# Co-Evolutionary Multi-Agent System for Portfolio Optimization

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**Abstract.** Co-evolutionary techniques for evolutionary algorithms help overcoming limited adaptive capabilities of evolutionary algorithms, and maintaining population diversity. In this paper the idea and formal model of agent-based realization of predator-prey co-evolutionary algorithm is presented. The presented system is applied to the problem of effective portfolio building and compared to classical multi-objective evolutionary algorithms.

## 1 Introduction

*Evolutionary Algorithms (EAs)* are the global search and optimization techniques based on analogies to Darwinian model of natural evolution [3]. Evolutionary algorithms have demonstrated in practice efficiency and robustness as global optimization techniques. However, in the case of some problems (for example multi-modal optimization, multi-objective optimization, dynamic problems, etc.) they show negative tendency to loss the diversity of population. Both the experiments and formal analysis show that for multi-modal problem landscapes (and such are most of the engineering and economic problems) a simple EA will locate a single solution [27]. If we are interested in localizing multiple solutions (like in the case of so called "multi-modal optimization problems"), some special techniques should be used. *Niching and speciation methods* for EAs [27] are aimed at forming and stably maintaining subpopulations (species) throughout the search process, thereby allowing to locate all or most of the basins of attraction of local minima. The loss of population diversity also limits the adaptive capabilities of EAs in dynamic environments.

In the evolutionary biology the process of co-evolution is defined as the prolonged mutual interactions between two (or more) species leading to the appearing of some features of the individuals coming from these species resulting from the interactions with other species. The examples of co-evolutionary interactions include competition for limited resources, predator-prey interactions, host-parasite interactions, mutualism, commensalism etc. Also sexual selection results from co-evolution of female mate choice and male displayed trait, where females evolve to reduce direct costs associated with mating and keep them on optimal level and males evolve to attract females to mating (*sexual conflict*) [15]. It is acknowledged that co-evolution is responsible for bio-diversity, and may lead to speciation (the new species formation processes).

In *co-evolutionary algorithms* (which are, generally speaking, evolutionary algorithms with co-evolutionary mechanisms) the fitness of each individual depends not only on the quality of solution to the given problem (like in the case of EAs) but also (or solely) on other individuals' fitness. Such techniques are applicable in the case of problems for which the fitness function formulation is difficult or impossible (like game strategies), there is need for improving adaptive capabilities of EA or maintaining useful population diversity and introducing speciation into EAs—as it was stated above the loss of population diversity is one of the main problems in some applications of EAs.

Because many financial and economic decision and optimization problems are multi-modal (there exist many comparable solutions) or/and multi-objective (there exist many objective functions) so different techniques for maintaining population diversity in EAs may be found useful and applicable. In the case of such problems intelligent computer system provides alternative solutions to the decision maker and he makes the final decision based on his experience. In order to do so evolutionary algorithm must keep rather high level of population diversity—otherwise it simply will not be able to provide many different solutions to the given problem.

Besides the positive effect of maintaining population diversity, co-evolutionary algorithms provides us also other useful analogies between co-evolution, financial markets, and generally speaking market-oriented economic systems. These include for example "arms races" between capitalist enterprises and financial institutions (comparable to predator-prey or host-parasite interactions). Such "arms races" lead to avoiding the economic stagnation like in evolutionary systems they lead to avoiding stagnation in evolutionary sense. Also "Red Queen effect" ("It takes all the running you can do, to keep in the same place."—what in the case of co-evolutionary systems means that in order to keep the fitness of the given species relative to other species at the same level, continuing development is needed) can be observed in the market-economic processes. Capitalist enterprises and commodities must be continuously developed in order to "keep in the same place".

Co-evolutionary mechanisms can also be found useful when we are interested in socio-economic modeling and simulations, for example simulation of antagonistic and non-antagonistic interactions between different classes and groups in society (generally speaking problems of social stratification).

In the case of multi-objective optimization problems, which are the main subject of this paper, the loss of population diversity may cause that the population locates in the areas faraway from the Pareto frontier or that individuals are located only in selected areas of Pareto frontier. In the case of multi-objective problems with many local Pareto frontiers (defined by Deb in [7]) the loss of population diversity may result in locating only local Pareto frontier instead of a global one.

The notion "agent" is now very well established in the area of social science (psychology, sociology, and economy), artificial intelligence, and computer modeling and simulation. According to J. Ferber ([13]) the agent can be defined as the physical or virtual entity which can act within the environment, can communicate with other agents, tries to realize some goals or optimize its fitness function, possesses some resources, may observe the environment (but only in a restricted way), possesses restricted knowledge about the environment, has some abilities and may offer some services to other agents, may reproduce, acts in the way that leads to the realization of its own goals taking into account the possessed resources, abilities, and knowledge acquired during the observation of the environment and communication with other agents.

Multi-agent system is composed of the following elements ([13]): the environment, the set of objects situated within the system which can be observed, created, destroyed and modified by agents (which are active entities), the set of agents, the set of relations between objects (including agents), the set of operations which allow agents to observe, create, destroy, "consume", and modify objects, and finally the operators which represent the operations performed by agents and the reaction of the environment. The above features of multi-agent systems makes them ideal tool for social and economic simulations. We have here all tools necessary for modeling and simulation of different kinds of societies, social structures, modes of production, competing or co-operating enterprises, social mechanisms of conflict and co-operation, and so on.

*Evolutionary multi-agent systems (EMAS)* are multi-agent systems, in which the population of agents evolves (agents can die, reproduce and compete for limited resources). The model of *co-evolutionary multi-agent system (CoEMAS)* [8] introduces additionally the notions of species, sexes, and interactions between them. CoEMAS allows modeling and simulation of different co-evolutionary interactions, which can serve as the basis for constructing the techniques of maintaining population diversity and improving adaptive capabilities of such systems. CoEMAS systems with sexual selection and host-parasite mechanisms have already been applied with promising results to multi-objective optimization problems ([9, 10]).

Co-evolutionary multi-agent systems have of course all the advantages and mechanisms of multi-agent systems, which can be used in artificial life modeling and simulations (especially in the area of psychology, sociology and economy). Additionally, we can utilize the evolutionary optimization of agents and co-evolutionary interactions between them. The very promising area for future interdisciplinary research include psychological, social and economic simulations, considering for example all kinds of the emergent phenomena in society and economy, the problems of social stratification, the role of conflict in the society, antagonistic and non-antagonistic conflicts between classes and groups, the effects of particular economic policy, the role of the state and institutions in economy and society, the role of ideology, its role in the reproduction of relations of production, social power, and stratification, etc.

In the following sections the introduction to multi-objective optimization problems is presented. Then, we concentrate on the previous research on techniques for maintaining population diversity in multi-objective evolutionary algorithms. Next, the co-evolutionary multi-agent system with population diversity maintaining technique based on predator-prey interactions is formally described. The presented system is applied to problem of effective portfolio building. Results from the experiments with the CoEMAS system are then compared to other classical evolutionary techniques' results.

## 2 Multi-Objective Optimization

The most natural process of decision making for human being consists in analyzing many—often contradictory—factors and searching for peculiar compromise among them. Such decisive process is known as a *multi-criteria decision making (MCDM)*.

