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Agent-Based Co-Evolutionary Techniques for Solving Multi-Objective Optimization Problems

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1. Introduction

Evolutionary algorithms (EAs) are optimization and search techniques inspired by the Darwinian model of biological evolutionary processes (Bäck et al., 1997). EAs are robust and efficient techniques, which find approximate solutions to many problems which are difficult or even impossible to solve with the use of "classical" techniques. There are many different types of evolutionary algorithms developed during over 40 years of research.

One of the branches of EAs are *co-evolutionary algorithms (CEAs)* (Paredis, 1998). The main difference between EAs and CEAs is the way in which the fitness of an individual is evaluated in each approach. In the case of evolutionary algorithms each individual has the solution of the given problem encoded within its genotype and its fitness depends only on how "good" is that solution. In the case of co-evolutionary algorithms of course there is also obviously solution to the given problem encoded within the individual's genotype but the fitness is estimated on the basis of interactions of the given individual with other individuals present in the population. Thus co-evolutionary algorithms are applicable in the case of problems for which it is difficult or even impossible to formulate explicit fitness function—in such cases we can just encode the solutions within the individuals' genotypes and individuals compete—or co-operate—with each other, and such process of interactions leads to the fitness estimation. Co-evolutionary interactions between individuals have also other positive effects. One of them is maintaining the population diversity, another one are "arms races"—continuous "progress" toward better and better solutions to the given problem via competition between species.

Co-evolutionary algorithms are classified into two general categories: competitive and cooperative (Paredis, 1998). The main difference between these two types of co-evolutionary algorithms is the way in which the individuals interact during the fitness estimation. In the case of competitive co-evolutionary algorithms the value of fitness is estimated as a result of the series of tournaments, in which the individual for which the fitness is estimated and some other individuals from the population are engaged. The way of choosing the competitors for tournaments may vary in different versions of algorithms—for example it may be the competition with the best individual from the other species or competition with several randomly chosen individuals, etc.

On the other hand, co-operative co-evolutionary algorithms (CCEAs) are CEAs in which there exist several sub-populations (species) (Potter & De Jong, 2000). Each of them solves

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only one sub- problem of the given problem. In such a case the whole solution is the group of individuals composed of the representants of all sub-populations. Individuals interact only during the fitness estimation process. In order to evaluate the given individual, representants from the other sub-populations are chosen (different ways of choosing such representants may be found in (Potter & De Jong, 2000)). Within the group the given individual is evaluated in such a way that the fitness value of the whole solution (group) becomes the fitness value of the given individual. Individuals coming from the same species are evaluated within the group composed of the same representants of other species.

Sexual selection is another mechanism used for maintaining population diversity in EAs. Sexual selection results from the co-evolution of female mate choice and male displayed trait (Gavrilets & Waxman, 2002). Sexual selection is considered to be one of the ecological mechanisms responsible for biodiversity and sympatric speciation (Gavrilets &Waxman, 2002; Todd & Miller, 1997). The research on sexual selection mechanism generally concentrated on two aspects. The first one was modeling and simulation of sexual selection as speciation mechanism and population diversity mechanism (for example see (Gavrilets &Waxman, 2002; Todd & Miller, 1997)). The second one was the application of sexual selection in evolutionary algorithms as a mechanism for maintaining population diversity. The applications of sexual selection include multi-objective optimization (Allenson, 1992; Lis & Eiben, 1996) and multimodal optimization (Ratford et al., 1997).

In the case of evolutionary multi-objective optimization (Deb, 1999), high quality approximation of *Pareto frontier* (basic ideas of multi-objective optimization are introduced in Section 2) should fulfill at least three distinguishing features. First of all, the population should be "located" as close to the ideal Pareto frontier as possible. Secondly it should include as many alternatives (individuals) as possible and, last but not least, all proposed non-dominated alternatives should be evenly distributed over the whole true Pareto set. In the case of multi-objective optimization maintaining of population diversity plays the crucial role. Premature loss of population diversity can result not only in lack of drifting to the true Pareto frontier but also in obtaining approximation of Pareto set that is focused around its selected area(s), what is very undesirable. In the case of multi-objective problems with many local Pareto frontiers (so called "multi-modal multi-objective problems" defined by Deb in (Deb, 1999)) the loss of population diversity may result in locating only a local Pareto frontier instead of a global one.

Co-evolutionary multi-agent systems (CoEMAS) are the result of research on decentralized models of co-evolutionary computations. CoEMAS model is the extension of "basic" model of evolution in multi-agent system—*evolutionary multi-agent systems (EMAS)* (Cetnarowicz et al., 1996). The basic idea of such an approach is the realization of evolutionary processes in multi-agent system—the population of agents evolves, agents live within the environment, they can reproduce, die, compete for resources, observe the environment, communicate with other agents, and make autonomously all their decisions concerning reproduction, choosing partner for reproduction, and so on. Co-evolutionary multi-agent systems additionally allow us to define many species and sexes of agents and to introduce interactions between them (Dreżewski, 2003).

All these features lead to completely decentralized evolutionary processes and to the class of systems that have very interesting features. It seems that the most important of them are the following:

- synchronization constraints of the computations are relaxed because the evolutionary processes are decentralized—individuals are agents, which act independently and do not need synchronization,
- there exists the possibility of constructing hybrid systems using many different computational intelligence techniques within one single, coherent multi-agent architecture,
- there are possibilities of introducing new evolutionary and social mechanisms, which were hard or even impossible to introduce in the case of classical evolutionary algorithms.

The possible areas of application of CoEMAS include multi-modal optimization (for example see (Dreżewski, 2006)), multi-objective optimization (the review of selected results is presented in this chapter), and modeling and simulation of social and economical phenomena.

This chapter starts with the overview of multi-objective optimization problems. Next, introduction to the basic ideas of CoEMAS systems—the general model of co-evolution in multi-agent system—is presented. In the following parts of the chapter the agent-based co-evolutionary systems for multi-objective optimization are presented. Each system is described with the use of notions and formalisms introduced in the general model of coevolution in multi-agent system. Each of the presented systems uses different coevolutionary interactions and mechanisms: sexual selection mechanism, and host-parasite co-evolution. For all the systems results of experiments with commonly used multi-objective test problems are presented. The results obtained during the experiments are the basis for comparisons of agent-based co-evolutionary techniques with "classical" evolutionary approaches.

2. An introduction to multi-objective optimization

During most real-life decision processes many different (often contradictory) factors have to be considered, and the decision maker has to deal with an ambiguous situation: the solutions which optimize one criterion may prove insufficiently good considering the others. From the mathematical point of view such multi-objective (or multi-criteria) problem can be formulated as follows (Coello Coello et al., 2007; Abraham et al., 2005; Zitzler, 1999; Van Veldhuizen, 1999).

Let the problem variables be represented by a real-valued vector:

$$\vec{x} = [x_1, x_2, \ldots, x_m]^T \in \mathbb{R}^m \tag{1}$$

where m is the number of variables. Then a subset of $\mathbb{R}^m$ of all possible (feasible) decision alternatives (options) can be defined by a system of:

- inequalities (constraints): $g_k(\vec{x}) \geq 0$ and $k = 1, 2, \ldots, K$
- equalities (bounds): $h_l(\vec{x}) = 0$, $l = 1, 2, \ldots, L$

and denoted by $\mathcal{D}$. The alternatives are evaluated by a system of n functions (objectives) denoted here by vector $F = [f_1, f_2, \ldots, f_n]^T$:

$$f_i : \mathbb{R}^m \to \mathbb{R}, \quad i = 1, 2, \ldots, n \tag{2}$$

Because there are many criteria–to indicate which solution is better than the other–specialized ordering relation has to be introduced. To avoid problems with converting minimization to maximization problems (and vice versa of course) additional operator $\lhd$ can be defined. Then, notation $\overline{x}_1 \lhd \overline{x}_2$ indicates that solution $\overline{x}_1$ is simply better than solution $\overline{x}_2$ for particular objective. Now, the crucial concept of Pareto optimality (what is the subject of our research) i.e. so called dominance relation can be defined. It is said that solution $\overline{x}_A$ dominates solution $\overline{x}_B$ ($\overline{x}_A \prec \overline{x}_B$) if and only if:

$$\overline{x}_A \prec \overline{x}_B \Leftrightarrow \begin{cases} f_j(\overline{x}_A) \not\rhd f_j(\overline{x}_B) \quad for \quad j = 1, 2 \ldots, n \\ \exists i \in \{1, 2, \ldots, n\} : \quad f_i(\overline{x}_A) \lhd f_i(\overline{x}_B) \end{cases}$$

A solution in the Pareto sense of the multi-objective optimization problem means determination of all non-dominated alternatives from the set $\mathcal{D}$. The Pareto-optimal set consists of globally optimal solutions and is defined as follows. The set $\mathcal{P} \subseteq D$ is global Pareto-optimal set if (Zitzler, 1999):

$$\forall \vec{x}^a \in \mathcal{P} : \nexists \vec{x}^b \in D \text{ such that } \vec{x}^b \geq \vec{x}^a \tag{3}$$

There may also exist locally optimal solutions, which constitute locally non-dominated set (*local Pareto-optimal set*) (Deb, 2001). The set $\mathcal{P}_{local} \subseteq D$ is local Pareto-optimal set if (Zitzler, 1999):

$$\begin{aligned} \forall \vec{x}^a \in \mathcal{P}_{local} : \nexists \vec{x}^b \in D \text{ such that} \\ \vec{x}^b \geq \vec{x}^a \wedge \|\vec{x}^b - \vec{x}^a\| < \varepsilon \wedge \|F(\vec{x}^b) - F(\vec{x}^a)\| < \delta \end{aligned}$$

where $\|\cdot\|$ is a distance metric and $\varepsilon > 0$, $\delta > 0$.

