Multi-Objective Optimization Technique Based on Co-Evolutionary Interactions in Multi-Agent System

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Abstract. Co-evolutionary techniques for evolutionary algorithms help overcoming limited adaptive capabilities of evolutionary algorithms, and maintaining population diversity. In this paper the idea and formal model of agent-based realization of predator-prey co-evolutionary algorithm is presented. The effect of using such approach is not only the location of Pareto frontier but also maintaining of useful population diversity. The presented system is compared to classical multi-objective evolutionary algorithms with the use of Kursawe test problem and the problem of effective portfolio building.

1 Introduction

Co-evolutionary techniques for *evolutionary algorithms (EAs)* are applicable in the case of problems for which the fitness function formulation is difficult or impossible, there is need for improving adaptive capabilities of EA or maintaining useful population diversity and introducing speciation into EAs—loss of population diversity is one the main problems in some applications of EAs (for example multi-modal optimization, multi-objective optimization, dynamic problems, etc.)

In the case of multi-objective optimization problems loss of population diversity may cause that the population locates in the areas far from Pareto frontier or that individuals are located only in selected areas of Pareto frontier. In the case of multi-objective problems with many local Pareto frontiers (defined by Deb in [2]) the loss of population diversity may result in locating only local Pareto frontier instead of a global one.

One of the first attempts to apply competitive co-evolutionary algorithm to multi-objective problems was predator-prey evolutionary strategy (PPES) [6]. This algorithm was then modified by Deb [2] in order to introduce some mechanisms of maintaining population diversity and evenly distributing individuals over the Pareto frontier, but this is still an open issue and the subject of ongoing research.

Evolutionary multi-agent systems (EMAS) are multi-agent systems, in which the population of agents evolve (agents can die, reproduce and compete for limited resources). The model of *co-evolutionary multi-agent system (CoEMAS)* [3] introduces additionally the notions of species, sexes, and interactions between them. CoEMAS allows modeling and simulation of different co-evolutionary interactions, which can serve as the basis for constructing the techniques of maintaining population diversity and improving adaptive capabilities of such systems. CoEMAS systems with sexual selection and host-parasite mechanisms have already been applied with good results to multi-objective optimization problems ([4, 5]). In the following sections the introduction to multi-objective optimization problems is presented. Next, the co-evolutionary multi-agent system with predator-prey mechanism is formally described. The system is applied to one standard multi-objective optimization test problem and to problem of effective portfolio building. Results from the experiments with the CoEMAS system are then compared to other classical evolutionary techniques' results.

2 Multi-Objective Optimization

Multi-criteria Decision Making (MCDM) is the most natural way of making decision for human beings. *Multi-criteria* means that during the decision process a lot of factors and objectives (often contradictory) are taken into consideration. Human being is equipped with natural gifts for multi-criteria decision making, however such abilities are not sufficient in more complex technical, business or scientific decisions. In such cases decision maker has to be equipped with efficient information systems able to support his decision making process.

MCDM process is based most frequently on *Multi-objective Optimization* formulated formally only in 19th century, but actual progress in solving *Multi-objective Optimization Problems (MOOP)* ensued after formulating by Vilfredo Pareto his optimality theory in 1906. Following [2]—*Multi-objective Optimization Problem* in its general form can be formulated as follows:

$$MOOP \equiv \begin{cases} Minimize/Maximize \quad f_m(\overline{x}), \quad m = 1,2\ldots,M \\ Subject\ \ to \quad g_j(\overline{x}) \geq 0, \quad j = 1,2\ldots,J \\ \qquad\qquad\qquad h_k(\overline{x}) = 0, \quad k = 1,2\ldots,K \\ \qquad\qquad\qquad x_i^{(L)} \leq x_i \leq x_i^{(U)}, \quad i = 1,2\ldots,N \end{cases}$$

The set of constraints—both constraint functions (equalities $h_k(\overline{x})$) and inequalities $g_k(\overline{x})$) and decision variable bounds (lower bounds $x_i^{(L)}$ and upper bounds $x_i^{(U)}$)—define all possible (feasible) decision alternatives ($\mathcal{D}$).

