Co-Evolutionary Multi-Agent System with Sexual Selection Mechanism for Multi-Objective Optimization

Rafał Dreżewski, and Leszek Siwik

***Abstract*— Co-evolutionary techniques for evolutionary algorithms are aimed at overcoming their limited adaptive capabilities and allow for the application of such algorithms to problems for which it is difficult or even impossible to formulate explicit fitness function. Sexual selection resulting from sexual conflict and co-evolution of female mate choice and male displayed trait is considered to be one of the ecological interactions responsible for maintaining population diversity. In this paper the idea of *co-evolutionary multi-agent system with sexual selection mechanism for multi-objective optimization* is introduced. In presented system the Pareto frontier is located by the population of agents as a result of co-evolutionary interactions between sexes. Also, results from runs of presented system against test functions and comparison to classical multi-objective evolutionary algorithms are presented.**

I. Introduction

Evolutionary Algorithms (EAs) are techniques for finding (sub)optimal solutions of global optimization and adaptation problems, which are based on analogies to biological evolutionary processes. Evolutionary algorithms, however, often suffer from premature loss of population diversity. This results in premature convergence and may lead to locating a local optimum instead of a global one. In the case of multi-modal problem landscapes EA without any special mechanisms will inevitably locate a basin of attraction of single optimum. The loss of diversity also limits the adaptive capabilities of EAs in dynamic environments.

In the case of multi-objective optimization, high quality approximation of *Pareto frontier* should fulfill at least three distinguishing features: first of all it should be "located" as close to the ideal Pareto frontier as possible (what is very natural and common condition for both single- and multi- objective optimization), secondly it should include as many alternatives as possible and, at last, all proposed non-dominated alternatives should be evenly distributed over the whole ideal Pareto set. In consequence in the case of multi-objective optimization, premature loss of population diversity can result not only in lack of drifting to the ideal Pareto frontier but also in obtaining approximation of Pareto set that is focused around its selected area(s), what is very undesirable assuming that preference-based multi-objective optimization is not considered in this place. Additionally, in the case of multi-objective problems with many local Pareto frontiers (so called multi-modal multi-objective problems defined by Deb in [1]) the loss of population diversity may result in locating only a local Pareto frontier instead of a global one.

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Co-evolutionary techniques (including sexual selection) are aimed at improving adaptive capabilities and introducing open-ended evolution into EAs by maintaining population diversity [2]. Sexual selection results from co-evolution of female mate choice and male displayed trait, where females evolve to reduce direct costs associated with mating and keep them on optimal level and males evolve to attract females to mating (*sexual conflict*) [3]. The proportion of two sexes (females and males) in population is almost always $1:1$. This fact combined with higher females' reproduction costs causes, that in the majority of cases, females choose males in the reproduction process according to some males' features. In fact, different variants of the sexual conflict are possible. For example there can be higher females' reproduction costs, equal reproduction costs (no sexual conflict), equal number of females and males in population, higher number of males in population (when the costs of producing a female are higher than producing a male), higher number of females in population (when the costs of producing a male are higher than producing a female).

Evolutionary multi-agent systems (EMAS) have proved their great usefulness for solving a lot of different discrete, continuous, combinatorial and non-combinatorial multi-objective optimization problems [4], [5], [6]. Co-evolutionary mechanisms introduced into EMAS are aimed at maintaining population diversity and improving adaptive capabilities of such systems—especially in dynamic environments.

In the following sections the introduction to multi-objective optimization problems, and the previous work on sexual selection as a mechanism for maintaining population diversity are presented. Next, the formal model of co-evolutionary multi-agent system based on the sexual conflict is presented. In such system several sexes co-evolve. The system is applied to multi-objective optimization problems and compared to other evolutionary techniques.

II. Evolutionary Multi-Objective Optimization

During most real-life decision processes a lot of different (often contradictory) factors have to be considered, and the decision maker has to deal with an ambiguous situation: the solutions which optimize one criterion may prove insufficiently good considering the others. From the mathematical point of view such multi-objective (or multi-criteria) problem can be formulated as follows [7], [8], [9]:

Let the problem variables be represented by a real-valued vector:

$$\vec{x} = [x_1, x_2, \ldots, x_N]^T \in \mathbb{R}^N \tag{1}$$

where N is the number of variables. Then a subset of $\mathbb{R}^N$ of all possible (feasible) decision alternatives (options) can be defined by a system of:

- inequalities (constraints): $g_k(\vec{x}) \geq 0$ and $k = 1, 2, \ldots, K$,
- equalities (bounds): $h_l(\vec{x}) = 0$, $l = 1, 2, \ldots, L$

and denoted by $\mathcal{D}$. The alternatives are evaluated by a system of M functions (objectives) denoted here by vector $F = [f_1, f_2, \ldots, f_M]^T$:

$$f_m : \ \mathbb{R}^N \to \mathbb{R}, \quad m = 1, 2, \ldots, M \tag{2}$$

The key issue of optimality in the Pareto sense is the *weak domination relation*. Alternative $\vec{x}^a$ is dominated by $\vec{x}^b$ (which is often denoted by $\vec{x}^b \succeq \vec{x}^a$) if and only if (assuming maximization of all the objectives):

$$\forall m \quad f_m(\vec{x}^a) \leq f_m(\vec{x}^b) \text{ and } \exists m \quad f_m(\vec{x}^a) < f_m(\vec{x}^b) \tag{3}$$

A solution in the Pareto sense of the multi-objective optimization problem means determination of all non-dominated (in the sense of the defined above *weak domination relation*) alternatives from the set $\mathcal{D}$, which is sometimes called a *Pareto set*:

$$\mathcal{P} = \{\vec{x} \in \mathcal{D} \ \mid \ \neg\exists \vec{x}^a \in \mathcal{D} \quad \vec{x}^a \succeq \vec{x}\} \tag{4}$$

At the same time the non-dominated alternatives create in criteria space a set called a *Pareto frontier*:

$$\mathcal{PF} = \{\vec{y} = F(\vec{x}) \in \mathbb{R}^M \quad | \quad \vec{x} \in \mathcal{P}\} \tag{5}$$

Unfortunately, when searching for the approximation of the Pareto frontier in the whole, classical computational methods often prove ineffective for many (real) decision problems. The corresponding models are too big or the formulas applied too complicated, or it can even occur that some formulations must be rejected in the face of numerical instability of available solvers. That is why so much attention is paid to methods based on evolutionary algorithms. These methods are relatively insensitive to complexity of the problem and give the approximation of the whole Pareto frontier with controllable adequacy, which means that a solving process can be stopped by a decision maker anytime he is satisfied.

For the last 20 years a variety of evolutionary multi-criteria optimization techniques have been proposed [10], [11], [12], [13], [14]. In the Deb's typology of evolutionary multi-objective algorithms (EMOAs) firstly the elitist and non-elitist ones are distinguished[1] [15]. Each of these groups include many practically used algorithms such as:

- elitist EMOAs: Rudolph's algorithm [16], distance-based Pareto GA [17], strength Pareto EA [18], Pareto-archived evolution strategy [19], multi-objective messy GA [9], multi-objective micro GA [20] etc.
- non-elitist EMOAs: vector-optimized evolution strategy [21], random weighted GA [22], weight-based GA [23], niched-pareto GA [24], non-dominated sorting GA [25], multiple objective GA [26], distributed sharing GA [27] etc.

The main difference between these two groups of techniques consists in utilizing the so-called elite-preserving operators that give the best individuals (the elite of the population) the opportunity to be directly carried over to the next generation regardless of the actual selection mechanism used. Of course, if the algorithm finds a better solution than the one in the elite, this solution becomes a new elitist solution.

III. Previous Research on Sexual Selection Mechanism

Sexual selection is considered to be one of the ecological mechanisms responsible for biodiversity and sympatric speciation. The research on sexual selection mechanism, which we will present shortly in this section, focuses on, generally, two aspects: modeling and simulation of sexual selection and investigating whether it can cause speciation or population diversity (artificial life simulations) and the application of sexual selection in evolutionary algorithms for multi-modal and multi-objective optimization in order to maintain population diversity and cause the population to split into sub-populations (species) located in the basins of attraction of different local optima or different parts of Pareto frontier.

In the recent years there has been observed growing interest in modeling and simulation of sexual selection mechanism and investigating the effects of co-evolution of sexes. Gavrilets experimented with sexual selection as a mechanism of sympatric speciation. He presented a model ([3]), which exhibits three general dynamic regimes. In the first one there is endless co-evolutionary chase between the sexes, where females evolve to decrease the mating rate and males evolve to increase it. In the second regime females' alleles split into two clusters both at the optimum distance from the males' alleles and males get trapped between the two female clusters with relatively low mating success. In the third regime males answer the diversification of females by splitting into two clusters that evolve toward the corresponding female clusters. As a result, the initial population splits into two species that are reproductively isolated.