These locally or globally non-dominated solutions define in the criteria space so-called local ($\mathcal{PF}_{local}$) or global ($\mathcal{PF}$) Pareto frontiers that can be defined as follows:

$$\mathcal{PF}_{local} = \{\vec{y} = F(\vec{x}) \in \mathbb{R}^n \mid \vec{x} \in \mathcal{P}_{local}\} \tag{4a}$$

$$\mathcal{PF} = \{\vec{y} = F(\vec{x}) \in \mathbb{R}^n \mid \vec{x} \in \mathcal{P}\} \tag{4b}$$

Multi-objective problems with one global and many local Pareto frontiers are called *multimodal multi-objective problems* (Deb, 2001).

3. General model of co-evolution in multi-agent system

As it was said, co-evolutionary multi-agent systems are the result of research on decentralized models of evolutionary computations which resulted in the realization of evolutionary processes in multi-agent system and the formulation of model of co-evolution in such system. The basic elements of CoEMAS are environment with some topography, agents (which are located and can migrate within the environment, which are able to reproduce, die, compete for limited resources, and communicate with each other), the selection mechanism based on competition for limited resources, and some agent-agent and agent-environment relations defined (see Fig. 1).

The selection mechanism in such systems is based on the resources defined in the system. Agents collect such resources, which are given to them by the environment in such a way that "better" agents (i.e. which have "better" solutions encoded within their genotypes) are given more resources and "worse" agents are given less resources. Agents then use such resources for every activity (like reproduction and migration) and base all their decisions on the possessed amount of resources.

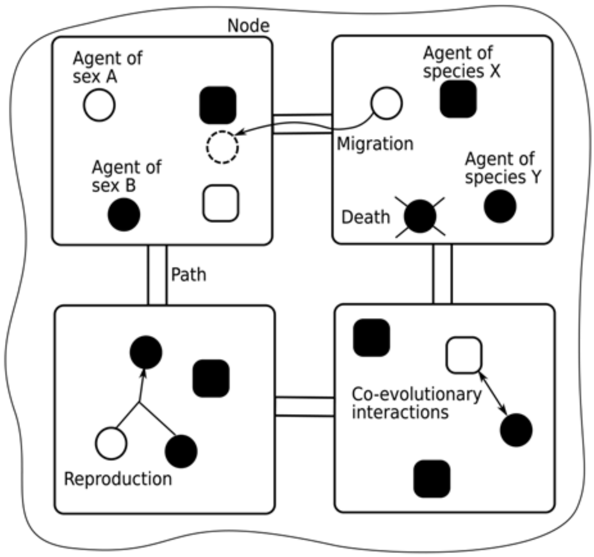


Fig. 1. The idea of co-evolutionary multi-agent system

In this section the general model of co-evolution in multi-agent system (CoEMAS) is presented. We will formally describe the basic elements of such systems and present the algorithm of agent's basic activities.

3.1 The co-evolutionary multi-agent system

The *CoEMAS* is described as 4-tuple:

$$CoEMAS = \langle E, S, \Gamma, \Omega \rangle \tag{5}$$

where E is the environment of the $CoEMAS$, S is the set of species ($s \in S$) that co-evolve in $CoEMAS$, Γ is the set of resource types that exist in the system, the amount of type γ resource will be denoted by r^{γ}, Ω is the set of information types that exist in the system, the information of type ω will be denoted by i^{ω}.

3.2 The environment

The environment of *CoEMAS* may be described as 3-tuple:

$$E = \left\langle T^E, \Gamma^E, \Omega^E \right\rangle \tag{6}$$

where T^E is the topography of environment E, Γ^E is the set of resource types that exist in the environment, Ω^E is the set of information types that exist in the environment. The topography of the environment is given by:

$$T^E = \langle H, l \rangle \tag{7}$$

where H is directed graph with the cost function c defined: $H = \langle V, B, c \rangle$, V is the set of vertices, B is the set of arches. The distance between two nodes is defined as the length of the shortest path between them in graph H.
The l function makes it possible to locate particular agent in the environment space:

$$l: \quad A \rightarrow V \tag{8}$$

where A is the set of agents, that exist in $CoEMAS$.
Vertice v is given by:

$$v = \langle A^v, \Gamma^v, \Omega^v, \varphi \rangle \tag{9}$$

A^v is the set of agents that are located in the vertice v, Γ^v is the set of resource types that exist within the v ($\Gamma^v \subseteq \Gamma^E$), Ω^v is the set of information types that exist within the v ($\Omega^v \subseteq \Omega^E$), φ is the fitness function.

3.3 The species

Species $s \in S$ is defined as follows:

$$s = \langle A^s, SX^s, Z^s, C^s \rangle \tag{10}$$

where:

- A^s is the set of agents of species s (by a^s we will denote the agent, which is of species s, $a^s \in A^s$);
- SX^s is the set of sexes within the s;
- Z^s is the set of actions, which can be performed by the agents of species s ($Z^s = \bigcup_{a \in A^s} Z^a$, where Z^a is the set of actions, which can be performed by the agent a);
- C^s is the set of relations with other species that exist within $CoEMAS$.

The set of relations of s_i with other species (C^{s_i}) is the sum of the following sets of relations:

$$C^{s_i} = \left\{ \xrightarrow{s_i, z-} : z \in Z^{s_i} \right\} \cup \left\{ \xrightarrow{s_i, z+} : z \in Z^{s_i} \right\} \tag{11}$$

where $\xrightarrow{s_i, z-}$ and $\xrightarrow{s_i, z+}$ are relations between species, based on some actions $z \in Z^{s_i}$, which can be performed by the agents of species s_i:

$$\xrightarrow{s_i, z-} = \{ \langle s_i, s_j \rangle \in S \times S\text{: agents of species } s_i \text{ can decrease the fitness of agents of species } s_j \text{ by performing the action } z \in Z^{s_i} \} \tag{12}$$

$$\xrightarrow{s_i, z+} = \{ \langle s_i, s_j \rangle \in S \times S\text{: agents of species } s_i \text{ can increase the fitness of agents of species } s_j \text{ by performing the action } z \in Z^{s_i} \} \tag{13}$$

If $s_i \xrightarrow{s_i, z_k -} s_i$ then we are dealing with the intra-species competition, for example the competition for limited resources, and if $s_i \xrightarrow{s_i, z_l +} s_i$ then there is some form of co-operation within the species s_i.

With the use of the above relations we can define many different co-evolutionary interactions e.g.: predator-prey, host-parasite, mutualism, etc. For example, host-parasite interactions between two species, s_i (parasites) and s_j (hosts) ($i \neq j$) take place if and only if $\exists z_k \in Z^{s_i} \wedge \exists z_l \in Z^{s_i}$, such that $s_i \xrightarrow{s_i, z_k -} s_j$ and $s_j \xrightarrow{s_j, z_l +} s_i$, and parasite can only live in tight co-existence with the host.

3.4 The sex

The sex $sx \in SX^s$ which is within the species s is defined as follows:

$$sx = \langle A^{sx}, Z^{sx}, C^{sx} \rangle \tag{14}$$

where A^{sx} is the set of agents of sex sx and species s ($A^{sx} \subseteq A^s$):

$$A^{sx} = \left\{ a : a \in A^s \wedge a \text{ is the agent of sex } sx \right\} \tag{15}$$

With a^{sx} we will denote the agent of sex sx ($a^{sx} \in A^{sx}$). Z^{sx} is the set of actions which can be performed by the agents of sex sx, $Z^{sx} = \bigcup_{a \in A^{SX}} Z^a$, where Z^a is the set of actions which can be performed by the agent a. And finally C^{sx} is the set of relations between the sx and other sexes of the species s.