Obviously, human being is equipped with natural abilities for making multi-criteria decisions. As far as such natural gifts are—as the matter of fact—sufficient in every-day life they are not sufficient in more complex technical, business or scientific decisive processes. In such cases *decision maker*—to make a proper decision has to be equipped with appropriate mathematical apparatus and efficient computing units and algorithms built on the basis of this very apparatus. The most frequently, *MCDM* process is based on appropriately defined *multi-objective optimization problem (MOOP)*. Following [7]—*multi-objective optimization problem* in its general form is being defined as follows:

$$MOOP \equiv \begin{cases} Minimize/Maximize \quad f_m(\bar{x}), \quad m = 1,2\ldots,M \\ Subject\ \ to \qquad g_j(\bar{x}) \geq 0, \quad j = 1,2\ldots,J \\ \qquad\qquad\qquad h_k(\bar{x}) = 0, \quad k = 1,2\ldots,K \\ \qquad\qquad x_i^{(L)} \leq x_i \leq x_i^{(U)}, \quad i = 1,2\ldots,N \end{cases}$$

The set of constraints—both constraint functions (equalities $h_k(\bar{x})$), inequalities $g_j(\bar{x})$) and decision variable bounds (lower bounds $x_i^{(L)}$ and upper bounds $x_i^{(U)}$)—define all possible (feasible) decision alternatives ($\mathcal{D}$).

Because there are many criteria—to indicate which solution is better than the other—specialized ordering relation has to be introduced. To avoid problems with converting minimization to maximization problems (and vice versa of course) additional operator $\triangleleft$ can be defined. Then, notation $\bar{x}_1 \triangleleft \bar{x}_2$ indicates that solution $\bar{x}_1$ is simply better than solution $\bar{x}_2$ for particular objective. Now, the crucial concept of Pareto optimality i.e. so called dominance relation can be defined. It is said that solution $\bar{x}_A$ dominates solution $\bar{x}_B$ ($\bar{x}_A \prec \bar{x}_B$) if and only if:

$$\bar{x}_A \prec \bar{x}_B \Leftrightarrow \begin{cases} f_j(\bar{x}_A) \ntriangleright f_j(\bar{x}_B) \quad for \quad j = 1,2\ldots,M \\ \exists i \in \{1,2,\ldots,M\}: \quad f_i(\bar{x}_A) \triangleleft f_i(\bar{x}_B) \end{cases}$$

A solution in the Pareto sense of the multi-objective optimization problem means determination of all non-dominated alternatives from the set $\mathcal{D}$. The Pareto-optimal set consists of globally optimal solutions, however there may also exist locally optimal solutions, which constitute locally non-dominated set (*local Pareto-optimal set*) [7]. The set $\mathcal{P}_{local} \subseteq D$ is local Pareto-optimal set if ([41]):

$$\begin{aligned} &\forall x^a \in \mathcal{P}_{local} : \nexists x^b \in D \text{ such that} \\ &\qquad x^b \geq x^a \wedge \left\| x^b - x^a \right\| < \varepsilon \wedge \left\| F(x^b) - F(x^a) \right\| < \delta \end{aligned}$$

where $\|\cdot\|$ is a distance metric and $\varepsilon > 0$, $\delta > 0$.

The set $\mathcal{P} \subseteq D$ is global Pareto-optimal set if [41]:

$$\forall x^a \in \mathcal{P} : \nexists x^b \in D \text{ such that } x^b \geq x^a \tag{1}$$

These locally or globally non-dominated solutions create (in the criteria space) so-called local ($\mathcal{PF}_{local}$) or global ($\mathcal{PF}$) Pareto frontiers that can be defined as follows:

$$\mathcal{PF}_{local} = \left\{ y = F(x) \in \mathrm{I\!R}^M \;\mid\; x \in \mathcal{P}_{local} \right\} \tag{2a}$$

$$\mathcal{PF} = \left\{ y = F(x) \in \mathbb{R}^M \;\mid\; x \in \mathcal{P} \right\} \tag{2b}$$

Multi-objective problems with one global and many local Pareto frontiers are called *multi-modal multi-objective problems* [7].

During over twenty years of research on evolutionary multi-objective algorithms (EMOAs) quite many techniques have been proposed. Generally all of these techniques and algorithms can be classified as elitist (which give the best individuals the opportunity to be directly carried over to the next generation) or non-elitist ones [7].

## 3 Selected Issues of Maintaining Population Diversity in Evolutionary Multi-Objective Algorithms

In order to maintain useful population diversity and introduce speciation (processes of forming species—subpopulations—located in different areas of solutions' space) special techniques—like niching mechanisms and co-evolutionary models—are used.

Niching techniques are primarily applied in problems of multi-modal optimization, but they are also used in evolutionary multi-objective algorithms. Such techniques promote useful population diversity and make possible creating species located within the basins of attraction of local minima or in different parts of Pareto frontier. During the years of research various niching techniques have been proposed. All these techniques promote niche formation via the modification of mechanism of selecting individuals for new generation (*crowding model* [26]), the modification of the parent selection mechanism (*fitness sharing technique* [16] or *sexual selection mechanism* [33]), or restricted application of selection and/or recombination mechanisms (by *grouping* individuals into subpopulations [20] or by introducing the environment with some topography, in which the individuals are located [1, 5]).

Fitness sharing technique was used in Hajela and Lin genetic algorithms for multi-objective optimization based on weighting method [17]. The weights were encoded in genotype and the fitness sharing was used in objective space in order to introduce the diversity of the weights. Fitness sharing in the objective space was also used by Fonseca and Fleming in their multi-objective genetic algorithm using Pareto-based ranking procedure [14]. In the niched Pareto genetic algorithm (NPGA) [18] fitness sharing mechanism is used in objective space during the tournament selection in order to decide which individual wins (when the mechanism based on domination relation fails to choose the winner). In non-dominated sorting genetic algorithm (NSGA) [37] the fitness sharing is performed in decision space, within each set of non-dominated individuals separately, in order to maintain high population diversity. In strength Pareto evolutionary algorithm (SPEA) [41] special type of fitness sharing is used in order to maintain diversity. The fitness sharing in SPEA forms niches not on the basis of distance but on the basis of Pareto dominance.

As it was said, co-evolutionary techniques for EAs are applicable in the cases where the fitness function formulation is difficult (or even impossible). Co-evolutionary algorithms are also applicable in the cases when We want to maintain population diversity, introduce speciation, open-ended evolution, "arms races", and improve adaptive capabilities of EAs—especially in dynamic environments. As the result of ongoing research quite many co-evolutionary models and techniques have been proposed. Generally, each of theme belongs to one of two classes: competitive ([30]) or co-operative ([32]). In competitive co-evolution based systems two (or more) individuals compete in a game and their "competitive fitness functions" are calculated based on their relative performance in that game [6]. In co-operative co-evolutionary algorithms a problem is decomposed into sub-problems and each of them is then solved by different subpopulation [32]. Each individual from the given subpopulation is evaluated within a group of randomly chosen individuals coming from different sub-populations. Its fitness value depends on how well the group solved the problem and on how well the individual assisted in the solution.

Laumanns, Rudolph and Schwefel ([22]) proposed co-evolutionary algorithm with predator-prey model and spatial graph-like structure for multi-objective optimization. Deb introduced modified algorithm in which predators eliminated preys not only on the basis of one criteria but on the basis of the weighted sum of all criteria [7]. Li proposed other modifications to this algorithm [23]. The main difference was that not only predators were allowed to migrate within the graph but also preys could do it. The model of cooperative co-evolution was also applied to multi-objective optimization ([19]).