Todd and Miller [28] showed that the natural selection and the sexual selection play complementary roles and both processes together are capable of generating evolutionary innovations and biodiversity much more efficiently. Sexual selection allows species to create its own optima in fitness landscapes. This aspect of sexual selection can result in rapidly shifting adaptive niches what allows the population to explore different regions of phenotype space and to escape from local optima. The authors also presented the model of sympatric speciation via sexual selection.

Sánchez-Velazco and Bullinaria [29] proposed *gendered selection strategies for genetic algorithms*. The main goal of the application of gendered selection in their algorithm

[1] Deb's typology includes also so-called *constrained EMOAs*—techniques that support handling constraints

was maintaining population diversity, avoiding premature convergence, and escaping local optima. They introduced sexual selection mechanism, where the males are selected on the basis of their fitness value and the females on the basis of the so called *indirect fitness*. Female's indirect fitness is the weighted average of her fitness value, age, and the potential to produce fit offspring (when compared to her partner). For each gender different mutation rates were used. The authors applied their algorithm to Traveling Salesman Problem and function optimization.

Sexual selection as the mechanism for multi-modal function optimization was studied by Ratford, Tuson and Thompson [30]. The main goal of their research was to investigate whether the sexual selection may cause speciation i.e. division of population into sub-populations located within basins of attraction of global and local minima. In their technique sexual selection is based on the so called *seduction function*. This function gives a low measure when two individuals are very similar or dissimilar and high measure for individuals fairly similar. The Hamming distance in genotype space was used as a distance metric for two individuals. The authors applied their mechanism alone and in combination with the crowding technique and the spatial population model. Although in most cases their technique was successful in locating multiple basins of attraction of local optima, the strong tendency to lose all basins of attraction except one after several hundreds simulation steps was observed.

Allenson proposed genetic algorithm with sexual selection for multi-objective optimization [31]. In his technique the number of sexes was the same as the number of criteria of the given problem. Individuals of the given sex were evaluated only according to one criterion (associated with their sex). The offspring was created with the use of tournament selection, crossover and mutation operators. Two individuals were selected at random from the population and one of them became first parent (in such a way that the "better" one had more chances to became the parent). Analogically the second parent was chosen. Sex of the child was determined randomly and then he replaced the worst individual from its sex. Allenson also introduced sexual selection mechanism. In this version of algorithm selection mechanism from evolutionary strategies was used. Every individual from population became parent and the partner for reproduction was selected on the basis of individual's preferences coded within its genotype (for example individual preferred top 10% of the individuals from the opposite sex).

Lis and Eiben proposed multi-sexual GA (MSGA) for multi-objective optimization [32] in which they used sexual selection mechanism. They also used one sex for each criterion. Each individual had the marker which indicated to which sex it belongs. Individual's fitness value was the value of criterion associated with the sex of this individual. Ranking selection mechanism was used (individuals were sorted on the basis of fitness value within each sex separately). If recombination operator was used during the reproduction (this was decided randomly) then partners for reproduction were chosen from each sex separately with the use of ranking mechanism and the offspring was created with the use of special multi-parent crossover operator. The sex of generated offspring was determined on the basis of genetic material (it had the same sex as the parent that provided most of genes). If recombination was not used then firstly the sex was determined randomly, next the parent from that sex with the use of ranking selection mechanism. The mutation operator applied during the creation of descendant could not change the sex of child. After the population of next generation was created the group of Pareto-optimal individuals was selected and this group was merged with the group of Pareto-optimal individuals from previous generations. During this phase dominated individuals were removed from the set of Pareto-optimal individuals.

Bonissone and Subbu continued work on Lis and Eiben's algorithm. They proposed additional mechanisms for determining the sex of offspring [33]. The first one was based on phenotype (child had the sex associated with the criterion for which it had the best fitness) and the second one was random (the sex of child was determined randomly).

All the works on sexual selection mechanism for multi-objective evolutionary algorithms were focused on using this mechanism for maintaining population diversity, which causes that individuals are evenly distributed over the Pareto frontier.