Analogically as in the case of species, we can define the relations between the sexes of the same species. The set of all relations of the sex $sx_i \in SX^s$ with other sexes of species s (C^{sx_i}) is the sum of the following sets of relations:

$$C^{sx_i} = \left\{ \xrightarrow{sx_i, z-} : z \in Z^{sx_i} \right\} \cup \left\{ \xrightarrow{sx_i, z+} : z \in Z^{sx_i} \right\} \tag{16}$$

where $\xrightarrow{sx_i, z-}$ and $\xrightarrow{sx_i, z+}$ are the relations between sexes, in which some actions $z \in Z^{sx_i}$ are used:

$$\xrightarrow{sx_i, z-} = \{ \langle sx_i, sx_j \rangle \in SX^s \times SX^s : \text{agents of sex } sx_i \text{ can decrease the fitness of agents of sex } sx_j \text{ by performing the action } z \in Z^{sx_i} \} \tag{17}$$

$$\xrightarrow{sx_i, z+} = \{ \langle sx_i, sx_j \rangle \in SX^s \times SX^s : \text{agents of sex } sx_i \text{ can increase the fitness of agents of sex } sx_j \text{ by performing the action } z \in Z^{sx_i} \} \tag{18}$$

If performing the action $z_k \in Z^{sx_i}$ (which permanently or temporally increases the fitness of the agent a^{sx_j} of sex $sx_j \in SX^s$) by the agent a^{sx_i} of sex $sx_i \in SX^s$ results in performing the action $z_l \in Z^{sx_i}$ by the agent a^{sx_i} and performing the action $z_m \in Z^{sx_j}$ by the agent a^{sx_j}, what results in decreasing of the fitness of agents a^{sx_i} and a^{sx_j} then such relation $\xrightarrow[z_l -, z_m -]{sx_i, z_k +}$ will be defined in the following way:

$$\xrightarrow[z_l-,z_m-]{sx_i,z_k+} = \{\langle sx_i, sx_j\rangle \in SX^s \times SX^s : \text{agents of sex } sx_i \text{ can increase (permanently or temporally) the fitness of the agents of sex } sx_j\text{, by performing the action } z_k \in Z^{sx_i}\text{, which results in performing the action } z_l \in Z^{sx_i} \text{ and the action } z_m \in Z^{sx_j}\text{, which decrease the fitness of the agents of sex } sx_i \text{ and } sx_j\} \quad (19)$$

Such relation represents the sexual selection mechanism, where the action $z_k \in Z^{sx_i}$ is the action of choosing the partner for reproduction, the action $z_l \in Z^{sx_i}$ is the action of reproduction performed by the agent of sex sx_i (with high costs associated with it) and the $z_m \in Z^{sx_j}$ is the action of reproduction performed by the agent of sex sx_j (with lower costs than in the case of z_i action).

3.5 Agent

Agent a (see Fig. 2) of sex sx and species s (in order to simplify the notation we assume that $a \equiv a^{sx,s}$) is defined as follows:

$$a = \langle gn^a, Z^a, \Gamma^a, \Omega^a, PR^a \rangle \quad (20)$$

where:

- gn^a is the genotype of agent a, which may be composed of any number of chromosomes (for example: $gn^a = \langle (x_1, x_2, \ldots, x_k) \rangle$, where $x_i \in \mathbb{R}$, $gn^a \in \mathbb{R}^k$
- Z^a is the set of actions, which agent a can perform;
- Γ^a is the set of resource types, which are used by agent a ($\Gamma^a \subseteq \Gamma$);
- Ω^a is the set of informations, which agent a can possess and use ($\Omega^a \subseteq \Omega$);
- PR^a is partially ordered set of profiles of agent a ($PR^a \equiv \langle PR^a, \trianglelefteq \rangle$) with defined partial order relation $\trianglelefteq$.

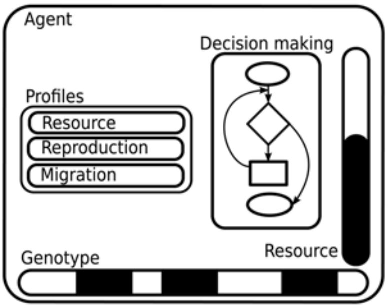


Fig. 2. Agent in the *CoEMAS*

Relation $\trianglelefteq$ is defined in the following way:

$$\trianglelefteq = \{\langle pr_i, pr_j\rangle \in PR^a \times PR^a : \text{realization of active goals of profile } pr_i \text{ has equal or higher priority than the realization of active goals of profile } pr_j\} \quad (21)$$

The active goal (which is denoted as gl^*) is the goal gl, which should be realized in the given time. The relation $\trianglelefteq$ is reflexive, transitive and antisymmetric and partially orders the set PR^a:

$$pr \trianglelefteq pr \qquad \text{for every } pr \in PR^a \tag{22a}$$

$$(pr_i \trianglelefteq pr_j \wedge pr_j \trianglelefteq pr_k) \Rightarrow pr_i \trianglelefteq pr_k \qquad \text{for every } pr_i, pr_j, pr_k \in PR^a \tag{22b}$$

$$(pr_i \trianglelefteq pr_j \wedge pr_j \trianglelefteq pr_i) \Rightarrow pr_i = pr_k \qquad \text{for every } pr_i, pr_j \in PR^a \tag{22c}$$

The set of profiles PR^a is defined in the following way:

$$PR^a = \{pr_1, pr_2, \ldots, pr_n\} \tag{23a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq \cdots \trianglelefteq pr_n \tag{23b}$$

Profile pr_1 is the basic profile—it means that the realization of its goals has the highest priority and they will be realized before the goals of other profiles.
Profile pr of agent a ($pr \in PR^a$) can be the profile in which only resources are used:

$$pr = \langle \Gamma^{pr}, ST^{pr}, RST^{pr}, GL^{pr} \rangle \tag{25}$$

in which only informations are used:

$$pr = \langle \Omega^{pr}, M^{pr}, ST^{pr}, RST^{pr}, GL^{pr} \rangle \tag{26}$$

or resources and informations are used:

$$pr = \langle \Gamma^{pr}, \Omega^{pr}, M^{pr}, ST^{pr}, RST^{pr}, GL^{pr} \rangle \tag{27}$$

where:

- Γ^{pr} is the set of resource types, which are used within the profile pr ($\Gamma^{pr} \subseteq \Gamma^a$);
- Ω^{pr} is the set of information types, which are used within the profile pr ($\Omega^{pr} \subseteq \Omega^a$);
- M^{pr} is the set of informations, which represent the agent's knowledge about the environment and other agents (it is the model of the environment of agent a);
- ST^{pr} is the partially ordered set of strategies ($ST^{pr} \equiv \langle ST^{pr}, \preccurlyeq \rangle$),which can be used by agent within the profile pr in order to realize an active goal of this profile;
- RST^{pr} is the set of strategies that are realized within the profile pr—generally, not all of the strategies from the set ST^{pr} have to be realized within the profile pr, some of them may be realized within other profiles;
- GL^{pr} is partially ordered set of goals ($GL^{pr} \equiv \langle GL^{pr}, \preccurlyeq \rangle$), which agent has to realize within the profile pr.

The relation $\preccurlyeq$ is defined in the following way:

$$\preccurlyeq = \{\langle st_i, st_j \rangle \in ST^{pr} \times ST^{pr} : \text{ strategy } st_i \text{ has equal or higher priority than strategy } st_j\} \tag{27}$$

This relation is reflexive, transitive and antisymmetric and partially orders the set ST^{pr}. Every single strategy $st \in ST^{pr}$ is consisted of actions, which ordered performance leads to the realization of some active goal of the profile *pr*:

$$st = \langle z_1, z_2, \ldots, z_k \rangle, \quad st \in ST^{pr}, \quad z_i \in Z^a \tag{28}$$

The relation $\preccurlyeq$ is defined in the following way:

$$\preccurlyeq \; = \{\langle gl_i, gl_j \rangle \in GL^{pr} \times GL^{pr} : \text{ goal } gl_i \text{ has equal or higher priority than the goal } gl_j\} \tag{29}$$

This relation is reflexive, transitive and antisymmetric and partially orders the set GL^{pr}.
The partially ordered sets of profiles PR^a, goals GL^{pr} and strategies ST^{pr} are used by the agent in order to make decisions about the realized goal and to choose the appropriate strategy in order to realize that goal. The basic activities of the agent *a* are shown in Algorithm 1.

Algorithm 1. Basic activities of agent *a* in *CoEMAS*

$r^\gamma \leftarrow r^\gamma_{init}$; /* r^γ_{init} is the initial amount of resource given to the agent */
while $r^\gamma > 0$ **do**
  activate the profile $pr_i \in PR^a$ with the highest priority and with the active goal $gl^*_j \in GL^{pr_i}$;
  if *pr_i is the resource profile* **then**
    if $0 < r^\gamma < r^\gamma_{min}$ **then** ; /* r^γ_{min} is the minimal amount of resource needed by the agent to realize its activities */
      choose the strategy $st_k \in ST^{pr_i}$ with the highest priority that can be used to take some resources from the environment or other agent;
      perform actions contained within the st_k;
    else if $r^\gamma = 0$ **then**
      execute $\langle die \rangle$ strategy;
    end
  else if *pr_i is the reproduction profile* **then**
    if $r^\gamma > r^{rep,\gamma}_{min}$ **then** ; /* $r^{rep,\gamma}_{min}$ is the minimal amount of resource needed for reproduction */
      choose the strategy $st_k \in ST^{pr_i}$ with the highest priority that can be used to reproduce;
      perform actions contained within the st_k;
    end
  else if *pr_i is the migration profile* **then**
    if $r^\gamma > r^{mig,\gamma}_{min}$ **then** ; /* $r^{mig,\gamma}_{min}$ is the minimal amount of resource needed for migration */
      choose the strategy $st_k \in ST^{pr_i}$ with the highest priority that can be used to migrate;
      perform actions contained within the st_k;
      give $r^{mig,\gamma}_{min}$ amount of resource to the environment;
    end
  end
end

In *CoEMAS* systems the set of profiles is usually composed of resource profile (pr_1), reproduction profile (pr_2), and migration profile (pr_3):

$$PR^a = \{pr_1, pr_2, pr_3\} \tag{30a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \tag{30b}$$

The highest priority has the resource profile, then there is reproduction profile, and finally migration profile.