The crucial concept of Pareto optimality is so called dominance relation that can be formulated as follows: to avoid problems with converting minimization to maximization problems (and vice versa of course) additional operator $\lhd$ can be introduced. Then, notation $\overline{x}_1 \lhd \overline{x}_2$ indicates that solution $\overline{x}_1$ is simply better than solution $\overline{x}_2$ for particular objective. It is said that solution $\overline{x}_A$ dominates solution $\overline{x}_B$ ($\overline{x}_A \prec \overline{x}_B$) then and only then if:

$$\overline{x}_A \prec \overline{x}_B \Leftrightarrow \begin{cases} f_j(\overline{x}_A) \ntriangleright f_j(\overline{x}_B) \quad for \quad j = 1,2\ldots,M \\ \exists i \in \{1,2,\ldots,M\} \quad : \quad \overline{x}_A \lhd \overline{x}_B \end{cases}$$

A solution in the Pareto sense of the multi-objective optimization problem means determination of all non-dominated alternatives from the set $\mathcal{D}$.

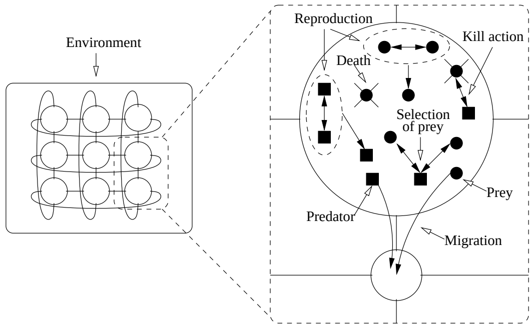

Fig. 1. CoEMAS with predator-prey mechanism

3 Co-Evolutionary Multi-Agent System with Predator-Prey Mechanism for Multi-Objective Optimization

The system presented in this paper is based on the CoEMAS model, which is the general model of co-evolution in multi-agent system. In order to maintain population diversity predator-prey co-evolutionary mechanism is used (see fig. 1). Prey represent solutions of the multi-objective problem. The main goal of predators is to eliminate "weak" (ie. dominated) prey.

The **co-evolutionary multi-agent system with predator-prey mechanism** is described as 4-tuple: $CoEMAS = \langle E, S, \Gamma, \Omega \rangle$ where S is the set of species ($s \in S$) that co-evolve in $CoEMAS$, Γ is the set of resource types that exist in the system, the amount of type γ resource will be denoted by r^{γ}, Ω is the set of information types that exist in the system, the information of type ω will be denoted by i^{ω}.

$E = \left\langle T^E, \Gamma^E, \Omega^E \right\rangle$ is the **environment** of the $CoEMAS$, where T^E is the topography of the environment E (directed graph with the cost function defined), Γ^E is the set of resource types that exist in the environment—in our case $\Gamma^E = \Gamma$, Ω^E is the set of information types that exist in the environment—in the described system $\Omega^E = \Omega$.

There are two **information types** ($\Omega = \{\omega_1, \omega_2\}$) and one resource type ($\Gamma = \{\gamma\}$) in *CoEMAS*. Informations of type ω_1 contain nodes to which agent can migrate, when it is located in particular node of the graph. Informations of type ω_2 contain agents-prey which are located in the particular node in time t. There is one **resource type** ($\Gamma = \{\gamma\}$) in *CoEMAS*, and there is closed circulation of resource within the system.

The **set of species** is given by: $S = \{prey, pred\}$. The prey species is defined as follows: $prey = \langle A^{prey}, SX^{prey} = \{sx\}, Z^{prey}, C^{prey} \rangle$, where A^{prey} is the set of agents that belong to the *prey* species, SX^{prey} is the set of sexes which exist within the *prey* species, Z^{prey} is the set of actions that agents of species *prey* can perform, and C^{prey} is the set of relations of species *prey* with other species that exist in the $CoEMAS$. There is only one sex sx ($sx \equiv sx^{prey}$) within the *prey* species, which is defined as follows: $sx = \langle A^{sx} = A^{prey}, Z^{sx} = Z^{prey}, C^{sx} = \emptyset \rangle$.