As it was presented here, the co-evolution of sexes is the biological mechanism responsible for biodiversity and sympatric speciation. However it was not widely used as a mechanism of maintaining useful genetic diversity of population for evolutionary algorithms. It seems that sexual selection should introduce open-ended evolution, improve adaptive capabilities of EA (especially in dynamic environments) and allow speciation (the formation of species located within basins of attraction of different local optima, in different areas of Pareto frontier or at different local Pareto frontiers in case of multi-modal multi-objective problems [1]) but this is still an open issue and the subject of ongoing research.

IV. Co-Evolutionary Multi-Agent System with Sexual Selection for Multi-Objective Optimization

The main idea of *co-evolutionary multi-agent system (CoEMAS)* is the realization of species and sexes co-evolution in *multi-agent system (MAS)* [34]. CoEMAS model, as opposed to the basic *evolutionary multi-agent system (EMAS)* model [35], allows for the co-existence of several species and sexes which can interact with each other and co-evolve. CoEMAS is especially suited for modeling different co-evolutionary interactions, such as resource competition, predator-prey and host-parasite co-evolution, sexual preferences, etc.

Systems based on CoEMAS model can be applied, for example, to multi-modal function optimization [36] and multi-objective optimization because such systems maintain population diversity and easily adapt to the changing environment.

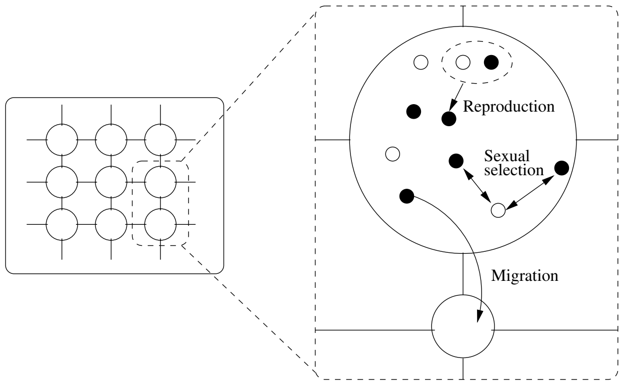

Fig. 1. CoEMAS with co-evolving sexes (SCoEMAS)

The system presented in this paper is the CoEMAS with sexual selection (*SCoEMAS*—see fig. 1). The mechanisms used in such system include: co-evolution of sexes, and sexual selection based on Pareto domination. All agents live within the environment, which has the graph-like structure. The number of sexes corresponds with the number of criteria (each sex has the criteria assigned to it and agents that belong to that sex are evaluated with the assigned criteria).

There is one resource defined in the system. The resource can be possessed by the agents and the environment (there is closed circulation of resource in the system). This resource is distributed (proportionally to the fitness values of the agents) by each node of the graph among the agents that are located in that node.

Each time step, the agents can migrate within the environment (they lose some resource during the migration). The agent can migrate only to the node connected with the one within which it is located. The agent chooses the node to which it will migrate on the basis of the amount of resource of that node.

Each time step, when the agent is ready for reproduction (i.e. the amount of its resource is above the given level) it sends the information to the agents of other sexes located within the same node. The other agents can response to this information when they are also ready for reproduction. Next, the agent which initiated the reproduction process chooses one (or more—it depends on the number of sexes in the system) of the agents of opposite sex on the basis of the amounts of their resources (the probability of choosing the agent is proportional to the amount of its resource). The offspring is created with the use of intermediate recombination [37] and Gaussian mutation [38]. Next, the child is compared to the individuals from the non-dominated individuals set of the node in which parents and child are located. If none of the individuals from this set is dominating the child then the child is copied to the set (all individuals dominated by the child are removed from the set).

In the following sections we will present the formal model of *SCoEMAS* system.

A. SCoEMAS

The *SCoEMAS* may be described as 4-tuple:

$$SCoEMAS = \langle E, S, \Gamma, \Lambda \rangle \tag{6}$$

where E is the environment of the $SCoEMAS$, S is the set of species ($s \in S$) that co-evolve in $SCoEMAS$, Γ is the set of resource types that exist in the system, the amount of type γ resource will be denoted by r^{γ}, Λ is the set of information types that exist in the system, the information of type λ will be denoted by i^{λ}. There are four information types ($\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$) and one resource type ($\Gamma = \{\gamma\}$) in *SCoEMAS*.