4. Co-evolutionary multi-agent systems for multi-objective optimization

In this section we will describe two co-evolutionary multi-agent systems used in the experiments. Each of these systems uses different co-evolutionary mechanism: sexual selection, and host-parasite interactions. All of the systems are based on general model of co-evolution in multi-agent system described in Section 3—in this section only such elements of the systems will be described that are specific for these instantiations of the general model. In all the systems presented below, real-valued vectors are used as agents' genotypes. Mutation with self-adaptation and intermediate recombination are used as evolutionary operators (Bäck et al., 1997).

4.1 Co-evolutionary multi-agent system with sexual selection mechanism (SCoEMAS)

The co-evolutionary multi-agent system with sexual selection mechanism is described as 4-tuple (see Eq. (5)):

$$CoEMAS = \langle E, S, \Gamma = \{\gamma\}, \Omega = \{\omega_1, \omega_2\}\rangle \tag{31}$$

The informations of type ω_1 represent all nodes connected with the given node. The informations of type ω_2 represent all agents located within the given node.

4.1.1 Species

The set of species $S = \{s\}$. The only species s is defined as follows:

$$s = \langle A^s, SX^s, Z^s, C^s\rangle \tag{32}$$

where SX^s is the set of sexes which exist within the s species, Z^s is the set of actions that agents of species s can perform, and C^s is the set of relations of s species with other species that exist in the *SCoEMAS*.

Actions The set of actions Z^s is defined as follows:

$$\begin{aligned} Z^s = \{&die, searchDominated, get, giveDominating, searchPartner, choose,\\ &clone, rec, mut, give, accept, selNode, migr\} \end{aligned} \tag{33}$$

where:

- *die* is the action of death (agent dies when it is out of resources);
- *searchDominated* finds the agents that are dominated by the given agent;
- *get* is used to get the resources from a dominated agent;

- *giveDominating* gives some resources to the dominating agent;
- *searchPartner* is used to find candidates for reproduction partners;
- *choose* realizes the mechanism of sexual selection—the partner is chosen on the basis of individual preferences;
- *clone* is used to make the new agent—offspring;
- *rec* realizes the recombination (intermediate recombination is used (Bäck et al., 1997));
- *mut* realizes the mutation (mutation with self-adaptation is used (Bäck et al., 1997));
- *give* is used to give the offspring some amount of the parent's resources;
- *accept* action accepts the agent performing *choose* action as the partner for reproduction;
- *selNode* chooses the node (from the nodes connected with the current node) to which the agent will migrate;
- *migr* allows the agent to migrate from the given node to another node of the environment. The migration causes the lose of some amount of the agent's resources.

Relations The set of relations is defined as follows:

$$C^s = \left\{ \xrightarrow{s,get-} \right\} \tag{34}$$

The relation models intra species competition for limited resources ("-" denotes that as a result of performing *get* action the fitness of another agent of species *s* is decreased):

$$\xrightarrow{s,get-} = \{\langle s, s\rangle\} \tag{35}$$

4.1.2 The sexes

The number of sexes within the *s* species corresponds with the number of criteria (n) of the multi-objective problem being solved:

$$SX^s = \{sx_1, \ldots, sx_n\} \tag{36}$$

Actions The set of actions of sex *sx* is defined in the following way: $Z^{sx} = Z^s$.
Relations The set of relations of sex sx_i is defined as follows:

$$C^{sx_i} = \left\{ \xrightarrow[give-,give-]{sx_i,choose+} \right\} \tag{37}$$

The relation $\xrightarrow[give-,give-]{sx_i,choose+}$ realizes the sexual selection mechanism (see Eq. (19)). Each agent has its own preferences, which are composed of the vector of weights (each weight for one of the criteria of the problem being solved). These individual preferences are used during the selection of partner for reproduction (*choose* action).

4.1.3 The agent

Agent *a* of sex *sx* and species *s* (in order to simplify the notation we assume that $a \equiv a^{sx,s}$) is defined as follows:

$$a = \langle gn^a, Z^a = Z^s, \Gamma^a = \Gamma, \Omega^a = \Omega, PR^a \rangle \tag{38}$$

In the case of *SCoEMAS* system the genotype of each agent is composed of three vectors (chromosomes): $\vec{x}$ of real-coded decision parameters' values, $\vec{\sigma}$ of standard deviations' values, which are used during mutation with self-adaptation, and $\vec{w}$ of weights used during selecting partner for reproduction ($gn^a = \langle \vec{x}, \vec{\sigma}, \vec{w} \rangle$). Basic activities of agent *a* with the use of profiles are presented in Alg. 2.

Algorithm 2. Basic activities of agent a in $SCoEMAS$

$r^\gamma \leftarrow r^\gamma_{init}$;
while $r^\gamma > 0$ **do**
  activate the profile $pr_i \in PR^a$ with the highest priority and with the active goal $gl^*_j \in GL^{pr_i}$;
  if pr_1 *is activated* **then**
    if $0 < r^\gamma < r^\gamma_{min}$ **then**
      $\langle searchDominated, get \rangle$;
      $r^\gamma \leftarrow \left(r^\gamma + r^\gamma_{get}\right)$;
    else if $r^\gamma = 0$ **then**
      $\langle die \rangle$;
    end
    if $\langle giveDominating \rangle$ *is executed* **then**
      $r^\gamma \leftarrow \left(r^\gamma - r^\gamma_{get}\right)$;
    end
  else if pr_2 *is activated* **then**
    if $r^\gamma > r^{rep,\gamma}_{min}$ **then**
      if $\langle searchPartner, choose, clone, rec, mut, give \rangle$ *is activated* **then**
        $r^\gamma \leftarrow \left(r^\gamma - r^{clone,\gamma}_{give}\right)$;
      else if $\langle accept, give \rangle$ *is activated* **then**
        $r^\gamma \leftarrow \left(r^\gamma - r^{accept,\gamma}_{give}\right)$; /* $r^{clone,\gamma}_{give} \gg r^{accept,\gamma}_{give}$ */
      end
    end
  else if pr_3 *is activated* **then**
    if $r^\gamma > r^{mig,\gamma}_{min}$ **then**
      $\langle selNode, migr \rangle$;
      $r^\gamma \leftarrow \left(r^\gamma - r^{mig,\gamma}_{min}\right)$;
    end
  end
end

Profiles The set of profiles $PR^a = \{pr_1, pr_2, pr_3\}$, where pr_1 is the resource profile, pr_2 is the reproduction profile, and pr_3 is the migration profile. The resource profile is defined in the following way:

$$pr_1 = \langle \Gamma^{pr_1} = \Gamma, \Omega^{pr_1} = \{\omega_2\}, M^{pr_1} = \{i^{\omega_2}\}, ST^{pr_1}, RST^{pr_1} = ST^{pr_1}, GL^{pr_1} \rangle \tag{39}$$

The set of strategies includes two strategies:

$$ST^{pr_1} = \{\langle die \rangle, \langle searchDominated, get \rangle, \langle giveDominating \rangle\} \tag{40}$$

The goal of the profile is to keep the amount of resource above the minimal level.

The reproduction profile is defined as follows:

$$pr_2 = \langle \Gamma^{pr_2} = \Gamma, \Omega^{pr_2} = \{\omega_2\}, M^{pr_2} = \{i^{\omega_2}\}, ST^{pr_2}, RST^{pr_2} = ST^{pr_2}, GL^{pr_2} \rangle \tag{41}$$

The set of strategies includes two strategies:

$$ST^{pr_2} = \{\langle searchPartner, choose, clone, rec, mut, give \rangle, \langle accept, give \rangle\} \tag{42}$$

The goal of the profile is to reproduce when the amount of resource is above the minimal level needed for reproduction.
The migration profile is defined as follows:

$$pr_3 = \langle \Gamma^{pr_3} = \Gamma, \Omega^{pr_3} = \{\omega_1\}, M^{pr_3} = \{i^{\omega_1}\}, ST^{pr_3} = \{\langle selNode, migr \rangle\}, RST^{pr_3} = ST^{pr_3}, GL^{pr_3} \rangle \tag{43}$$

The goal of the profile is to migrate to another node when the amount of resource is above the minimal level needed for migration.

4.2 Co-evolutionary multi-agent system with host-parasite interactions (HPCoEMAS)

The co-evolutionary multi-agent system with host-parasite interactions is defined as follows (see Eq. (5)):

$$HPCoEMAS = \langle E, S, \Gamma, \Omega \rangle \tag{44}$$

The set of species includes two species, hosts and parasites: $S = \{host, par\}$. One resource type exists within the system ($\Gamma = \{\gamma\}$). Three information types ($\Omega = \{\omega_1, \omega_2, \omega_3\}$) are used. Information of type ω_1 denotes nodes to which each agent can migrate when it is located within particular node. Information of type ω_2 denotes such host-agents that are located within the particular node in time t. Information of type ω_3 denotes the host of the given parasite.