The **set of actions** $Z^{prey} = \{die, get, give, accept, seek, clone, rec, mut, migr\}$, where *die* is the action of death, which is performed when prey is out of resources, *get* action gets some resource from another a^{prey} agent located in the same node (this agent must be dominated by the agent that performs *get* action or is too close to him in the criteria space—*seek* action allows to find such agents), *give* actions gives some resource to another agent (which performs *get* action), *accept* action accepts partner for reproduction (partner is accepted when the amount of resource possessed by the prey agent is above the given level), *seek* action also allows the prey agent to find partner for reproduction when the amount of its resource is above the given level, *clone* is the action of cloning prey (new agent with the same genotype as parent's one is created), *rec* is the recombination operator (intermediate recombination is used [1]), *mut* is the mutation operator (mutation with self-adaptation is used [1]), *migr* action allows prey to migrate between the nodes of the graph (migrating agent loses some resource).

The **set of relations of** *prey* **species** with other species that exist within the system is defined as follows: $C^{prey} = \left\{ \xrightarrow{prey,get-} = \{\langle prey, prey\rangle\}, \xrightarrow{pred,give+} = \{\langle prey, pred\rangle\} \right\}$. The first relation models intra species competition for limited resources (prey can decrease ("-") the fitness of another prey with the use of *get* action). The second one models predator-prey interactions: prey gives all the resource it owes to predator (which fitness is increased: "+") and then dies.

The **predator species** (*pred*) is defined analogically as *prey* species with the following differences. The set of actions $Z^{pred} = \{seek, get, migr\}$, where *seek* action seeks for the "worst" (according to the criteria associated with the given predator) prey located in the same node, *get* action gets all resource from chosen prey, *migr* action is analogical as in the case of prey species. The **set of relations of** *pred* **species** with other species is limited to one relation, which models predator-prey interactions: $C^{pred} = \left\{ \xrightarrow{prey,get-} = \{\langle pred, prey\rangle\} \right\}$.

Agent a **of species** *prey* is given by: $a = \langle gn^a, Z^a = Z^{prey}, \Gamma^a = \Gamma, \Omega^a = \Omega, PR^a \rangle$. Genotype gn^a is consisted of two vectors (chromosomes): x of real-coded decision parameters' values and σ of standard deviations' values, which are used during mutation. $Z^a = Z^{prey}$ is the set of actions which agent a can perform. Γ^a is the set of resource types, and Ω is the set of information types, which agent can possess.

The set of profiles PR^a includes resource profile (pr_1, which goal is to maintain the amount of resource above the minimal level), reproduction profile (pr_2, which goal is agent's reproduction), interaction profile (pr_3, which goal is to interact with agents from the same and another species), and migration profile (pr_4, which goal is to migrate to another node). Each time step agent tries to realize goals of the profiles taking into account their priorities: $pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \trianglelefteq pr_4$ (pr_1 has the highest priority). In order to realize goal of the given profile agent uses actions which can be realized within the given profile. For example within pr_1 profile all actions connected with type γ resource (*die*, *seek*, *get*) can be used in order to realize the goal of this profile. This profile uses informations of type ω_2.

Agent a **of species** *pred* is defined analogically to *prey* agent. The main differences are genotype and the set of profiles. Genotype of agent a is consisted of the information about the criterion associated with this agent. The set of profiles PR^a includes only resource profile (pr_1, which goal is to "kill" prey and collect their resources), and migration profile (pr_2, which goal is to migrate within the environment).

4 Test Problems

The experimental and comparative studies presented in this paper are based on well known *Kursawe* multi-objective test problem (the formal definition may be found in [7]) and the problem of effective portfolio building.

The Pareto set and Pareto frontier for the *Kursawe* problem are presented in fig. 2. In this case optimization algorithm has to deal with disconnected two-dimensional Pareto frontier and disconnected three dimensional Pareto set. Additionally, a specific definition of f_1 and f_2 functions causes that even very small changes in the space of decision variables can cause big differences in the space of objectives. All of these causes that *Kursawe* problem is quite difficult for solving in general—and for solving using evolutionary techniques in particular.