Sexual selection resulting from female-male co-evolution is considered to be one of the ecological mechanisms responsible for biodiversity and sympatric speciation [39, 15]. All the works on sexual selection mechanism for multi-objective evolutionary algorithms were focused on using this mechanism for maintaining population diversity, which causes that individuals are evenly distributed over the Pareto frontier. Allenson proposed genetic algorithm with sexual selection for multi-objective optimization [2]. In his technique the number of sexes was the same as the number of criteria of the given problem and individuals of the given sex were evaluated only according to one criterion (associated with their sex). Sex of the child was determined randomly and it replaced the worst individual from its sex. Allenson also introduced sexual selection mechanism. For each individual the partner for reproduction was selected on the basis of individual's preferences coded within its genotype. Lis and Eiben proposed multi-sexual genetic algorithm (MSGA) for multi-objective optimization [25]. They also used one sex for each criterion. If recombination operator was used during the reproduction (this was decided randomly) then partners for reproduction were chosen from each sex separately with the use of ranking mechanism and the offspring was created with the use of special multi-parent crossover operator. The sex of generated offspring was the same as the sex of the parent that provided most of genes. After the population of next generation was created the group of Pareto-optimal individuals was selected and this group was merged with the group of Pareto-optimal individuals from previous generations. During this phase dominated individuals were removed from the set of Pareto-optimal individuals. Bonissone and Subbu [4] continued work on Lis and Eiben's algorithm. They proposed additional mechanisms for determining the sex of offspring: random and based on phenotype (child had the sex associated with the criterion for which it had the best fitness).

Co-evolution of species and sexes is the biological mechanism responsible for biodiversity and sympatric speciation. However it was not widely used as a mechanism of maintaining useful genetic diversity of population for evolutionary algorithms. It seems that co-evolution and sexual selection can be used as a basis for constructing niching and speciation mechanisms (which promote the formation of species located within basins of attraction of different local optima or in different areas of Pareto frontier) but this is still an open issue and the subject of ongoing research.

## 4 Co-Evolutionary Multi-Agent System with Population Diversity Maintaining Mechanism

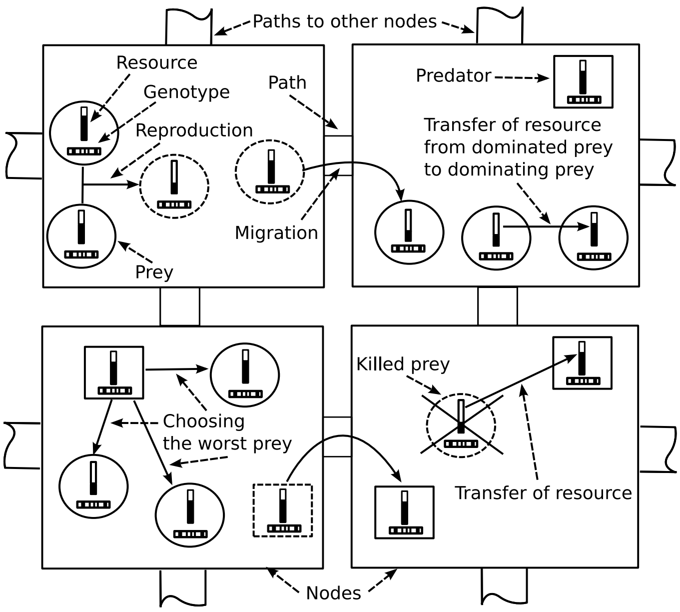


**Fig. 1.** CoEMAS with predator-prey mechanism

The system presented in this section is based on the CoEMAS model—the general model of co-evolution in multi-agent system [8]. The most important component of the population diversity maintaining mechanism are predator-prey co-evolutionary interactions (see fig. 1). The spatial structure of EMAS systems also plays the role of diversity maintaining mechanism but it is rather the mechanism of secondary importance. First prototypes of the CoEMAS with predator-prey interactions were presented also in [11, 12]. In the following sections the system used in experiments is described with the use of ideas, notions, and relations introduced in the general model for co-evolution in multi-agent system.

### 4.1 CoEMAS

The co-evolutionary multi-agent system with predator-prey interactions (*CoEMAS*) is defined as follows [8]:

$$CoEMAS = \langle E, S, \Gamma, \Omega \rangle \tag{3}$$

$E$ is the environment of the $CoEMAS$ system, $S$ is the set of species ($s \in S$) that exist and co-evolve in $CoEMAS$, $\Gamma$ is the set of resource types (the amount of type $\gamma$ resource which is possessed by the given element of the system will be denoted by $r^{\gamma}$), $\Omega$ is the set of information types (the information of type $\omega$, which can be used or possessed by the given element of the system is denoted by $i^{\omega}$). Two information types ($\Omega = \{\omega_1, \omega_2\}$) and one resource type ($\Gamma = \{\gamma\}$) are used. Information of type $\omega_1$ denote nodes to which agent can migrate. Information of type $\omega_2$ denote such prey that are located within the particular node in time $t$.

The selection mechanism is based on the closed circulation of resource within the system. The whole amount of resource is constant, the resource can be possessed by the agents, and is transferred from dominated prey to dominating prey, and from prey to predators during killing prey.

The environment $E$ is defined in the following way:

$$E = \left\langle T^E, \Gamma^E = \emptyset, \Omega^E = \Omega \right\rangle \tag{4}$$

$T^E$ is the topography of the environment $E$. $\Gamma^E$ is the set of resource types that exist within the environment. $\Omega^E$ is the set of information types that exist within the environment. The topography of the environment $T^E = \langle H, l \rangle$, where $H$ is directed graph with the cost function $c$ defined ($H = \langle V, B, c \rangle$, $V$ is the set of vertices, $B$ is the set of arches). In the case of the presented system every node is connected with its four neighbors, which results in the torus-like environment. The $l: \quad A \to V$ ($A$ is the set of agents) function makes it possible to locate particular agent in the environment space.

Vertice $v$ is given by:

$$v = \left\langle A^v, \Gamma^v = \Gamma^E, \Omega^v = \Omega^E \right\rangle \tag{5}$$

$A^v$ is the set of agents that are located within the vertice $v$. There are two types of information in the vertice. The first one includes all vertices that are connected with the vertice $v$:

$$i^{\omega_1, v} = \{u : u \in V \wedge \langle v, u \rangle \in B\} \tag{6}$$

The second one includes all agents of species *prey* that are located within the vertice $v$:

$$i^{\omega_2, v} = \{a^{prey} : a^{prey} \in A^v\} \tag{7}$$

### 4.2 Species

The set of species $S = \{prey, pred\}$. The prey species (*prey*) is defined as follows:

$$prey = \langle A^{prey}, SX^{prey} = \{sx\}, Z^{prey}, C^{prey} \rangle \tag{8}$$

where $SX^{prey}$ is the set of sexes which exist within the *prey* species, $Z^{prey}$ is the set of actions that agents of species *prey* can perform, and $C^{prey}$ is the set of relations of *prey* species with other species that exist in the $CoEMAS$.

The set of actions $Z^{prey}$ is defined as follows:

$$Z^{prey} = \{die, get, give, accept, seek, clone, rec, mut, migr\} \tag{9}$$

where:

- *die* is the action of death (prey dies when it is out of resources);
- *get* action gets some resource from another $a^{prey}$ agent located within the same node, which is dominated by the agent that performs *get* action or is too close to it in the criteria space;
- *give* action gives some resource to another agent (which performs *get* action);
- *accept* action accepts partner for reproduction when the amount of resource possessed by the prey agent is above the given level;
- *seek* action seeks for another prey agent that is dominated by the prey performing this action or is too close to it in criteria space. This action is also used in order to find the partner for reproduction when the amount of resource is above the given level and agent can reproduce;
- *clone* is the action of producing offspring (parents give some of their resources to the offspring during this action);
- *rec* is the recombination operator (intermediate recombination is used [3]);
- *mut* is the mutation operator (mutation with self-adaptation is used [3]);
- The *migr* is the action of migrating from one node to another. During this action agent loses some of its resource.