B. Environment

The environment of *SCoEMAS* may be described as 3-tuple:

$$E = \langle T^E, \Gamma^E = \Gamma, \Lambda^E = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} \rangle \tag{7}$$

where T^E is the topography of environment E, Γ^E is the set of resource types that exist in the environment, Λ^E is the set of information types that exist in the environment. The topography of the environment is given by:

$$T^E = \langle D, l \rangle \tag{8}$$

where D is directed graph with the cost function c defined: $D = \langle V, F, c \rangle$, V is the set of vertices, F is the set of arches. The distance between two nodes is defined as the length of the shortest path between them in graph D.

The l function makes it possible to locate particular agent in the environment space:

$$l: \quad A \rightarrow V \tag{9}$$

where A is the set of agents, that exist in $SCoEMAS$.

Vertice v is given by:

$$v = \langle A^v, \Gamma^v = \Gamma^E, \Lambda^v = \Lambda^E \rangle \tag{10}$$

A^v is the set of agents that are located in the vertice v. Agents can collect three types of informations from the vertice. The first one includes all vertices that are connected with the vertice v (information i^{λ_1}), the second one includes the amounts of resources within vertices connected with the vertice v (information i^{λ_2}), and the third one includes all agents of opposite sexes that are located in the vertice v (information i^{λ_3}). Information of type i^{λ_4} includes all non-dominated agents from previous generations (these are agents that represent non-dominated solutions already found by system).

C. Species

The set of species is given by:

$$S = \{s\} \tag{11}$$

Species s is defined as follows:

$$s = \langle A^s, SX^s, Z^s, C^s \rangle \tag{12}$$

where A^s is the set of agents that belong to species s. The number of sexes within species s depends on the problem being solved (each sex is assigned to one criterion): $SX^s = \{sx_1, \ldots, sx_M\}$, where M is the number of criteria. The set of actions for species s is defined as follows:

$$Z^s = \{die, get, seek, accept, clone, rec, mut, give, migr\} \quad (13)$$

where *die* is the action of removing agent from the system (when it runs out of resource), *get* action allows agent to get some resource from the environment (the resource γ is given to the agents proportionally to their fitness values—each sex is evaluated with the use of criterion associated with it), *seek* is the action that sends messages to agents of opposite sex(es) located in the vertice $v = l(a^{sx})$, when agent a^{sx} (of sex sx) is ready for reproduction (the amount of resource is above the given level). *accept* is the action of accepting the agent of another sex as a partner for reproduction (agents of opposite sexes are accepted with probability proportional to the amount of resource they posses). *clone*, *rec*, and *mut* actions are responsible for, respectively, child creation, Gaussian mutation [38] and intermediate recombination [37] of its genotype. *give* action gives some resource of type γ to the child. *migr* action allows the migration within the environment.

The set of relations of species s with other species that exist in the *SCoEMAS* (C^s) is given by:

$$C^s = \{\xrightarrow{s,get-}\} \quad (14)$$

The $\xrightarrow{s,get-}$ relation models the intra-species competition for limited resources:

$$\xrightarrow{s,get-} = \{\langle s, s\rangle : s, \in S\} \quad (15)$$

where *get* is the action of taking resource from the environment and the "–" sign indicates that action *get* performed by individuals of species s has the negative effect on the fitness of individuals that belongs to the same species.

D. Sex sx

The sx sex of species s is defined as follows:

$$sx = \langle A^{sx}, Z^{sx}, C^{sx}\rangle \quad (16)$$

where A^{sx} is the set of agents of sex sx ($A^{sx} \subseteq A^s$). The set of actions that agent a^{sx} can perform $Z^{sx} = Z^s$.

The set of relations of sex sx_i with opposite sex sx_j is defined as follows:

$$C^{sx_i} = \left\{ \xrightarrow[give-]{sx_i,accept+} \right\} \quad (17)$$

$$\xrightarrow[give-]{sx_i,accept+} = \{\langle sx_i, sx_j\rangle : sx_i, sx_j \in SX^s\} \quad (18)$$

where *accept* is the action of choosing individual a^{sx_j} for reproduction (which has the positive effect on its fitness) by agent a^{sx_i}. The action *accept* results in performing action *give* by both agents. These actions transfer some amount of resource γ to the child, what results in decreasing the fitness of agents a^{sx_i} and a^{sx_j}.

E. Agent

Agent a of sex sx, that belongs to the species $s \in S$ ($a \equiv a^{s,sx}$) is defined as follows:

$$a = \langle GN^a, Z^a, \Gamma^a = \Gamma, \Lambda^a, PR^a\rangle \quad (19)$$

where GN^a is the genotype (consisted of real-valued vector of objective variables). The set of agent's actions $Z^a = Z^s$, see equation (13). The set of informations used by agent a $\Lambda^a = \{\lambda_1, \lambda_2, \lambda_3\}$.