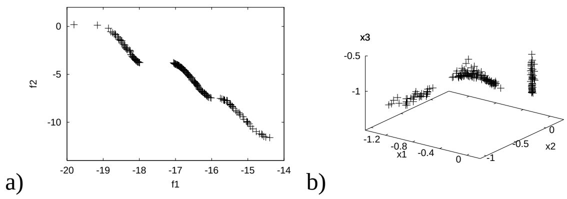

Fig. 2. *Kursawe test problem*: a) Pareto frontier and b) Pareto set

Proposed co-evolutionary agent-based approach has also been preliminarily assessed using the problem of effective portfolio building. Below, there are presented consecutive steps (based on the Sharp model) during computing the expectation of the risk level and generally speaking income expectation related to the wallet of p shares: 1. Computing of arithmetic means on the basis of rate of returns; 2. Computing the value of α coefficient: $\alpha_i = \overline{R_i} - \beta_i \overline{R_m}$, where R_i is the rate of return of i-th share, and R_m is the rate of return of market index; 3. Computing the value of β coefficient: $\beta_i = \frac{\sum_{t=1}^{n}(R_{it}-\overline{R_i})(R_{mt}-\overline{R_m})}{\sum_{t=1}^{n}(R_{mt}-\overline{R_m})^2}$, where n is the number of rate of return, R_{it} is the rate of return in the period t, R_{mt} is the rate of return related to market index in period t; 4. Computing the share expectation: $R_i = \alpha_i - \beta_i R_m + e_i$, where e_i is the random component of the equation; 5. Computing the variance of random index of the $i-th$ share: ${s_{e_i}}^2 = \frac{\sum_{t=1}^{n}(R_{it}-\alpha_i-\beta_i R_m)^2}{n-1}$; 6. Computing the variance of market index: ${s_m}^2 = \frac{\sum_{t=1}^{n}(R_{mt}-\overline{R_m})^2}{n-1}$; 7. Computing the risk level of the investing wallet: $risk = \beta_p^2 {s_m}^2 + {s_{e_p}}^2$, where $\beta_p = \sum_{i=1}^{p}(\omega_i\beta_i)$, p is the number of shares in the wallet, ω_i is the percentage participation of i-th share in the wallet, ${s_{e_p}}^2 = \sum_{i=1}^{p}(\omega_i^2 {s_{e_i}}^2)$ is the variance of the wallet; 8. Computing the investing wallet expectation: $R_p = \sum_{i=1}^{p}(\omega_i R_i)$. The goal of optimization is to maximize the investing wallet expectation along with minimizing the risk level. Model Pareto frontiers related to two cases taken into consideration in the course of this paper are presented in fig 3.

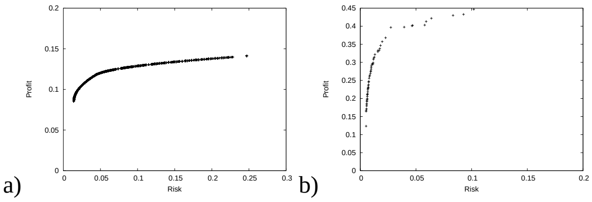

Fig. 3. *Building of effective portfolio*: the model Pareto frontier for a) 3 and b) 17 stocks set

5 Results of Experiments

As it was mentioned in sec. 4 proposed CoEMAS system with predator-prey mechanism has been evaluated using inter-alia Kursawe test problem. To give a kind of reference point, results obtained by CoEMAS are compared with results obtained by "classical" (i.e. non agent-based) predator-prey evolutionary strategy (PPES) [6] and another classical evolutionary algorithm for multi-objective optimization: niched pareto genetic algorithm (NPGA) [2]. In fig. 4 approximations of Pareto frontier obtained by all three algorithms are presented. As one may notice initially, i.e. after 1, 10 and partially after 20 (see fig. 4a, 4b and 4c) steps, Pareto frontiers obtained by all three algorithms are quite similar if the number of found non-dominated individuals, their distance to the model Pareto frontier and their dispersing over the whole Pareto frontier are considered. Afterwards yet, definitely higher quality of CoEMAS-based Pareto frontier approximation is more and more distinct. The NPGA-based Pareto frontier almost completely disappears after about 30 steps, and although PPES-based Pareto frontier is better and better this improving process is quite slow and not so clear as in the case of CoEMAS-based solution.