4.2.1 Host species

The host species is defined as follows:

$$host = \left\langle A^{host}, SX^{host} = \{sx\}, Z^{host}, C^{host} \right\rangle \tag{45}$$

where SX^{host} is the set of sexes which exist within the *host* species, Z^{host} is the set of actions that agents of species *host* can perform, and C^{host} is the set of relations of *host* species with other species that exist in the *HPCoEMAS*.

Actions The set of actions Z^{host} is defined as follows:

$$Z^{host} = \{die, get, give, accept, seek, clone, rec, mut, giveChild, migr\} \tag{46}$$

where:

- *die* is the action of death (host dies when it is out of resources);
- *get* action gets some resource from the environment;
- *give* action gives some resource to the parasite;
- *accept* action accepts other agent as a reproduction partner;
- *seek* action seeks for another host agent that is able to reproduce;

- *clone* is the action of producing offspring (parents give some of their resources to the offspring during this action);
- *rec* is the recombination operator (intermediate recombination is used (Bäck et al., 1997));
- *mut* is the mutation operator (mutation with self-adaptation is used (Bäck et al., 1997));
- *giveChild* action gives some resource to the offspring;
- *migr* is the action of migrating from one node to another. During this action agent loses some of its resource.

Relations The set of relations of *host* species with other species that exist within the system is defined as follows:

$$C^{host} = \left\{ \xrightarrow{host,get-}, \xrightarrow{host,give+} \right\} \tag{47}$$

The first relation models intra species competition for limited resources given by the environment:

$$\xrightarrow{host,get-} = \{\langle host, host \rangle\} \tag{48}$$

The second one models host-parasite interactions:

$$\xrightarrow{host,give+} = \{\langle host, par \rangle\} \tag{49}$$

4.2.2 Parasite species

The parasite species is defined as follows:

$$par = \langle A^{par}, SX^{par} = \{sx\}, Z^{par}, C^{par} \rangle \tag{50}$$

Actions The set of actions Z^{par} is defined as follows:

$$Z^{par} = \{die, seekHost, get, clone, mut, giveChild, migr\} \tag{51}$$

where:

- *die* is the action of death;
- *seekHost* is the action used in order to find the host. Test that is being performed by parasite-agent on host-agent before infection consists in comparing—in the sense of Pareto domination relation—solutions represented by assaulting parasite-agent and host-agents that is being assaulted. The more solution represented by host-agent is dominated by parasite-agent the higher is the probability of infection.
- *get* action gets some resource from the host;
- *clone* is the action of producing two offspring;
- *mut* is the mutation operator (mutation with self-adaptation is used (Bäck et al., 1997));
- *giveChild* action gives all the resources to the offspring—after the reproduction parasite agent dies;
- *migr* is the action of migrating from one node to another. During this action agent loses some of its resource.

Relations The set of relations of *par* species with other species that exist within the system are defined as follows:

$$C^{par} = \left\{ \xrightarrow{par,get-} \right\} \tag{52}$$

This relation models host-parasite interactions:

$$\xrightarrow{par,get-} = \{\langle par, host\rangle\} \tag{53}$$

As a result of performing *get* action some amount of the resources is taken from the host.

4.2.3 Host agent

Agent a of species *host* ($a \equiv a^{host}$) is defined as follows:

$$a = \left\langle gn^a, Z^a = Z^{host}, \Gamma^a = \Gamma, \Omega^a = \{\omega_1, \omega_2\}, PR^a \right\rangle \tag{54}$$

Genotype of agent a is consisted of two vectors (chromosomes): $\vec{x}$ of real-coded decision parameters' values and $\vec{\sigma}$ of standard deviations' values, which are used during mutation with self-adaptation. $Z^a = Z^{host}$ (see Eq. (46)) is the set of actions which agent a can perform. Γ^a is the set of resource types used by the agent, and Ω^a is the set of information types. Basic activities of the agent a are presented in Alg. 3.

Profiles The partially ordered set of profiles includes resource profile (pr_1), reproduction profile (pr_2), interaction profile (pr_3), and migration profile (pr_4):

$$PR^a = \{pr_1, pr_2, pr_3, pr_4\} \tag{55a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \trianglelefteq pr_4 \tag{55b}$$

The resource profile is defined in the following way:

$$pr_1 = \langle \Gamma^{pr_1} = \Gamma, \Omega^{pr_1} = \emptyset, M^{pr_1} = \emptyset, ST^{pr_1}, RST^{pr_1} = ST^{pr_1}, GL^{pr_1} \rangle \tag{56}$$

The set of strategies includes two strategies:

$$ST^{pr_1} = \{\langle die\rangle, \langle get\rangle\} \tag{57}$$

The goal of the pr_1 profile is to keep the amount of resources above the minimal level or to die when the amount of resources falls to zero.

The reproduction profile is defined as follows:

$$pr_2 = \langle \Gamma^{pr_2} = \Gamma, \Omega^{pr_2} = \{\omega_2\}, M^{pr_2} = \{i^{\omega_2}\}, ST^{pr_2}, RST^{pr_2} = ST^{pr_2}, GL^{pr_2} \rangle \tag{58}$$

The set of strategies includes two strategies:

$$ST^{pr_2} = \{\langle seek, clone, rec, mut, giveChild\rangle, \langle accept, giveChild\rangle\} \tag{59}$$

The only goal of the pr_2 profile is to reproduce. In order to realize this goal agent can use strategy of reproduction ⟨*seek*, *clone*, *rec*, *mut*, *giveChild*⟩ or can accept other agent as a reproduction partner ⟨*accept*, *giveChild*⟩.

The interaction profile is defined as follows:

$$pr_3 = \langle \Gamma^{pr_3} = \Gamma, \Omega^{pr_3} = \emptyset, M^{pr_3} = \emptyset, ST^{pr_3} = \{\langle give \rangle\}, RST^{pr_3} = ST^{pr_3}, GL^{pr_3} \rangle \tag{60}$$

The goal of the pr_3 profile is to interact with parasites with the use of strategy $\langle give \rangle$, which gives some of the host's resources to the parasite.
The migration profile is defined as follows:

$$pr_4 = \langle \Gamma^{pr_4} = \Gamma, \Omega^{pr_4} = \{\omega_1\}, M^{pr_4} = \{i^{\omega_1}\}, ST^{pr_4} = \{\langle migr \rangle\}, RST^{pr_4} = ST^{pr_4}, GL^{pr_4} \rangle \tag{61}$$

The goal of the pr_4 profile is to migrate within the environment. In order to realize such a goal the migration strategy is used, which firstly chooses the node and then realizes the migration. Agent loses some of its resources in order to migrate.

Algorithm 3. Basic activities of agent $a \equiv a^{host}$ in $HPCoEMAS$

$r^{\gamma} \leftarrow r^{\gamma}_{init}$;
while $r^{\gamma} > 0$ **do**
  activate the profile $pr_i \in PR^a$ with the highest priority and with the active goal $gl^*_j \in GL^{pr_i}$;
  if *pr_1 is activated* **then**
    if $0 < r^{\gamma} < r^{\gamma}_{min}$ **then**
      $\langle get \rangle$;
      $r^{\gamma} \leftarrow \left(r^{\gamma} + r^{env,\gamma}_{get}\right)$; /* $r^{env,\gamma}_{get}$ is the amount of resource given by the environment */
    else if $r^{\gamma} = 0$ **then**
      $\langle die \rangle$;
    end
  else if *pr_2 is activated* **then**
    if $r^{\gamma} > r^{rep,\gamma}_{min}$ **then**
      if *$\langle seek, clone, rec, mut, giveChild \rangle$ is performed* **then**
        $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{\gamma}_{giveChild}\right)$;
      else if *$\langle accept, giveChild \rangle$ is performed* **then**
        $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{\gamma}_{giveChild}\right)$;
      end
    end
  else if *pr_3 is activated* **then**
    $\langle give \rangle$;
    $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{\gamma}_{give}\right)$;
  else if *pr_4 is activated* **then**
    if $r^{\gamma} > r^{mig,\gamma}_{min}$ **then**
      $\langle migr \rangle$;
      $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{mig,\gamma}_{min}\right)$;
    end
  end
end

4.2.4 Parasite agent

Agent a of species par ($a \equiv a^{par}$) is defined as follows:

$$a = \langle gn^a, Z^a = Z^{par}, \Gamma^a = \Gamma, \Omega^a = \Omega, PR^a \rangle \tag{62}$$

Genotype of agent a is consisted of two vectors (chromosomes): $\vec{x}$ of real-coded decision parameters' values and $\vec{\sigma}$ of standard deviations' values. $Z^a = Z^{par}$ (see eq. (51)) is the set of actions which agent a can perform. Γ^a is the set of resource types used by the agent, and Ω^a is the set of information types. Basic activities of the agent a are presented in Alg. 4.