The set of relations of *prey* species with other species that exist within the system is defined as follows:

$$C^{prey} = \left\{ \xrightarrow{prey,get-}, \xrightarrow{pred,give+} \right\} \tag{10a}$$

The first relation models intra species competition for limited resources ("-" denotes that as a result of performing *get* action the fitness of another prey is decreased):

$$\xrightarrow{prey,get-} = \{\langle prey, prey \rangle\} \tag{10b}$$

The second one models predator-prey interactions ("+" denotes that when prey gives all its resources to the predator, the predator fitness is increased):

$$\xrightarrow{pred,give+} = \{\langle prey, pred \rangle\} \tag{10c}$$

The predator species (*pred*) is defined as follows:

$$pred = \left\langle A^{pred}, SX^{pred} = \{sx\}, Z^{pred}, C^{pred} \right\rangle \tag{11}$$

All the symbols used have analogical meaning as in the case of *prey* species—see eq. (8). The set of actions $Z^{pred}$ is defined as follows:

$$Z^{pred} = \{seek, get, migr\} \tag{12}$$

where:

– The *seek* action allows finding the “worst” (according to the criteria associated with the given predator) prey located within the same node as the predator;
– *get* action gets all resources from the chosen prey,
– *migr* action allows predator to migrate between nodes of the graph $H$—this results in losing some of the resources.

The set of relations of *pred* species with other species that exist within the system are defined as follows:

$$C^{pred} = \left\{ \xrightarrow{prey,get-} \right\} \tag{13a}$$

This relation models predator-prey interactions:

$$\xrightarrow{prey,get-} = \{\langle pred, prey \rangle\} \tag{13b}$$

As a result of performing *get* action and taking all resources from selected prey, it dies.

### 4.3 Prey Agents

Agent $a$ of species *prey* ($a \equiv a^{prey}$) is defined as follows:

$$a = \langle gn^a, Z^a = Z^{prey}, \Gamma^a = \Gamma, \Omega^a = \Omega, PR^a \rangle \tag{14}$$

Genotype of agent $a$ is consisted of two vectors (chromosomes): $x$ of real-coded decision parameters' values and $\sigma$ of standard deviations' values, which are used during mutation with self-adaptation. $Z^a = Z^{prey}$ (see eq. (9)) is the set of actions which agent $a$ can perform. $\Gamma^a$ is the set of resource types used by the agent, and $\Omega^a$ is the set of information types.

The partially ordered set of profiles includes resource profile ($pr_1$), reproduction profile ($pr_2$), interaction profile ($pr_3$), and migration profile ($pr_4$):

$$PR^a = \{pr_1, pr_2, pr_3, pr_4\} \tag{15a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \trianglelefteq pr_4 \tag{15b}$$

Each profile $pr$ is defined as follows:

$$pr = \langle \Gamma^{pr}, \Omega^{pr}, M^{pr}, ST^{pr}, GL^{pr} \rangle \tag{16}$$

$\Gamma^{pr}$ is the set of resource types used in the $pr$ profile ($\Gamma^{pr} \subseteq \Gamma^a$). $\Omega^{pr}$ is the set of information types ($\Omega^{pr} \subseteq \Omega^a$). $M^{pr}$ is the set of informations (the model) which represent the agent's knowledge about the environment and other agents.

$ST^{pr}$ is the partially ordered set ($ST^{pr} \equiv \langle ST^{pr}, \preccurlyeq \rangle$) of strategies which agent can use in order to realize the active goal of the given profile. The relation $\preccurlyeq$ is defined as follows:

$$\begin{aligned} \preccurlyeq = \{\langle st_i, st_j \rangle \in ST^{pr} \times ST^{pr} :\ & \text{strategy } st_i \text{ has equal or higher} \\ & \text{priority than strategy } st_j\} \end{aligned} \tag{17}$$

The single strategy $st \in ST^{pr}$ is composed of actions, which performing (in the given order) leads to the realization of a $pr$ profile's active goal:

$$st = \langle z_1, z_2, \ldots, z_k \rangle, \quad st \in ST^{pr}, \quad z_i \in Z^a \tag{18}$$

$GL^{pr}$ is the partially ordered ($GL^{pr} \equiv \langle GL^{pr}, \preccurlyeq \rangle$) set of goals. The relation $\preccurlyeq$ is defined in the following way:

$$\preccurlyeq = \{\langle gl_i, gl_j \rangle \in GL^{pr} \times GL^{pr} : \text{ the goal } gl_i \text{ has equal or higher priority, than the goal } gl_j\} \tag{19}$$

Now we can define the $\trianglelefteq$ relation (see eq. (15)):

$$\trianglelefteq = \{\left\langle pr_i, pr_j \right\rangle \in PR^a \times PR^a : \text{the realization of active goals of the profile } pr_i \text{ has the equal or higher priority than the realization of the active goals of profile } pr_j\} \tag{20}$$

By "active goal" (denoted by $gl^*$) we mean the goal $gl$ which should be realized in the given time step.

**The Process of Realizing Goals and Choosing the Strategies** The defined above partially ordered sets of profiles ($PR^a$), goals ($GL^{pr}$) and strategies ($ST^{pr}$) are used by agent for selecting the goal and strategy for its realization. The whole process of decision making is realized in the following way:

1) Agent $a$ activates the profile with highest priority ($pr_i \in PR^a$), which has the active goal $gl_j^* \in GL^{pr_i}$.
2) If there are more than one active goals in the set $GL^{pr_i}$ then the goal which has the highest priority is chosen for realization (let us assume that this is the goal $gl_j^*$).
3) Next, such strategy for the realization of the goal $gl_j^*$ is chosen from the set $ST^{pr_i}$ that it has the highest priority, it is possible to realize it in the given time, and it does not contradict with the goals of profiles with the lower priority than profile $pr_i$ (let us assume that this is the strategy $st_k \in ST^{pr_i}$).
4) If the realization of the chosen strategy is accomplished with success then the $gl_j$ becomes non-active goal.
5) Next, again activities from 1) are realized.

**The Profiles** The processes of realizing goals and choosing the strategies by prey agent are illustrated in the figure 2. The goal of the $pr_1$ (resource) profile is to keep the amount of resources above the minimal level or to die. In order to realize such goal agent can use the following strategies: $\langle die \rangle$, $\langle seek, get \rangle$. This profile uses the model $M^{pr_1} = \{i^{\omega_2}\}$ (see eq. (7)).

The only goal of the $pr_2$ (reproduction) profile is to reproduce. In order to realize this goal agent can use strategy of reproduction: $\langle seek, clone, rec, mut \rangle$. The model is defined in the following way: $M^{pr_2} = \{i^{\omega_2}\}$.