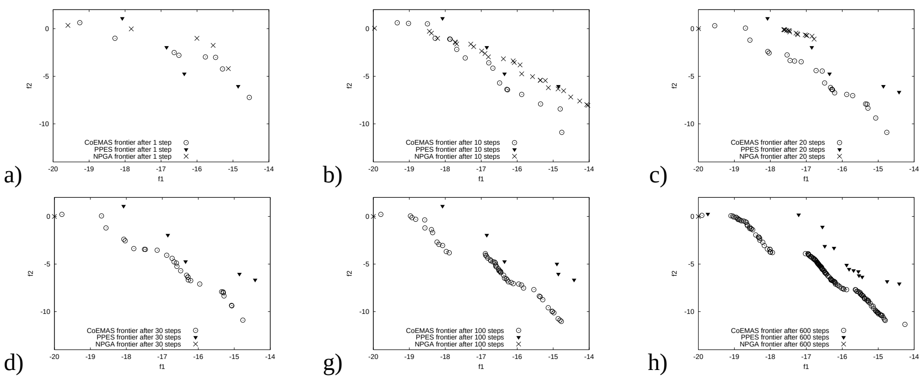

Fig. 4. Kursawe problem Pareto frontier approximations obtained by CoEMAS, PPES and NPGA after a) 1, b) 10, c) 20, d) 30, e) 100 and f) 600 steps

Because solutions presented in fig. 4 partially overlap, in fig. 5 there are presented separately Pareto frontiers obtained by analyzed algorithms after 2000, 4000 and 6000 time steps. There is no doubt that—what can be especially seen in fig. 5a, d and g—CoEMAS is definitely the best alternative since it is able to obtain Pareto frontier that is located very close to the model solution, that is very well dispersed and what is also very important—it is more numerous than PPES and NPGA-based solutions.

Table 1. The values of the *HV* and *HVR* metrics for compared systems (Kursawe problem)

HV / HVR			
Step	**CoEMAS**	**PPES**	**NPGA**
1	541.21 / 0.874	530.76 / 0.857	489.34 / 0.790
10	588.38 / 0.950	530.76 / 0.867	563.55 / 0.910
20	594.09 / 0.959	531.41 / 0.858	401.79 / 0.648
30	601.66 / 0.971	531.41 / 0.858	378.78 / 0.611
40	602.55 / 0.973	531.41 / 0.858	378.73 / 0.611
50	594.09 / 0.959	531.41 / 0.858	378.77 / 0.611
100	603.04 / 0.974	531.42 / 0.858	378.80 / 0.6117
600	603.79 / 0.975	577.44 / 0.932	378.80 / 0.611
200	611.43 / 0.987	609.47 / 0.984	378.80 / 0.611
4000	611.44 / 0.987	555.53 / 0.897	378.80 / 0.611
6000	613.10 / 0.990	547.73 / 0.884	378.80 / 0.611

It is of course quite difficult to compare algorithms only on the basis of qualitative results, so in Table 1 there are presented values of HV and HVR metrics ([2]) obtained during the experiments with Kursawe problem. The results presented in this table confirm that in the case of Kursawe problem CoEMAS is much better alternative than "classical" PPES or NPGA algorithms.

In the case of optimizing investing portfolio each individual in the prey population is represented as a p-dimensional vector. Each dimension represents the percentage participation of i-th ($i \in 1 \dots p$) share in the whole portfolio. Because of the space limitation in this paper only a kind of summary of two single experiments will be presented. During presented experiment quotations from 2003-01-01 until 2005-12-31 were taken into consideration. Simultaneously the portfolio consists of the following three (in experiment I) or seventeen (in experiment II) stocks quoted on the Warsaw Stock Exchange: in experiment I: RAFAKO, PONARFEH, PKOBP, in experiment II: KREDYTB, COMPLAND, BETACOM, GRAJEWO, KRUK, COMARCH, ATM, HANDLOWY, BZWBK, HYDROBUD, BORYSZEW, ARKSTEEL, BRE, KGHM, GANT, PROKOM, BPHPBK. As the market index WIG20 has been taken into consideration. In fig. 6 there are presented Pareto frontiers obtained using CoEMAS, NPGA and PPES algorithm after 100, 500 and 900 steps in experiment I. As one may notice in this case CoEMAS-based frontier is more numerous (especially initially) than NPGA-based and as numerous as PPES-based one. Unfortunately in this case diversity of population in CoEMAS approach is visibly worse than in the case of NPGA or PPES-based

