slave model).

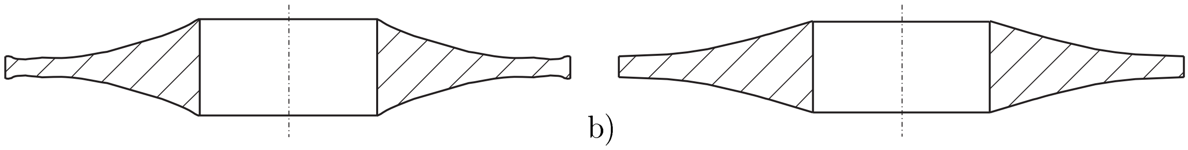
Below optimal shapes in the meaning of criterion (23) connected with different ratio Ω/p are presented. All results were obtained for the typical material of the disc for which:

$$\nu = 0.3, \quad \sigma_0/E = 0.001 \tag{24}$$

As regards geometrical parameters all calculations were performed for the following values:

$$\beta = b/a = 2, \quad h_{min} = 1, \quad H_{max} = 3, \quad k_d = 0.9, \quad k_g = 1.1 \tag{25}$$

Figure 3: Optimal disc for purely inertial loading



a) b)

Figure 4: Optimal disc for $\Omega/p = 2$ (a) and $\Omega/p = 1$ (b), $c = 0.4$

Figure 3 presents the shape of optimal disc when the external pressure is equal to zero. There are two interesting things in the shape: first it is the flat part near the center of the disc and the second one is the swelling near the outer radius. The flat segment results from the constraint connected with the minimal thickness, which becomes active near the center. The swelling results from the character of objective function — this swelling is the outcome of more equal distribution of the stress intensity. The elastic carrying capacity (ECC) is described by the pair: $\Omega = 1.21$; $p = 0$.

The optimal shape of the disc when the dimensionless angular velocity is twice greater than the dimensionless external pressure is shown in figure 4a. In this case the shape is more regular but a small swelling near the outer radius still remains. A more regular shape can be explained by the fact that a part of inertia forces in this case is replaced by the external pressure action. In this case the ECC is described by the following pair: $\Omega = 0.94$; $p = 0.47$. In figure 4b the optimal shape of the disc for dimensionless external pressure and rotation velocity being equal is shown. The elastic carrying capacity is described by the pair: $\Omega = 0.74$; $p = 0.74$. It is interesting that for such a ratio the shape is very smooth, which may be, in a way, optimal for the work of the disc.

4.1 The ES analysis

Figure 5 shows the minimum and maximum quality of the individuals and average standard deviation according to (5) in subsequent steps of the algorithm. One should mention the fast convergence of classic (μ, λ)-ES, which may be considered an advantage. Yet it may be dangerous for the evolution process which can stop too early (premature convergence). After about a hundredth step of the algorithm obtained solutions do not improve so much, and in fact the computation may be stopped. Values of the most important parameters used in calculations are presented in Fig. 5 and 6, where parameter σ_{min} stands for the minimum value of standard deviation (at a single parameter level).

4.2 Introduction of additional mutation operator

In order to prevent the premature convergence the additional mutation operator was introduced. Figure 6 shows the results of simulations with additional mutation operator. In the figure 6a the minimum and maximum value of the objective function in population is shown. When we compare these results to the ones presented in the figure 5a we can

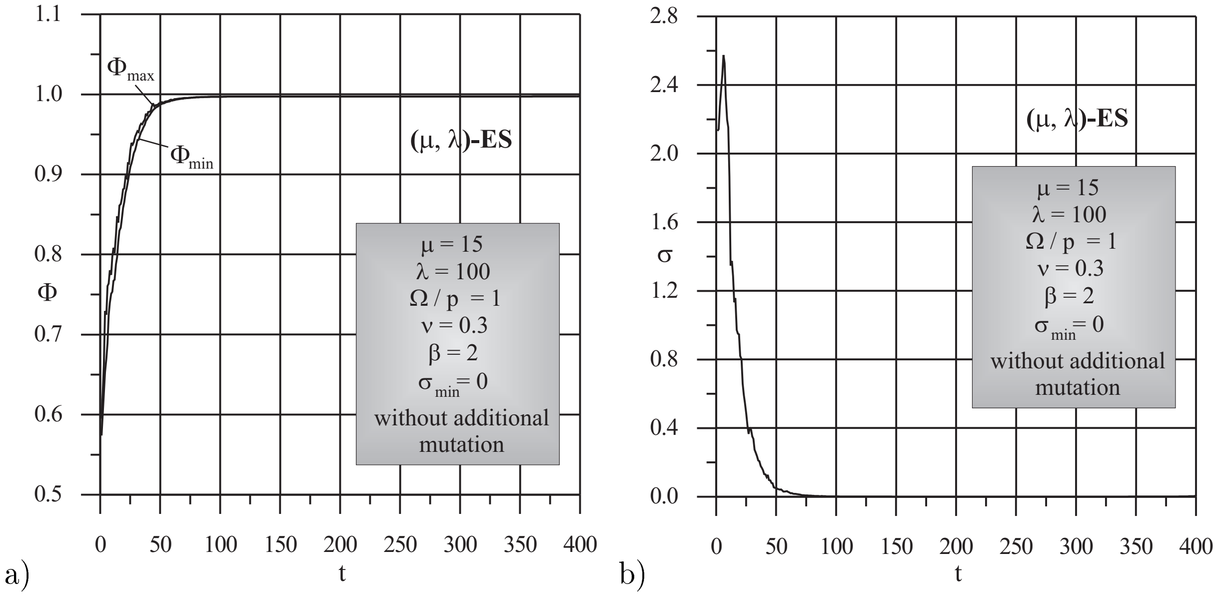


Figure 5: Minimal and maximal value of objective function (a) and average standard deviation (b) without additional mutation

notice that the introduction of additional mutation operator slowed down the convergence and allowed for about four times longer maintenance of genetic diversity of the population. This may also be seen in the figure 6b. One can notice that average standard deviation of the population needs over four times more simulation steps to approach zero value in comparison with the case when there were no additional mutation used.