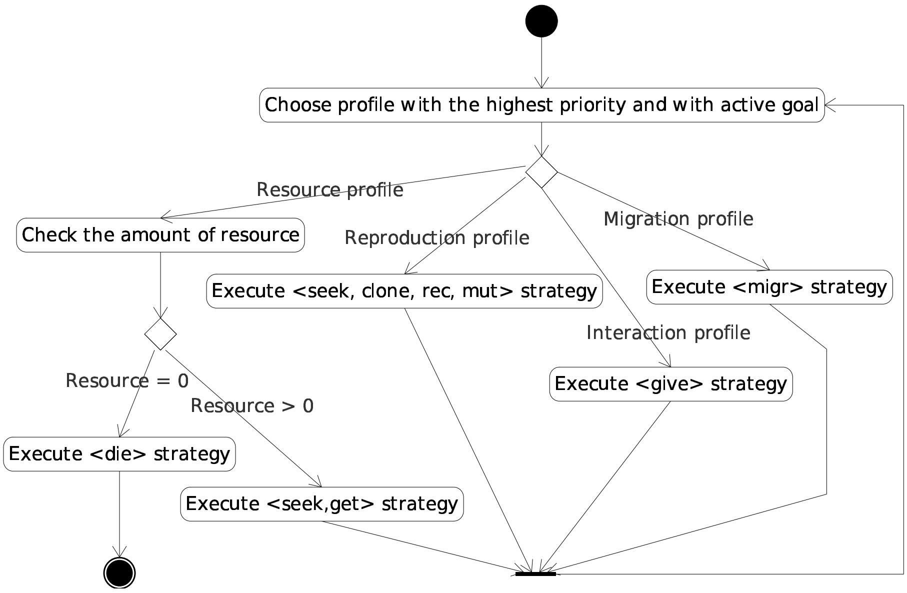


**Fig. 2.** The process of realizing goals and choosing the strategies by prey agent

The goal of the $pr_3$ (interaction) profile is to interact with predators with the use of strategy $\langle give \rangle$.

The goal of the $pr_4$ (migration) profile is to migrate within the environment. In order to realize such goal the migration strategy is used: $\langle migr \rangle$. The model used is defined as follows: $M^{pr_4} = \{i^{\omega_1}\}$ (see eq. (6).) As a result of migrating prey loses some resource.

### 4.4 Predator Agents

Agent $a$ of species *pred* is defined analogically to *prey* agent (see eq. (14)). There exist two main differences. Genotype of predator agent is consisted of the information about the criterion associated with the given agent. The set of profiles is consisted only of two profiles, resource profile ($pr_1$), and migration profile ($pr_2$): $PR^a = \{pr_1, pr_2\}$, where $pr_1 \trianglelefteq pr_2$.

The processes of realizing goals and choosing the strategies by predator agent are illustrated in the figure 3. The goal of the $pr_1$ (resource) profile is to keep the amount of resource above the minimal level with the use of strategy $\langle seek, get \rangle$. The model used within this profile is defined as follows: $M^{pr_1} = \{i^{\omega_2}\}$. The goal of $pr_2$ (migration) profile is to migrate within the environment. In order to realize this goal the migration strategy $\langle migr \rangle$) is used. The model of the environment is defined in the following way: $M^{pr_2} = \{i^{\omega_1}\}$. The realization of the migration strategy results in losing some of the resource possessed by the agent.

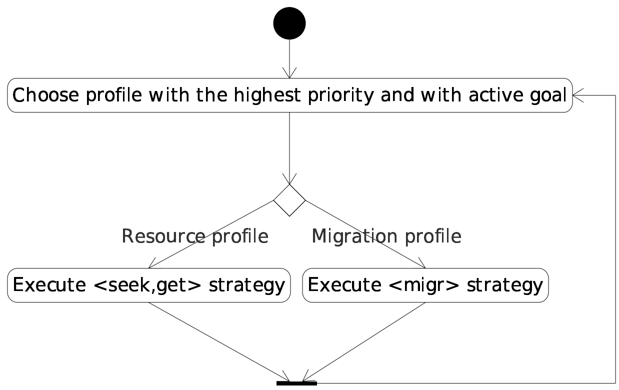


**Fig. 3.** The process of realizing goals and choosing the strategies by predator agent

## 5 Building effective investing portfolio

Proposed co-evolutionary agent-based system has been assessed preliminary [11, 12] using well known benchmark problems such as: Kursawe problem [21], Laumanns problem [22], and—recently also—the set of Zitzler test problems ZDT1—ZDT6 [41] where solving each next problem algorithm which is being tested has to deal with the more and more difficult and challenging characteristics starting from continuous and convex Pareto frontier, through concave or disconnected problems until multi-objective multi-modal problem (discussion about consequences of concavity, discontinuity or multimodality of the Pareto frontier can be found for instance in [7]).

Analyzing the behavior and characteristics of co-evolutionary computation techniques in general, and agent-based co-evolutionary techniques in particular (especially such approaches as predator-prey, or host parasite approaches)—it is natural that one of the first associations to such techniques (and obviously one of possible applications of such computational techniques) is the situation in economy, market economy and in the financial and investments markets in particular. Entrepreneurs, SMEs, corporations—all of them all the time have to be better, more innovative, cheaper, more effective etc. than the others. That is why, free market is so dynamic, all the time some enterprises introduce some organizational, financial or technological innovations and the rest of market-game participants has to respond to such changes introducing another innovations, products etc—so, all the time we are eye witnesses of a peculiar arms race. The range of dependencies that can be seen on the market can be pretty wide—from cooperation, through competition until antagonism. As it can be read in [31]—such situation is not the best one for all the market players (the situation when all participants of market game are the ”winners” is not possible—always some of them have to lose). There is no doubt however, that (only) thanks to such strong relationships, influences and interactions—the common organizational, technological and economical development and progress are possible—and in that way, extremely desirable phenomenon called ”invisible hand of market” by Adam Smith is realized. Of course, the most desirable situation is the perfect competition—but even the most developed markets only bring nearer and nearer to such a situation—mainly because of conditions (third condition in particular) required by ”perfect competition”. Mentioned three conditions of perfect competition are:

1. There are many buyers and many sellers in particular branch.
2. There are mainly small enterprises in the market.
3. The buyers and the sellers possess the full and perfect knowledge about the market (uncertainty and information asymmetry do not take place).

Fulfilling especially the third condition is very difficult (if possible at all), and if so, it is no wonder that both, competitive situation as well as possible interactions and relationships among market-players can vary in a (mentioned above) wide range. It is obvious however, that in the Darwin’s world—all activities of each participant of the market game are conformed to one overriding goal—to survive and to gain more and more wealth. From the interactions with another enterprises point of view it can be realized by: eliminating from the market as many weak rivals as possible and taking over their customers, products, delivery channels etc. (so by being ”predator”), by sucking out of another (stronger) enterprises’ customers, technologies, products etc. (so by being ”parasite”), by supplementing partners’ portfolio with additional products, technologies, customers etc.—and vice versa (so by living in symbiosis) etc. etc. It is seen clearly, that one of the most important activity of all market-game participants is co-existence with co-development—and from the computational intelligence point of view we would say—co-evolution. Because (generally speaking of course and under additional conditions) participants of the market game are autonomous entities (from the computational intelligence point of view we would say—agents), they are distributed, they act asynchronously, and they interact with another entities to achieve common goal—prosperity and wealth—in natural way applying co-evolutionary multi agent systems seems to be the perfect approach for modeling such phenomenons and environments. This is the first motivation of our experiments. But why ”building effective portfolio”. Well, we are working and perceiving co-evolutionary multi agent systems not only as modeling techniques but also as computational techniques. When we finished preliminary tests with benchmark problems—we wanted to run such systems against real—because of above stated motivation market-oriented—problems. Additionally, our goal was running one of proposed approaches against challenging, combinatorial, well defined and well-known multi-objective optimization problem where arm race interactions can be observed to test our predator-prey co-evolutionary multi-agent system. Building effective portfolio seems to be the perfect candidate test problem fulfilling all above mentioned requirements.