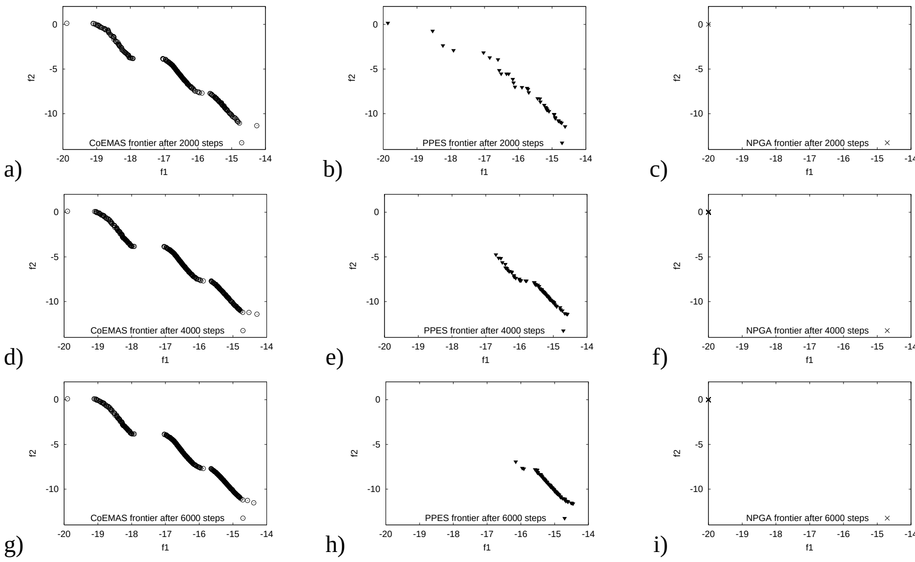

Fig. 5. Kursawe problem Pareto frontier approximations after 2000 (a), (b), (c), 4000 (d), (e),(f) and 6000 (g), (h), (i) steps obtained by CoEMAS, PPES, and NPGA

frontiers [1]. What is more, with time the tendency of CoEMAS-based solver for focusing solutions around small part of the whole Pareto frontier is more and more distinct. Similar situation can be also observed in fig. 7 presenting Pareto frontiers obtained by CoEMAS, NPGA and PPES—but this time portfolio that is being optimized consists of 17 shares. Also this time CoEMAS-based frontier is quite numerous and quite close to the model Pareto frontier but the tendency for focusing solutions around only selected part(s) of the whole frontier is very distinct [2].

6 Concluding Remarks

Co-evolutionary techniques for evolutionary algorithms are applicable in the case of problems for which it is difficult or impossible to formulate explicit fitness function, there is need for maintaining useful population diversity, forming species located in the basins of attraction of different local optima, or introducing open-ended evolution. Such techniques are also widely used in artificial life simulations. Although co-evolutionary algorithms has been recently the subject of intensive research their application to multi-modal and multi-objective optimization is still the open problem and many questions remain unanswered.

[1] It is also confirmed by values of HV or HVR metrics, but because of space limitations these characteristics are omitted in this paper.

[2] It is also confirmed by values of appropriate metrics but as it was said those characteristics are omitted in this paper.