PR is the set of agent's profiles with the order relation $\trianglelefteq$ defined:

$$PR^a = \{pr_1, pr_2, pr_3\} \quad (20a)$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \quad (20b)$$

where pr_1 is the resource profile (this is also the profile, which goal has the higher priority), pr_2 is the reproductive profile, and pr_3 is the migration profile. Within pr_1 profile all strategies connected with type γ resource are realized ($\langle die\rangle$, $\langle get\rangle$). Within pr_2 profile all strategies connected with the reproduction process ($\langle seek\rangle$, and $\langle accept, clone, rec, mut, give\rangle$) are realized. These strategies use information i^{λ_3}. Within pr_3 profile the migration strategy ($\langle migr\rangle$), which uses information i^{λ_1} and i^{λ_2} is realized.

In each time step the agent activates the profile with active goal and the highest priority. Next it chooses the strategy with highest priority that is realized within the chosen profile and which results in realizing the active goal.

V. Simulation experiments

First experiments, which results are presented in this section, were aimed at investigating if SCoEMAS can be applied to multi-objective optimization problems and whether it works properly (agents do not die off). Proposed *co-evolutionary multi-agent system with sexual selection mechanism for multi-objective optimization* has been tested using, inter alia, *Tamaki* and *Obayashi* test functions that can be defined as follows:

$$Obayashi = \begin{cases} f_1(x,y) = x \\ f_2(x,y) = y \\ x, y \geq 0 \quad and \quad x^2 + y^2 \leq 1 \end{cases}$$

$$Tamaki = \begin{cases} f_1(x,y,z) = x \\ f_2(x,y,z) = y \\ f_3(x,y,z) = z \\ x, y, z \geq 0 \quad and \quad x^2 + y^2 + z^2 \leq 1 \end{cases}$$

Additionally, on the same *JagWorld* platform, which was used for SCoEMAS, there have been implemented also some "classical" evolutionary algorithms for multi-objective optimization i.e. *Niched-Pareto Genetic Algorithm (NPGA)* [24] and *Strength Pareto Evolutionary Algorithm (SPEA)* [18].

To compare proposed approach with implemented classical algorithms some metrics (which may be found in [8]) have been used.[2] According to this thesis, if $A \subseteq X$ denotes a non-dominated set, $\sigma \geq 0$ denotes appropriately chosen neighborhood parameter and $\|\cdot\|$ denotes the given distance metric—then three functions $M_1(A)$, $M_2(A)$ and $M_3(A)$ can be introduced to asses the quality of A regarding the decision space:

- M_1—the average distance to the Pareto optimal set X_p:

$$M_1(A) = \frac{1}{|A|} \sum_{a \in A} min\{\|a - x\| \mid x \in X_p\} \tag{21}$$

- M_2—the distribution in combination with the number of non-dominated solutions found:

$$M_2(A) = \frac{1}{|A-1|} \sum_{a \in A} |\{b \in A \mid\, \| a - b \| > \sigma\}| \tag{22}$$

- M_3—the spread of non-dominated solutions over the set A:

$$M_3(A) = \sqrt{\sum_{i=1}^{N} max\{\|a_i - b_i\| \mid a, b \in A\}} \tag{23}$$

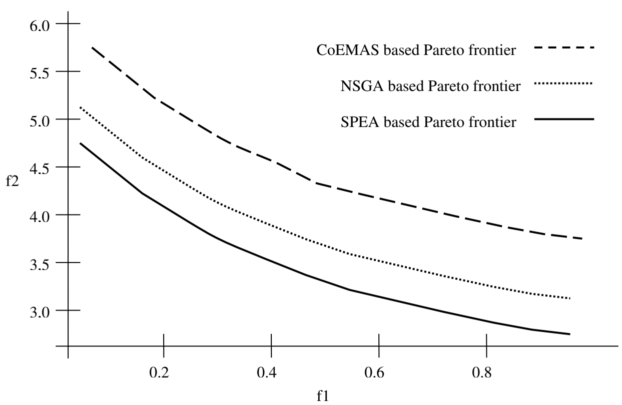

Fig. 2. Pareto frontier approximations for t_1 problem obtained by CoEMAS with sexual selection, SPEA, and NSGA algorithms

Thanks to this, some comparative studies of proposed co-evolutionary agent-based system and these very well known, and commonly used algorithms could be performed. Obtained values of these metrics are presented in fig. 5.