We know now why building effective portfolio problem has been selected as a test problem. Unfortunately, the next problem arises. How such a problem should be formally defined or which well-known definition should be chosen. Practically, there are some well known models describing building of effective portfolio i.e. Modern Portfolio Theory (MPT), one-factor Sharpe model, CAPM—Capital Asset Pricing Model, APT—Arbitrage Pricing Theory, Post Modern Portfolio Theory (PMPT) etc. The starting point for modern considerations about building efficient portfolio is the Nobel prize winner Harry Markowitz' Modern Portfolio Theory (MPT)(1952) [28, 29], or its extension proposed in 1958 by James Tobin [38]—consisting in introducing risk-free assets to the model. Those research resulted in defining for the first time formal foundations of *risk—rate of return* investing decision making and defining so-called Capital Market Line (CML) with the following equation:

$$R = R_f + (\frac{R_M - R_f}{S_M}) * S \tag{21}$$

where:

R - rate of return;
S - standard deviation;
$R_M$ - rate of return of market portfolio;
$S_M$ - standard deviation of market portfolio.

It turned out, after introducing to the model the risk-free assets that effective portfolio(s) belong(s) to the segment of the above defined line. Markowitz' portfolio analysis (and its expanded by J.Tobin with risk-free assets version) takes some strong and important assumptions. The most significant are:

– The goal of investor is to maximize of his wealth;
– Investors are characterized by risk aversion (their goal is to minimize the risk level);
– Investing horizon is the same for all investors;
– Suitable measure of risk level is standard deviation of rates of return from "average" rate of return of market portfolio;
– Investors make a decision on the basis of only rates of return and standard deviation;
– No taxes and transaction costs are assumed.

Although, described briefly above theory lays the foundations of modern capital investments. Practically it is nowadays rather only historically-important method of assets pricing.

Capital Asset Pricing Model (CAPM) was proposed by J.Traynor [40], J.Lintner [24], J.Mossin and formalized by W.Sharpe [36]—and it was based of course on previous work of Markowitz and his MPT theory. This time, in this model, not only Capital Market Line but also so-called Security Market Line is crucial. SML is defined as follows:

$$R_i = R_f + \beta_i * (R_M - R_f) \tag{22}$$

where $R_M - R_f$ - it is so-called prize for risk. CAPM is the most popular effective-portfolio building model. One may ask why this very model was not used during our tests. Well, mainly because of its complexity and shortcomings. On the basis of the critique of CAPM (e.g. so called Roll's Critique)—Arbitrage Pricing Theory (APT) was proposed by Stephen A. Ross in mid-1970s [35]. Again, being very general, APT can be described using the following equation:

$$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \cdots + b_{im}Fm + e_i \tag{23}$$

So, APT assumes that rates of return depends on $m$ factors. Coefficient $b_{ij}$ indicates how sensible is $R_i$ asset on changes of $F_{ij}$ factor. There are also another assumptions, the most important are the following:

– The number of F factors used in the model can not be higher than the number of assets and—more importantly
– In the market we have the perfect competition (how difficult for fulfilling is that assumption it was mentioned earlier).

In 1990s so-called Post Modern Portfolio Theory was proposed. The notion of PMPT was used for the first time probably by B.M. Rom and K.W. Ferguson in 1993 [34]. Generally, PMPT model is based on three main assumptions and observations:

1. Used in MPT (and in next theories) risk measure was symmetrical—i.e. returns above average or target rates of returns are as risky as returns below this value—whereas from investor's point of view—really risky are returns below the target (minimum or average) value, and the return above those values are perceived rather as prize for risk. It was observed and stated already by Markowitz, confirmed by Sharpe and another researchers—but mainly because of computational difficulties PMT was based on symmetrical measure.
2. Much better measure of risk (downside risk in this case) is continuous formula rather than its discrete version.
3. Much better index of rate of return is Sortino ratio rather than Sharpe ratio.

Taking all the pros and cons into consideration—because it was the first attempt of applying proposed algorithm to building effective portfolio—we decided to use during our experiments, and during preliminary assessing our co-evolutionary agent-based approach against building effective portfolio problem—one-factor Sharpe model, and this very model will be discussed below more precisely.

The meaning of symbols used in the definitions below, are as follows:

**p** - the number of assets in the portfolio;
**n** - the number of periods taken into consideration (the number of rates of return taken to the model);
$\alpha_i$,$\beta_i$ - coefficients of the equations;
$\omega_i$ - percentage participation of *i*-th asset in the portfolio;
$e_i$ - random component of the equation;
$R_{it}$ - the rate of return in the period $t$;
$R_{mt}$ - the rate of return of market index in period $t$;
$R_m$ - the rate of return of market index;
$R_i$ - the rate of return of the *i*-th asset;
$R_p$ - the rate of return of the portfolio;
${s_i}^2$ - the variance of the *i*-th asset;
${s_{e_i}}^2$ - the variance of the random index of the *i*-th asset;
${s_{e_p}}^2$ - the variance of the portfolio;
$\overline{R_i}$ - arithmetic mean of rate of return of the *i*-th asset;
$\overline{R_m}$ - arithmetic mean of rate of return of market index;

The algorithm (based on the one-factor Sharpe model) of computing the expected risk level and, generally speaking, income expectation related to the portfolio of $p$ assets is as follows:

1. Compute the arithmetic means on the basis of rate of returns;
2. Compute the value of $\alpha$ coefficient:

$$\alpha_i = \overline{R_i} - \beta_i \overline{R_m} \tag{24}$$

3. Compute the value of $\beta$ coefficient:

$$\beta_i = \frac{\sum_{t=1}^{n}(R_{it} - \overline{R_i})(R_{mt} - \overline{R_m})}{\sum_{t=1}^{n}(R_{mt} - \overline{R_m})^2} \tag{25}$$

4. Compute the expected rate of return of asset $i$:

$$R_i = \alpha_i + \beta_i R_m + e_i \tag{26}$$

5. Compute the variance of random index:

$${s_{e_i}}^2 = \frac{\sum_{t=1}^{n}(R_{it} - \alpha_i - \beta_i R_m)^2}{n-1} \tag{27}$$

6. Compute the variance of market index:

$${s_m}^2 = \frac{\sum_{t=1}^{n}(R_{mt} - \overline{R_m})^2}{n-1} \tag{28}$$

7. Compute the risk level of the investing portfolio:

$$\beta_p = \sum_{i=1}^{p}(\omega_i \beta_i) \tag{29}$$

$${s_{e_p}}^2 = \sum_{i=1}^{p}(\omega_i^2 {s_{e_i}}^2) \tag{30}$$

$$risk = \beta_p^2 {s_m}^2 + {s_{e_p}}^2 \tag{31}$$

8. Compute the portfolio rate of return:

$$R_p = \sum_{i=1}^{p}(\omega_i R_i) \tag{32}$$

The goal of the optimization is to maximize the portfolio rate of return and minimize the portfolio risk level. The task consists in determining values of decision variables $\omega_1 \ldots \omega_p$ forming the vector

$$\Omega = [\omega_1, \ldots, \omega_p]^T \tag{33}$$

where $0\% \leq \omega_i \leq 100\%$ and $\sum_{i=1}^{p} \omega_i = 100\%$ and $i = 1 \ldots p$ and which is the subject of minimization with respect of two criteria:

$$F = [R_p(\Omega) * (-1), risk(\Omega)]^T \tag{34}$$

Model Pareto frontiers for two cases (portfolios consisting of three and seventeen stocks set), which are the subject of analysis in the following section, are presented in fig. 4.