Algorithm 4. Basic activities of agent $a \equiv a^{par}$ in $HPCoEMAS$

$r^\gamma \leftarrow r^\gamma_{init}$;
while $r^\gamma > 0$ **do**
    activate the profile $pr_i \in PR^a$ with the highest priority and with the active goal $gl^*_j \in GL^{pr_i}$;
    if pr_1 *is activated* **then**
        if $0 < r^\gamma < r^\gamma_{min}$ **then**
            if $i^{\omega_3} = \emptyset$ **then**
                $\langle seekHost, get \rangle$;
                $r^\gamma \leftarrow \left(r^\gamma + r^\gamma_{get}\right)$; /* r^γ_{get} is the amount of resource taken from host */
            else
                $\langle get \rangle$;
                $r^\gamma \leftarrow \left(r^\gamma + r^\gamma_{get}\right)$;
            end
        else if $r^\gamma = 0$ **then**
            $\langle die \rangle$;
        end
    else if pr_2 *is activated* **then**
        if $r^\gamma > r^{rep,\gamma}_{min}$ **then**
            $\langle clone, mut, giveChild \rangle$;
            $r^\gamma \leftarrow 0$; /* All the resources are given to the offspring */
        end
    else if pr_3 *is activated* **then**
        if $r^\gamma > r^{mig,\gamma}_{min}$ **then**
            $\langle migr \rangle$;
            $r^\gamma \leftarrow \left(r^\gamma - r^{mig,\gamma}_{min}\right)$;
        end
    end
end

Profiles The partially ordered set of profiles includes resource profile (pr_1), reproduction profile (pr_2), and migration profile (pr_3):

$$PR^a = \{pr_1, pr_2, pr_3\} \tag{63a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \tag{63b}$$

The resource profile is defined in the following way:

$$pr_1 = \langle \Gamma^{pr_1} = \Gamma, \Omega^{pr_1} = \{\omega_2, \omega_3\}, M^{pr_1} = \{i^{\omega_2}, i^{\omega_3}\}, ST^{pr_1}, RST^{pr_1} = ST^{pr_1}, GL^{pr_1} \rangle \tag{64}$$

The set of strategies includes three strategies:

$$ST^{pr_1} = \{\langle die\rangle, \langle get\rangle, \langle seekHost, get\rangle\} \tag{65}$$

The goal of the pr_1 profile is to keep the amount of resources above the minimal level or to die when the amount of resources falls to zero. When the parasite has not infected any host (information i^{ω_3} is used), it uses strategy $\langle seekHost, get\rangle$ in order to find and infect some host and get its resources. If the parasite has already infected a host it can use $\langle get\rangle$ strategy in order to take some resources.
The reproduction profile is defined as follows:

$$pr_2 = \langle \Gamma^{pr_2} = \Gamma, \Omega^{pr_2} = \emptyset, M^{pr_2} = \emptyset, ST^{pr_2}, RST^{pr_2} = ST^{pr_2}, GL^{pr_2}\rangle \tag{66}$$

The set of strategies includes one strategy:

$$ST^{pr_2} = \{\langle clone, mut, giveChild\rangle\} \tag{67}$$

The only goal of the pr_2 profile is to reproduce. In order to realize this goal agent can use strategy of reproduction: $\langle clone, mut, giveChild\rangle$. Two offsprings are produced and the parent gives them all its resources and then dies.
The migration profile is defined as follows:

$$pr_3 = \langle \Gamma^{pr_3} = \Gamma, \Omega^{pr_3} = \{\omega_1\}, M^{pr_3} = \{i^{\omega_1}\}, ST^{pr_3} = \{\langle migr\rangle\}, RST^{pr_3} = ST^{pr_3}, GL^{pr_3}\rangle \tag{68}$$

The goal of the pr_3 profile is to migrate within the environment. In order to realize such a goal the migration strategy is used, which firstly chooses the node and then realizes the migration. During this some amount of the resource is given back to the environment.

5. Experimental results

Presented formally in section 4 agent-based co-evolutionary approaches for multi-objective optimization have been tentatively assessed. Obtained during experiments preliminary results were presented in some of our previous papers and in this section they are shortly summarized.

5.1 Performance metrics

Using only one single measure during assessing the effectiveness of (evolutionary) algorithms for multi-objective optimization is not enough (Zitzler et al., 2003) however it is impossible to present all obtained results (metrics as well as obtained Pareto frontiers and Pareto sets) discussing simultaneously (a lot of) ideas and issues related to the proposed new approach for evolutionary multi-objective optimization in one single article especially that the main goal of this chapter is to present coherent formal models of innovative agent-based co-evolutionary systems dedicated for multi-objective optimization rather than indepth results' analysis. Since hypervolume (HV) or hypervolume ratio (HVR) metrics allow to estimate both: the convergence to the true Pareto frontier as well as distribution of solutions over the whole approximation of the Pareto frontier, despite of its shortcomings it is one of the most commonly and most frequently used measure as the main metric for comparing the quality of obtained result sets—that is why results and comparisons presented in this paper are based mainly on this very measure.

Hypervolume or hypervolume ratio (Zitzler & Thiele, 1998) describes the area covered by solutions of obtained approximation of the Pareto frontier (*PF*). For each found nondominated solution, hypercube is evaluated with respect to the fixed reference point. In order to evaluate hypervolume ratio, value of hypervolume for obtained set is normalized with hypervolume value computed for true Pareto frontier. HV and HVR are defined as follows:

$$\mathrm{HV} = v\left(\bigcup_{i=1}^{N} v_i\right) \tag{69a}$$

$$\mathrm{HVR} = \frac{\mathrm{HV}(PF^{*})}{\mathrm{HV}(PF)} \tag{69b}$$

where v_i is hypercube computed for *i−th* found non-dominated solution, PF^* represents obtained approximation of the Pareto frontier and *PF* is the true Pareto frontier.

Assuming the following meaning of used below symbols: $\mathbb{P}$—Pareto set, $A, B \subseteq D$—two sets of decision vectors, $\sigma \geq 0$—appropriately chosen neighborhood parameter and $\|\cdot\|$—the given distance metric, then the following (used also in some of our experiments) measures can be defined (Zitzler, 1999):

- $\sigma(A, B)$—the coverage of two sets maps the ordered pair (A, B) to the interval [0, 1] in the following way:

$$\delta(A, B) = \frac{|\{b \in B \mid \exists a \in A : a \geq b\}|}{|B|} \tag{70}$$

- $\xi(A, B)$—the coverage difference of two sets ($\wp$ denotes value of the *size of dominated space* measure):

$$\xi(A, B) = \wp(A + B) - \wp(B) \tag{71}$$

- M_1—the average distance to the Pareto optimal set $\mathcal{P}$:

$$M_1(\mathcal{P}) = \frac{1}{|\mathcal{P}|} \sum_{p \in \mathcal{P}} min\,\{\|p - x\| \mid x \in \mathcal{P}\} \tag{72}$$

- M_2—the distribution in combination with the number of non-dominated solutions found:

$$M_2(\mathcal{P}) = \frac{1}{|\mathcal{P} - 1|} \sum_{p \in \mathcal{P}} |\{r \in \mathcal{P} \mid \|p - r\| > \sigma\}| \tag{73}$$

- M_3—the spread of non-dominated solutions over the set A:

$$M_3(\mathcal{P}) = \sqrt{\sum_{i=1}^{N} max\,\{\|p_i - r_i\| \mid p, r \in \mathcal{P}\}} \tag{74}$$

5.2 Test problems

Firstly, *Binh* (Binh & Korn, 1996; Binh & Korn, 1997) as well as *Schaffer* (Schaffer, 1985) problems were used. Binh problem is defined as follows:

$$Binh = \begin{cases} f_1(x,y) = x^2 + y^2 \\ f_2(x,y) = (x-5)^2 + (y-5)^2 \\ where \quad -5 \leq x, y \leq 10 \end{cases} \tag{75}$$

whereas used *modified Schaffer* problem is defined as follows:

$$Modified\,Schaffer = \begin{cases} f_1(x) = x^2 \\ f_2(x) = (x-2)^2 \\ where \quad -32 \leq x \leq 32 \end{cases} \tag{76}$$

Obviously during our experiments also well known and commonly used test suites were used. Inter alia such problems as ZDT test suite was used but because of its importance it is discussed wider in section 5.2.1.

5.2.1 ZDT (Zitzler-Deb-Thiele) test suite

One of test suites used during experiments presented and shortly discussed in the course of this section is Zitzler-Deb-Thiele test suite which in the literature it is known and identified as the set of test problems ZDT1-ZDT6 ((Zitzler, 1999, p. 57–63), (Zitzler et al., 2000), (Deb, 2001, p. 356–362), (Coello Coello et al., 2007, p. 194–199)). K. Deb in his work (Deb, 1998) tried to identify and systematize factors that can heighten difficulties in identifying by optimizing algorithm the true (model) Pareto frontier of multi-objective optimization problem that is being solved. The two main issues regarding the quality of obtained approximation of the Pareto frontier are: closeness to the true Pareto frontier as well as even dispersion of found non-dominated solution over the whole (approximation) of the Pareto frontier. Drifting to the Pareto frontier can be disturbed by such features of the problem as its multi-modality or isolated optima, what is known and can be observed also in the case of single-objective optimization. The other features that can (negatively) influence the ability of optimization algorithm for obtaining the high-quality Pareto frontier approximation are convex or concave character of the frontier or its discontinuity as well. Taking such observations into consideration the set of six test functions (ZDT1-ZDT6) was proposed. Each of them addresses and makes it possible to assess if algorithm that is being tested is able to overcome difficulties caused by each of mentioned feature. The whole ZDT test suite is constructed according to the following schema:

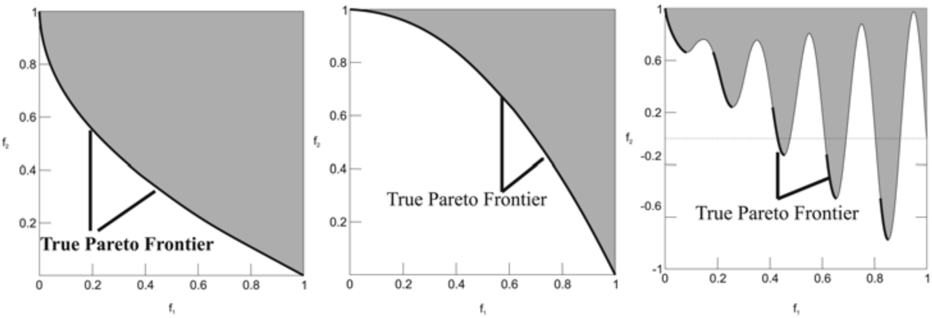
$$ZDT = \begin{cases} Minimize & F(x) & = & (f_1(x_1), f_2(x)) \\ On\ condition & f_2(x) & = & g(x_2, \ldots, x_n) \cdot h(f_1(x_1), g(x_2, \ldots, x_n)) \end{cases} \tag{77}$$

where: $x = (x_1, \ldots, x_n)$. Well, as one may see, ZDT1-ZDT6 problems are constructed on the basis of functions f_1, g and h as well, where f_1 is a function of one single (first) decision variable (x_1), function g is a function of the rest $n-1$ decision variables, and finally, function h is a function depending on values of functions f_1 and g. Particular problems ZDT1-ZDT6 assume different definitions of f_1, g and h functions as well as the number of decision variables n and the range of values of decision variables.

ZDT1 problem is the simplest (with continuous and convex true Pareto frontier) multi-objective optimization problem within the ZDT test-suite. The visualization of the true

Pareto frontier for ZDT1 problem (with $g(x) = 1$) is presented in Fig. 3a. Definitions of f_1, g and h functions in the case of ZDT1 problem are as follows:

$$ZDT1 = \begin{cases} f_1(x) = x_1 \\ g(x_2, ..., x_n) = 1 + \frac{9}{n-1} \sum_{i=2}^{n} x_i \\ h(f_1, g) = 1 - \sqrt{f_1/g(x)} \\ where\ n = 30, x_i \in [0, 1] \end{cases} \tag{78}$$



(a) (b) (c)

Fig. 3. Visualization of objective space and the true Pareto frontiers for problems ZDT1 (a) ZDT2 (b) and ZDT3 (c)

ZDT2 problem introduces the first potential difficulty for optimizing algorithm i.e. it is a problem with continuous but concave true Pareto frontier. The visualization of the true Pareto frontier for ZDT2 problem (with $g(x) = 1$) is presented in Fig. 3b. Definitions of f_1, g and h in this case are as follows:

$$ZDT2 = \begin{cases} f_1(x) = x_1 \\ g(x_2, ..., x_n) = 1 + \frac{9}{n-1} \sum_{i=2}^{n} x_i \\ h(f_1, g) = 1 - (f_1/g(x))^2 \\ where\ n = 30, x_i \in [0, 1] \end{cases} \tag{79}$$

ZDT3 problem introduces the next difficulty for optimization algorithm, this time it is discontinuity of the Pareto frontier. In the case of ZDT3 problem (defined obviously according to the (77) schema) the formulation of functions f_1, g and h are as follows:

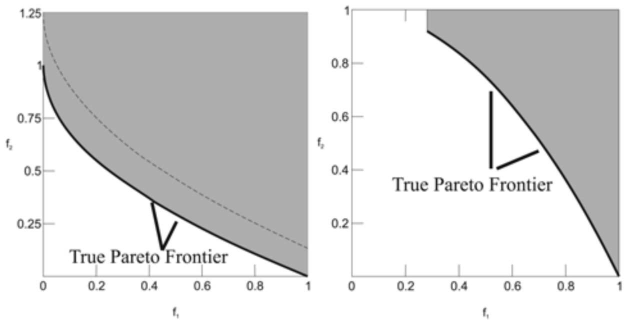
$$ZDT3 = \begin{cases} f_1(x) = x_1 \\ g(x_2, ..., x_n) = 1 + \frac{9}{n-1} \sum_{i=2}^{n} x_i \\ h(f_1, g) = 1 - \sqrt{f_1/g(x)} - \frac{f_1}{g(x)} sin(10\pi f_1) \\ where\ n = 30, x_i \in [0, 1] \end{cases} \tag{80}$$

Using *sinus* function in the case of ZDT3 problem in the definition of function h causes discontinuity in the Pareto frontier and simultaneously it does not cause discontinuity in the space of decision variables. The visualization of the true Pareto frontier for ZDT3 problem is presented in Fig. 3c.

ZDT4 problem makes it possible to assess the optimization algorithm in the case of solving multi-objective but simultaneously multi-modal optimization problem. The visualization of the true Pareto frontier for ZDT4 problem obtained with $g(x) = 1$) is presented in Fig. 4a.

ZDT4 problem introduces 21^9 local Pareto frontiers and the formulations of f_1, g and h in this case are as follows:

$$ZDT4 = \begin{cases} f_1(x) = x_1 \\ g(x_2, \ldots, x_n) = 1 + 10(n-1) + \sum_{i=2}^{n}(x_i^2 - 10cos(4\pi x_i)) \\ h(f_1, g) = 1 - \sqrt{f_1/g(x)} \\ where\ n = 10,\ x_1 \in [0,1],\ x_i \in [-5,5] \end{cases} \tag{81}$$



(a) (b)

Fig. 4. Visualization of objective space and the true Pareto frontiers for problems ZDT4 (a) and ZDT6 (b)

ZDT6 problem is a multi-objective optimization problem introducing several potential difficulties for optimization algorithm. It is a problem with non-convex Pareto frontier. Additionally, non-dominated solutions are dispersed not evenly. Next, in the space of decision variables, the "density" of solutions is less and less in the vicinity of the true Pareto frontier.

The visualization of the true Pareto frontier for ZDT6 problem is presented in Fig. 4b. Functions f_1, g and h defined obviously according to the schema (77) in the case of ZDT6 problem are formulated as follows:

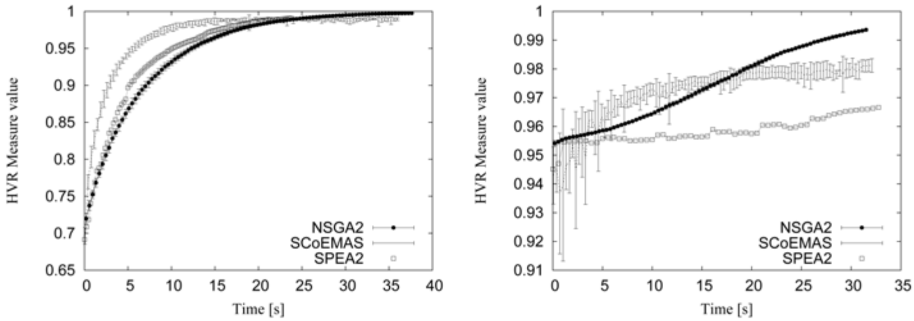
$$ZDT6 = \begin{cases} f_1(x) = 1 - exp(-4x_1)sin^6(6\pi x_1) \\ g(x_2, \ldots, x_n) = 1 + 9(\frac{(\sum_{i=2}^{n} x_i)}{(n-1)})^{0.25} \\ h(f_1, g) = 1 - (\frac{f_1(x)}{g(x)})^2 \\ where\ n = 10,\ x_i \in [0,1] \end{cases} \tag{82}$$

5.3 A glance at assessing sexual-selection based approach (SCoEMAS)

Sexual-selection co-evolutionary multi-agent system (SCoEMAS) presented in section 4.1 was preliminary assessed using inter alia presented in section 5.2.1 ZDT test suite. Also this time, SCoEMAS approach was compared among others with the state-of-the-art evolutionary algorithms for multi-objective optimization i.e. NSGA-II (Deb et al., 2002; Deb et al., 2000) and SPEA2 (Zitzler et al., 2001; Zitzler et al., 2002).

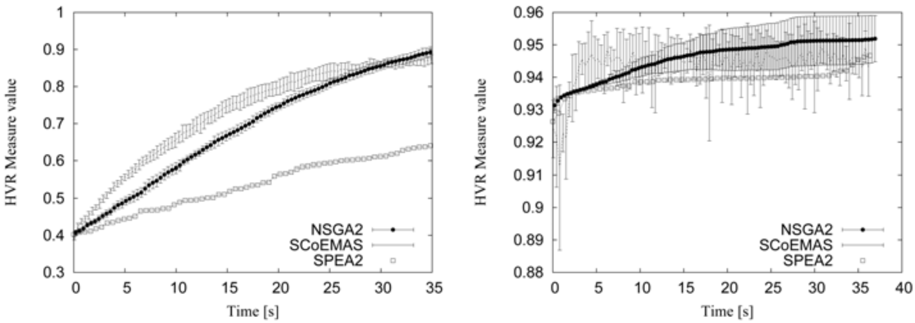
The size of population of SCoEMAS is 100, and the size of population of benchmarking algorithms are as follows: NSGA-II—300 and SPEA2—100. Selected parameters and their values assumed during presented experiments are as follows: $r^{\gamma}_{init} = 50$ (it represents the level of resources possessed initially by individual just after its creation), r^{γ}_{get} = 30 (it represents resources transferred in the case of domination), $r^{rep,\gamma}_{min}$ = 30 (it represents the level of resources required for reproduction), p_{mut} = 0.5 (mutation probability).