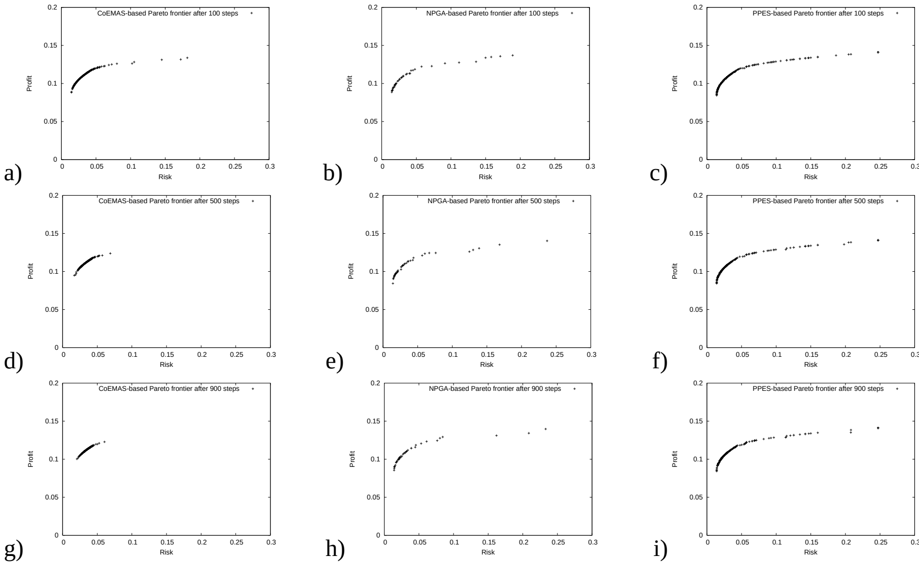

Fig. 6. Pareto frontier approximations after 100 (a), (b), (c), 500 (d), (e),(f), and 900 (g), (h), (i) steps obtained by CoEMAS, PPES, and NPGA for building effective portfolio consisting of 3 stocks

In this paper the agent-based realization of predator-prey model within the more general framework of *co-evolutionary multi-agent system* has been presented. The system was run against Kursawe test problem and hard real-life multi-objective problem—effective portfolio building—and then compared to two classical multi-objective evolutionary algorithms: PPES and NPGA. In the case of difficult Kursawe test problem CoEMAS with predator-prey mechanism properly located Pareto frontier, the useful population diversity was maintained and the individuals were evenly distributed over the whole frontier. In the case of this test problem the results obtained with the use of proposed system was clearly better than in the case of two other "classical" algorithms. It seems that the proposed predator-prey mechanism for evolutionary multi-agent systems may be very useful in the case of hard dynamic and multi-modal multi-objective problems (as defined by Deb [2]). In the case of effective portfolio building problem CoEMAS was able to form more numerous frontier, however negative tendency to lose population diversity during the experiment was observed. In this case PPES and NPGA were able to form better dispersed Pareto frontiers. The results of experiments show that still more research is needed on co-evolutionary mechanisms for maintaining population diversity used in CoEMAS, especially when we want to stably maintain diversity of solutions. Future work will include more detailed analysis of proposed co-evolutionary mechanisms, especially focused on problems of stable maintaining population diversity. Also the comparison of CoEMAS to other classical multi-objective evolutionary algorithms with the use of hard multi-modal multi-objective test problems, and the appli-

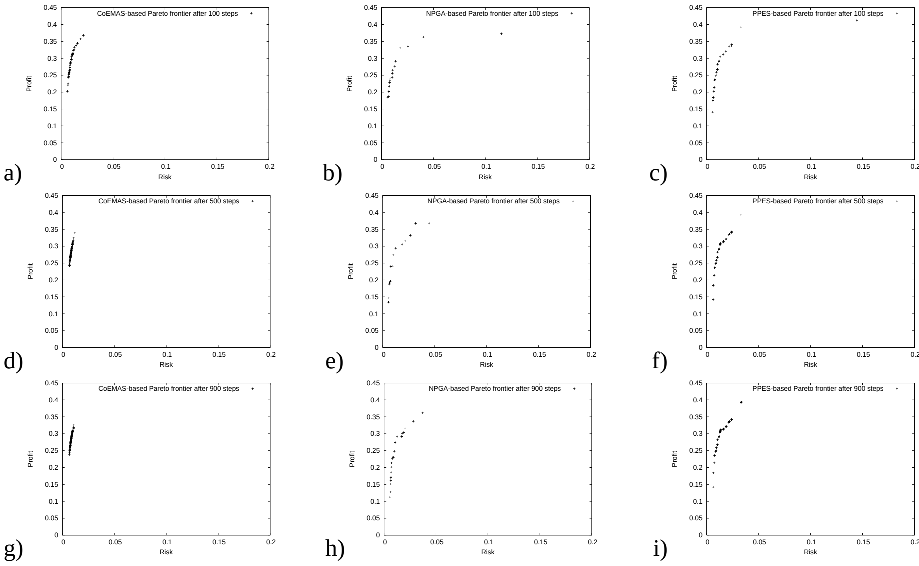

Fig. 7. Pareto frontier approximations after 100 (a), (b), (c), 500 (d), (e),(f), and 900 (g), (h), (i) steps obtained by CoEMAS, PPES, and NPGA for building effective portfolio consisting of 17 stocks

cation of other co-evolutionary mechanisms like symbiosis (co-operative co-evolution) are included in future plans.

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