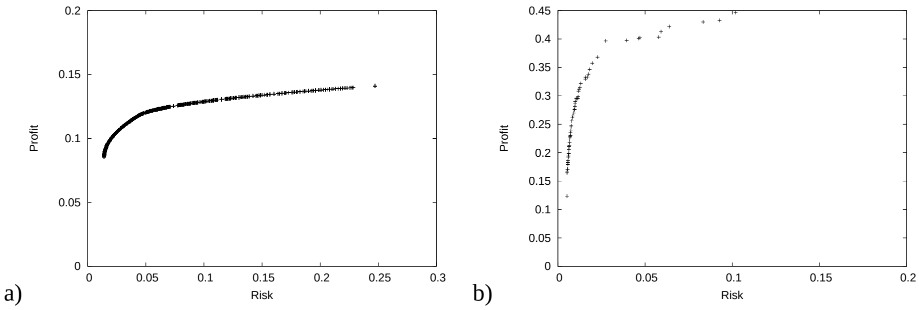


**Fig. 4.** *Building of effective portfolio*: visualization of the model Pareto frontier obtained using utter review method for a) three and b) seventeen stocks set

## 6 Results of Experiments

In this section the results of experiments are presented. The results obtained by proposed system are also compared with the results obtained by "classical" (i.e. non agent-based) predator-prey evolutionary strategy (PPES) [22] and another "classical" evolutionary algorithm for multi-objective optimization: niched pareto genetic algorithm (NPGA) [41]. In order to deeper analyze the results obtained by compared algorithms—values of HV and HVR metrics (their definitions can be found in [7]) are also presented.

In the case of optimizing investing portfolio each individual in the prey population is represented as a $p$-dimensional vector. Each dimension represents the percentage participation of $i$-th ($i \in 1 \dots p$) share in the whole portfolio. In this paper a kind of summary of two single experiments will be presented.

During presented experiments Warsaw Stock Exchange quotations from 2003-01-01 until 2005-12-31 were taken into consideration. Simultaneously, the portfolio consists of the following three (experiment I) or seventeen (experiment II) stocks quoted on the Warsaw Stock Exchange: in experiment I: RAFAKO, PONARFEH, PKOBP, in experiment II: KREDYTB, COMPLAND, BETACOM, GRAJEWO, KRUK, COMARCH, ATM, HANDLOWY, BZWBK, HYDROBUD, BORYSZEW, ARKSTEEL, BRE, KGHM, GANT, PROKOM, BPHPBK. As the market index WIG20 has been taken into consideration. In fig. 5 and fig. 6 there are presented Pareto frontiers obtained using CoEMAS, NPGA and PPES algorithm after 1, 300, 500, 700, 900 and 1000 steps in experiment I. As one may notice in this case CoEMAS-based frontier is more numerous (especially initially) than NPGA-based and as numerous as PPES-based one. Unfortunately in this case diversity of population in CoEMAS approach is visibly worse than in the case of NPGA or PPES-based frontiers. What is more, with time the tendency of CoEMAS-based solver for focusing solutions around small part of the whole Pareto frontier is more and more distinct. Similar situation can be also observed in fig. 7 and fig. 8 presenting Pareto frontiers obtained by CoEMAS, NPGA and PPES—but this time portfolio that is being optimized consists of 17 shares. Also this time CoEMAS-based frontier is quite numerous and quite close to the model Pareto frontier but the tendency for focusing solutions around only selected part(s) of the whole frontier is very distinct. In section 1 it was mentioned that proposed approach has been tested using such non-combinatorial test problems as Kursawe problem, Laumanns problem or the set of Zitzler problems. And it has to be underlined that using those problems Co-EMAS was definitely the better alternative than NPGA or PPES and the question appears why in the case of building effective portfolio the situation is the different one. Well, the explanation is as follows. With time, the population of agents consists mainly of mutually non-dominated agents and the situation that during the meetings agent dominates the opponent is more and more unlikely. If so, also gathering additional units of resources is more and more unlikely. Because agents pays in each step with resource for its life—with time the level of its energy falls below the death level and in the consequence it has to be removed from the system. The solution of such a situation is introducing to the system mechanisms similar to the elitism—where elitist agents for instance can migrate to the special island and can not be removed from the system as long as they are non-dominated. As it can be observed in this paper, mentioned phenomenon is much more dangerous during solving combinatorial problems, since meeting dominated agents is more unlikely (as simulation time passes) than in the case of continuous problems like Kursawe, Laumanns or Zitzler problems.

In this paper authors decided to present not only Pareto frontiers but also portfolio composition. It is of course impossible in the course of this paper to present consecutive portfolios proposed by all non-dominated solutions—that is why we decided to choose average non-dominated solution in first step and then to follow during consecutive steps solutions proposed by this very solution (or its descendant(s)). Such hypothetical non-dominated average portfolios for experiment I and II are presented in fig. 9 and in fig. 10 respectively (in fig. 10 shares are presented from left to right in the order in which they were mentioned above). Generally, it can be said that during experiment I—average solution proposed by CoEMAS system is a kind of balanced portfolio (percentage share of all three stocks are quite similar, but the percentage participation in the whole portfolio of PONAR is the lowest one and finally PKOBP became the most important "ingredient" of analyzed portfolio), whereas during experiment II there are more important stocks (with given assumptions and parameters of course)—i.e. HANDLOWY, HYDROBUD, ARKSTEEL.

## 7 Conclusions and Future Work

Co-evolutionary techniques for evolutionary algorithms are applicable in the case of problems for which it is difficult or impossible to formulate explicit fitness function, there is need for maintaining useful population diversity, forming species located in the basins of attraction of different local optima, or introducing open-ended evolution and "arms races". Such techniques are also widely used in artificial life simulations. Although co-evolutionary algorithms have been recently the subject of intensive research their application to multi-modal and multi-objective optimization is still the open problem and many questions remain unanswered.

In this paper the agent-based realization of predator-prey model within the more general framework of *co-evolutionary multi-agent system* has been presented. The system was run against hard real-life multi-objective problem (effective portfolio building) and then compared to two classical multi-objective evolutionary algorithms: PPES and

NPGA. CoEMAS was able to form more numerous frontier, however negative tendency to lose the population diversity during the experiment was observed. PPES and NPGA were able to form better dispersed Pareto frontiers. When the portfolio composition is considered the average solution proposed by CoEMAS system was rather a kind of balanced portfolio when it was composed of three stocks and portfolio with dominating elements when it was composed of seventeen stocks. The results of experiments with effective portfolio building problem show that still more research is needed on co-evolutionary mechanisms for maintaining population diversity used in CoEMAS, especially when we want to stably maintain diversity of solutions. It seems that the proposed predator-prey mechanism for evolutionary multi-agent systems may be very useful in the case of hard dynamic and multi-modal multi-objective problems (as defined by Deb [7]).

Future work will include more detailed analysis of the proposed co-evolutionary mechanisms, especially focused on problems of stable maintaining population diversity. The most important part of this research will be introduction of the elitism mechanism for decentralized agent-based evolutionary computation. Also the comparison of CoEMAS to other classical multi-objective evolutionary algorithms with the use of hard multi-modal multi-objective test problems, and the application of other co-evolutionary mechanisms like symbiosis (co-operative co-evolution) are included in future plans. Another, and very important, area of research on co-evolutionary multi-agent systems will be modeling and simulation of socio-economical mechanisms and emergent phenomena.