In [8] Eckart Zitzler presented, among others, six test functions that are constructed according to the following schema:

$$t_1 = \begin{cases} Minimize \quad t(x) = (f_1(x), f_2(x)) \\ Subject\ to \quad f_2(x) = g(x_2, \ldots, x_n) \cdot h(f_1(x_1), g(x_2, \ldots, x_n)) \\ Where \quad x = (x_1, \ldots, x_n) \end{cases}$$

Each of these functions tests algorithm against the ability for dealing with a specific difficulty caused by multi-objective problems. The difficulties that are there being taken

[2]In the literature there can be found arguments that metrics not based on Hypervolume measure are not suitable for assessing algorithms dedicated for solving multiobjective problems (see for example [39]). On the other hand however non hypervolume-based metrics are still quite commonly used ([20]). Further analysis of the approach that is being proposed in this paper will be based of course also on such metrics as HV, HVR etc.

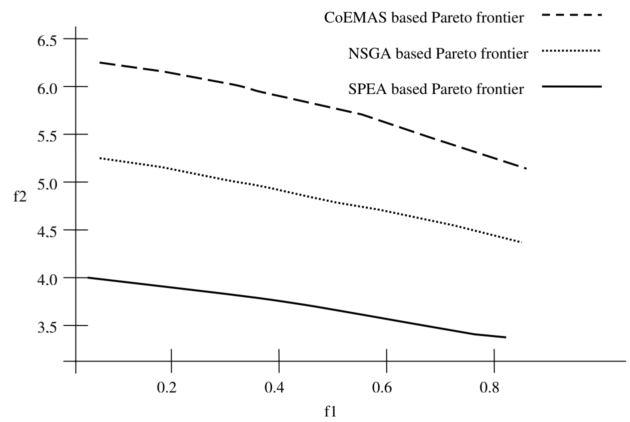

Fig. 3. Pareto frontier approximations for t_2 problem obtained by CoEMAS with sexual selection, SPEA, and NSGA algorithms

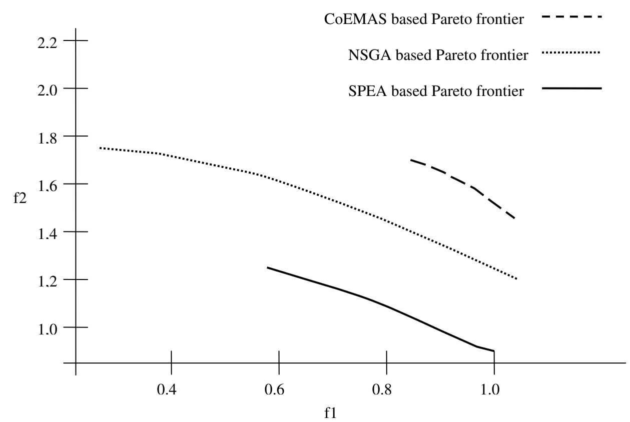

Fig. 4. Pareto frontier approximations for t_6 problem obtained by CoEMAS with sexual selection, SPEA, and NSGA algorithms

into considerations include: convexity or non-convexity, discreteness, non-uniformity etc. The comparison presented in this section is based on three of these six functions i.e. on functions t_1 (see fig. 2), t_2 (see fig. 3) and t_6 (see fig. 4), that can be defined as follows:

$$t_1 = \begin{cases} f_1(x_1) = x_1 \\ g(x_2, \ldots, x_n) = 1 + 9(\sum_{i=2}^{n} x_i)(n-1) \\ h(f_1, g) = 1 - \sqrt{f_1/g} \\ n = 30 \qquad x_i \in [0, 1] \end{cases}$$

$$t_2 = \begin{cases} f_1(x_1) = x_1 \\ g(x_2, \ldots, x_n) = 1 + 9(\sum_{i=2}^{n} x_i)(n-1) \\ h(f_1, g) = 1 - (f_1/g)^2 \\ n = 30 \qquad x_i \in [0, 1] \end{cases}$$

$$t_6 = \begin{cases} f_1(x_1) = 1 - \exp(-4x_1)\sin^6(6\pi x_1) \\ g(x_2, \ldots, x_n) = 1 + 9((\sum_{i=2}^{n} x_i)(n-1))^{0.25} \\ h(f_1, g) = 1 - (f_1/g)^2 \\ n = 10 \qquad x_i \in [0, 1] \end{cases}$$

a)