In Figure 5, Figure 6 and Figure 7 there are presented values of HVR measure obtained with time by SCoEMAS for ZDT1 (Figure 5a), ZDT2 (Figure 5b), ZDT3 (Figure 6a), ZDT4 (Figure 6b) and ZDT6 (Figure 7) problems. For comparison there are presented also results obtained by NSGA-II and SPEA2 algorithms.



(a) (b)

Fig. 5. HVR values obtained by SCoEMAS, NSGA-II and SPEA2 run against Zitzler's problems ZDT1 (a), and ZDT2 (b) (Siwik & Dreżewski, 2008)



(a) (b)

Fig. 6. HVR values obtained by SCoEMAS, NSGA-II and SPEA2 run against Zitzler's problems ZDT3 (a), and ZDT4 (b) (Siwik & Dreżewski, 2008)

On the basis of presented characteristics it can be said that initially co-evolutionary multi-agent system with sexual selection is faster than two other algorithms, it allows for obtaining better solutions—what can be observed as higher values of HVR(t) metrics but finally, the best results are obtained by NSGA-II algorithm. A little bit worse alternative than NSGA-II is SCoEMAS and finally SPEA2 is the third alternative—but obviously it depends on the problem that is being solved and differences between analyzed algorithms are not very distinctive.

Deeper analysis of obtained results can be found in (Dreżewski & Siwik, 2007; Dreżewski & Siwik, 2006*a*; Siwik & Dreżewski, 2008).

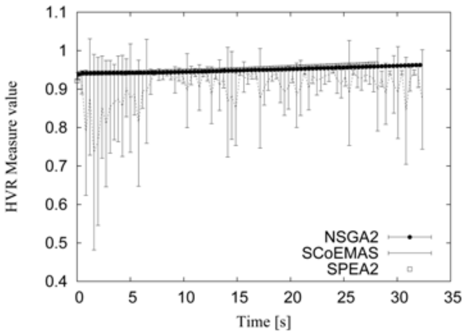


Fig. 7. HVR values obtained by SCoEMAS, NSGA-II and SPEA2 run against Zitzler's ZDT6 problem (Siwik & Dreżewski, 2008)

5.4 A glance at assessing host-parasite based approach (HPCoEMAS)

Discussed in section 4.2 co-evolutionary multi-agent system with host-parasite mechanism was tested using, inter alia, *Binh* and slightly modified *Schaffer* test functions that are defined as in equations (75) and (76).

Coverage of two sets $\delta(A, B)$				
	SPEA	**VEGA**	**NPGA**	**HPCoEMAS**
SPEA	✓	0.08	0.00	0.04
VEGA	0.92	✓	0.30	0.32
NPGA	1.00	0.62	✓	0.40
HPCoEMAS	0.96	0.70	0.58	✓

Table 1. Comparison of proposed HPCoEMAS approach with selected classical EMOAs according to the *Coverage of two sets* metrics (Dreżewski & Siwik, 2006*b*)

Coverage difference of two sets $\xi(A, B)$				
	SPEA	**VEGA**	**NPGA**	**HPCoEMAS**
SPEA	✓	8	0	6
VEGA	116	✓	3	13
NPGA	154	42	✓	25
HPCoEMAS	197	27	7	✓

Table 2. Comparison of proposed HPCoEMAS approach with selected classical EMOAs according to the *Coverage difference of two sets* metrics (Dreżewski & Siwik, 2006*b*)

	Size of dominated space ($\wp$)	**Average distance to the model Pareto set (M_1)**	**Distribution (M_2)**	**Spread (M_3)**
SPEA	39521	0.8	0.21	10.2
VEGA	39405	2.3	0.11	10.3
NPGA	39368	3.2	0.18	10.1
HPCoEMAS	39324	3.7	0.15	9.9

Table 3. Comparison of proposed HPCoEMAS approach with selected classical EMOAs according to other four metrics (Dreżewski & Siwik, 2006*b*)

This time, the following benchmarking algorithms were used: vector evaluated genetic algorithm (VEGA) (Schaffer, 1984; Schaffer, 1985), niched-pareto genetic algorithm (NPGA) (Horn et al., 1994) and strength Pareto evolutionary algorithm (SPEA) (Zitzler, 1999).
To compare proposed approach with implemented classical algorithms metrics defined in equations (70), (71), (72), (73) and (74) have been used. Obtained values of these metrics are presented in Table 1, Table 2 and Table 3.
Basing on defined above test functions and measures, some comparative studies of proposed co-evolutionary agent-based system with host-parasite interactions and well known and commonly used algorithms (i.e. VEGA, NPGA and SPEA) could be performed and the most important conclusion from such experiments can be formulated as follows: proposed HPCoEMAS system has turned out to be comparable to the *classical algorithms* according almost all considered metrics except for *Average distance to the model Pareto set* (see. Table 3). More conclusions and deeper analysis can be found in (Dreżewski & Siwik, 2006*b*).

6. Summary and conclusions

During last 25 years multi-objective optimization has been in the limelight of researchers. Because of practical importance and applications of multi-objective optimization as the most natural way of decision making and real-life optimizing method—growing interests of researchers in this very field of science was a natural consequence and extension of previous research on single-objective optimization techniques. Unfortunately, when searching for the approximation of the Pareto frontier, classical computational methods often prove ineffective for many (real) decision problems. The corresponding models are too complex or the formulas applied too complicated, or it can even occur that some formulations must be rejected in the face of numerical instability of available solvers. Also, because of such a specificity of multi-objective optimization (especially when—as in our case—we are considering multi-objective optimization in the Pareto sense) that we are looking for the whole set of nondominated solutions rather than one single solution—the special attention has been paid on population-based optimization techniques and if so, the most important techniques turned out here to be evolutionary-based methods. Research on applying evolutionary-based methods for solving multi-objective optimization tasks resulted in developing a completely new (and now commonly and very well known) science field: evolutionary multi-objective optimization (EMOO). To confirm above sentences, it is enough to mention statistics regarding at least the number of conference and journal articles, PhD thesis, conferences, books etc. devoted to EMOO and available at http://delta.cs.cinvestav.mx/~coello/EMOO.
After the first stage of research on EMOO when plenty of algorithms were proposed[1], simultaneously with introducing in early 2000s two the most important EMOO algorithms

[1] It is enough to mention such algorithms as: Rudolph's algorithm (Rudolph, 2001), distance-based Pareto GA (Osyczka & Kundu, 1995), strength Pareto EA (Zitzler & Thiele, 1998), multi-objective micro GA (Coello Coello & Toscano, 2005), Pareto-archived evolution strategy (Knowles & Corne, 2000), multi-objective messy GA (Van Veldhuizen, 1999), vector-optimized evolution strategy (Kursawe, 1991), random weighted GA (Murata & Ishibuchi, 1995), weight-based GA (Hajela et al., 1993), niched-pareto GA (Horn et al., 1994), non-dominated sorting GA (Srinivas & Deb, 1994), multiple objective GA (Fonseca & Fleming, 1993), distributed sharing GA (Hiroyasu et al., 1999)

i.e. NSGA-II and SPEA2 it seemed that no further research regarding new optimization techniques is needed. Unfortunately, in the case of really challenging problems (for instance in the case of multi-objective optimization in noisy environments, in the case of solving constrained problems, in the case of modeling market-related interactions etc.) mentioned algorithm turned out to be not efficient enough.
In this context, techniques with a kind of "soft selection" such as evolutionary multi-agent systems (EMAS), where in the population there can exist even not very strong individuals—seem to be very attractive alternatives. It turns out that "basic" EMAS model applied for multi-objective optimization can be improved significantly with the use of additional mechanisms and interactions among agents that can be introduced into such a system. In particular, as it is presented in the course of this chapter, some co-evolutionary interactions, mechanisms and techniques can be there successfully introduced. In section 5 there are presented results obtained with the use of two different co-evolutionary multi-agent systems. As one may see, presented results are not always significantly better than results obtained by "referenced" algorithms (in particular by state-of-the-art algorithms) but both, this chapter as well as presented results should be perceived as a kind of summary of the first stage of research on possibilities of developing co-evolutionary multi-agent systems for multi-objective optimization.
The most important conclusion of this very first stage of our research is as follows: on the basis of CoEMAS approach it is possible to model a wide range of co-evolutionary interactions. It is possible to develop such models as a distributed, decentralized and autonomous agent system. All proposed approaches can be modeled in a coherent way and can be derived from a basic CoEMAS model in a smooth and elegant way. So, in spite of not so high-quality results presented in previous section—after mentioned first stage of our research we know that both formal modeling as well as implementation of co-evolutionary multi-agent systems is possible in general. Because of their potential possibilities for modeling of (extremely) complex environments, problems, interactions, markets—further research on CoEMASes should result in plenty of their successful applications for solving real-life multi-objective optimization problems.

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