5 Concluding remarks

In many global optimization problems that appear in engineering design only experiments (or their simulation) seem to be the only way to evaluate the quality of particular solutions. In these cases one must apply optimization methods that do not require strict mathematical models of a problem like evolutionary algorithms or simulated annealing. In the paper the possible application of evolution strategies to the design of rotating variable-thickness annular elastic disc was described. The proposed representation of the shape (the use of 3rd order splines) makes it possible to describe the shape of the disc more precisely (giving many degrees of freedom to the optimization problem). This analytical-numerical approach may be helpful in many shape optimization problems e.g. looking for the best profile of a rotating wheel. Results obtained in this way may be treated as the first approach to the more exact solution based on FEM analysis.

The results of simulation experiments show that the proposed approach allows to increase (even up to 80%) the disc carrying capacity in comparison to the constant thickness (flat) shape. The use of additional mutation operator introduced as a means to preserve diversity in the population can improve this result by another 5% or more.

Further research should concentrate on different variants of evolution strategy used for the optimization including specialized operators and parallel (agent-based) implementation of the algorithm.

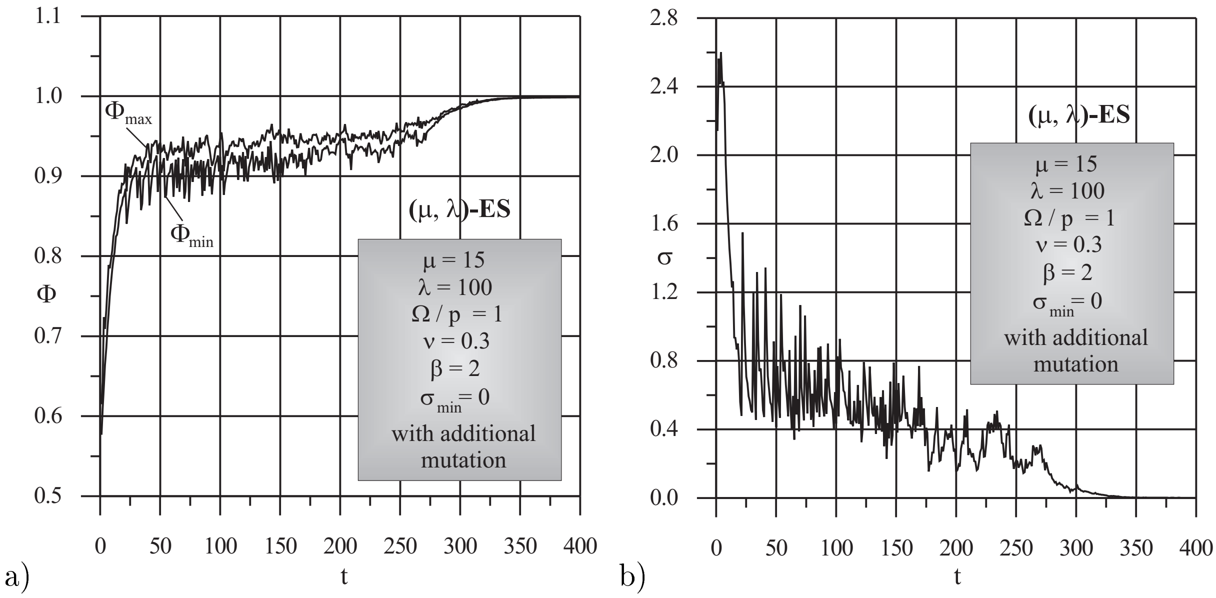


Figure 6: Minimal and maximal value of objective function (a) and average standard deviation (b) with additional mutation

References

[1] T. Bäck, D. Fogel, D. Whitley, and P. Angeline. Mutation. In T. Bäck, D. Fogel, and Z. Michalewicz, editors, *Handbook of Evolutionary Computation.* IOP Publishing and Oxford University Press, 1997.

[2] T. Bäck and H.-P. Schwefel. Evolutionary computation: An overview. In *Proceedings of the Third IEEE Conference on Evolutionary Computation*, Piscataway NJ, 1996. IEEE Press.

[3] L. Booker, D. Fogel, D. Whitley, and P. Angeline. Recombination. In T. Bäck, D. Fogel, and Z. Michalewicz, editors, *Handbook of Evolutionary Computation.* IOP Publishing and Oxford University Press, 1997.

[4] T. Bäck and H.-P. Schwefel. An overview of evolutionary algorithms for parameter optimisation. *Evolutionary Computation*, 1993.

[5] R. Dębski, G. Woźniak, R. Dreżewski, and M. Kisiel-Dorohinicki. Evolutionary strategy for shape design of rotating variable-thickness annular elastic disc. In T. Burczyński, W. Cholewa, and W. Moczulski, editors, *Methods of Artificial Intelligence.* Silesian University of Technology, Gliwice, Poland, 2002.

[6] G. Rudolph. Evolution strategies. In T. Bäck, D. Fogel, and Z. Michalewicz, editors, *Handbook of Evolutionary Computation.* IOP Publishing and Oxford University Press, 1997.

[7] H.-P. Schwefel. *Numerical Optimization of Computer Models.* John Willey & Sons, Ltd., Chichester, 1981.