Population size	500
Chromosome length	10
External set size	256
Crossover probability	0,3
Mutation probability	0,2

b)

Population size	100
Chromosome length	8
External set size	64
Crossover probability	0,1
Mutation probability	0,1

c)

σ	Metrics	SPEA		NSGA		CoEMAS	
		Obayashi problem	Tamaki problem	Obayashi problem	Tamaki problem	Obayashi problem	Tamaki problem
0,05	M_1	0.08	0.001	0.003	0.10	0.011	0.15
	M_2	1.75	1.49	1.84	0.41	1.37	0.39
	M_3	1.69	1.50	1.81	0.64	2.01	0.83
0,2	M_1	0.15	0.14	0.41	0.37	0.75	0.24
	M_2	3.47	5.80	1.58	5.20	1.64	5.98
	M_3	2.55	3.37	1.95	3.42	2.13	4.32
0,6	M_1	0.46	0.43	0.26	0.21	0.52	0.62
	M_2	2.38	18.67	4.38	1.09	4.03	2.24
	M_3	4.28	6.71	4.84	3.51	5.19	3.40

d)

σ	Metrics	SPEA		NSGA		CoEMAS	
		Obayashi problem	Tamaki problem	Obayashi problem	Tamaki problem	Obayashi problem	Tamaki problem
0,05	M_1	0.13	0.32	0.44	0.0	0.56	0.32
	M_2	0.55	1.28	2.55	0.15	2.79	0.29
	M_3	0.74	1.13	1.96	0.39	2.00	0.62
0,2	M_1	0.10	0.19	0.14	0.07	0.43	0.44
	M_2	4.38	7.17	3.95	2.98	3.88	3.14
	M_3	3.32	3.67	2.79	2.66	2.99	2.96
0,6	M_1	0.11	0.28	0.33	0.15	0.85	0.65
	M_2	0.54	14.29	7.63	2.11	8.21	2.87
	M_3	3.60	6.18	5.45	4.33	5.83	6.07

Fig. 5. Comparison of the proposed agent-based approach, SPEA and NSGA algorithms according to the M_1, M_2 and M_3 metrics (table a includes selected configuration parameters for results presented in table c, whereas table b includes selected configuration parameters for results presented in table d)

As it was mentioned above, these are results of preliminary experiments aimed at investigating if SCoEMAS can be applied to multi-objective optimization problems and whether it works properly. Some potential advantages of proposed co-evolutionary system could not be here observed but further experiments, especially with difficult multi-dimensional and dynamic testing problems will be conducted. Proposed approach should turn out especially useful in case of multi-modal multi-objective problems such as Zitzler's t_4 test function [8] because of the SCoEMAS's ability to maintain high population diversity and to form the species of agents located at the local and global Pareto frontiers.

VI. Concluding Remarks

Evolutionary algorithms often suffer from premature loss of population diversity what limits their adaptive capabilities and possible application to hard problems like multi-modal and multi-objective optimization. To avoid such problems co-evolutionary techniques for evolutionary algorithms are proposed and applied. However, co-evolutionary techniques (including sexual selection) are rather rarely used as the mechanisms of maintaining useful population diversity.

The model of *co-evolutionary multi-agent system* allows co-evolution of several species and sexes. This results in maintaining population diversity and improves adaptive capabilities of the systems based on *CoEMAS* model. In this paper the *co-evolutionary multi-agent system with sexual selection mechanism for multi-objective optimization* has been presented. The system was run against commonly used test problems and compared to classical SPEA and NSGA algorithms. Presented results show that SPEA is the best of all compared algorithms. It turned out that proposed SCoEMAS with sexual selection mechanism can be used for multi-objective problems however more research is needed to obtain better results. The fact that results were worse than in the case of classical evolutionary multi-objective algorithms results from the tendency to maintain high population diversity what could be very useful in the case of hard dynamic and multi-modal multi-objective problems (as defined by Deb [1]).

Future work will include more detailed comparison to other classical algorithms with the use of hard multi-dimensional, dynamic, and multi-modal multi-objective test problems. Also the application of other co-evolutionary mechanisms like predator prey, host-parasite and symbiosis (co-operative co-evolution) are included in future plans.

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