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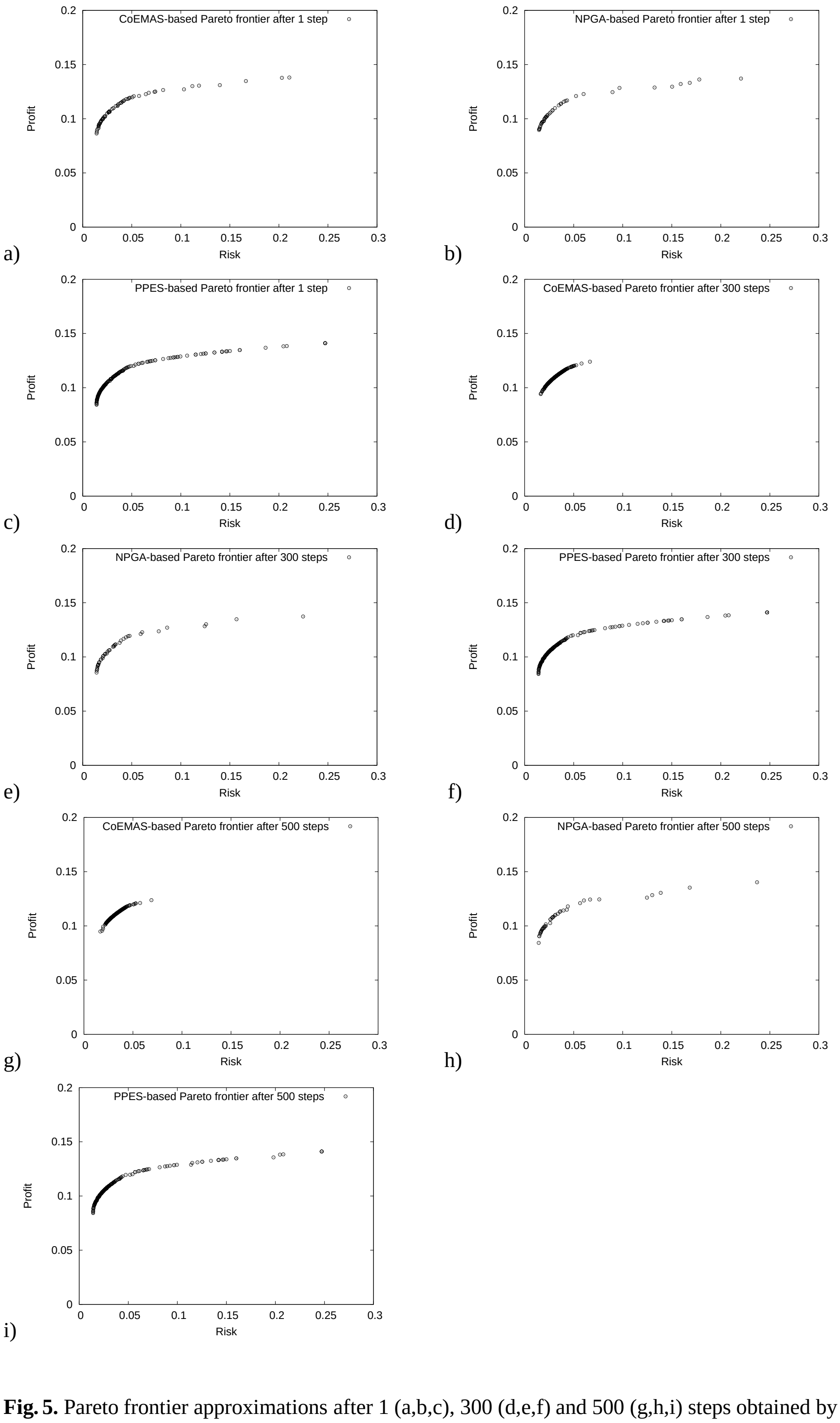


**Fig. 5.** Pareto frontier approximations after 1 (a,b,c), 300 (d,e,f) and 500 (g,h,i) steps obtained by CoEMAS, PPES, and NPGA for building effective portfolio consisting of 3 stocks

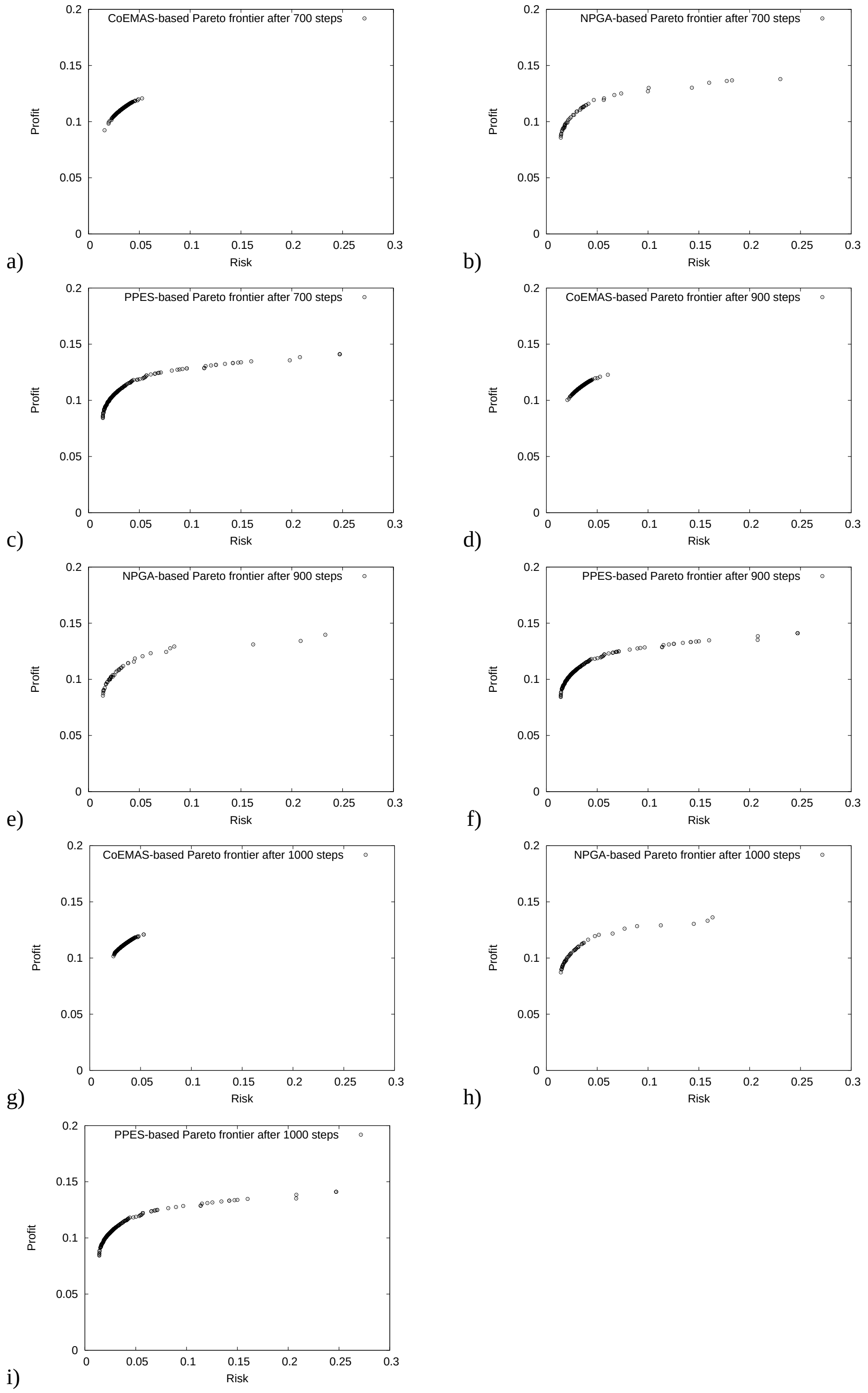


**Fig. 6.** Pareto frontier approximations after 700 (a,b,c), 900 (d,e,f), 1000 (g,h,i) steps obtained by CoEMAS, PPES, and NPGA for building effective portfolio consisting of 3 stocks

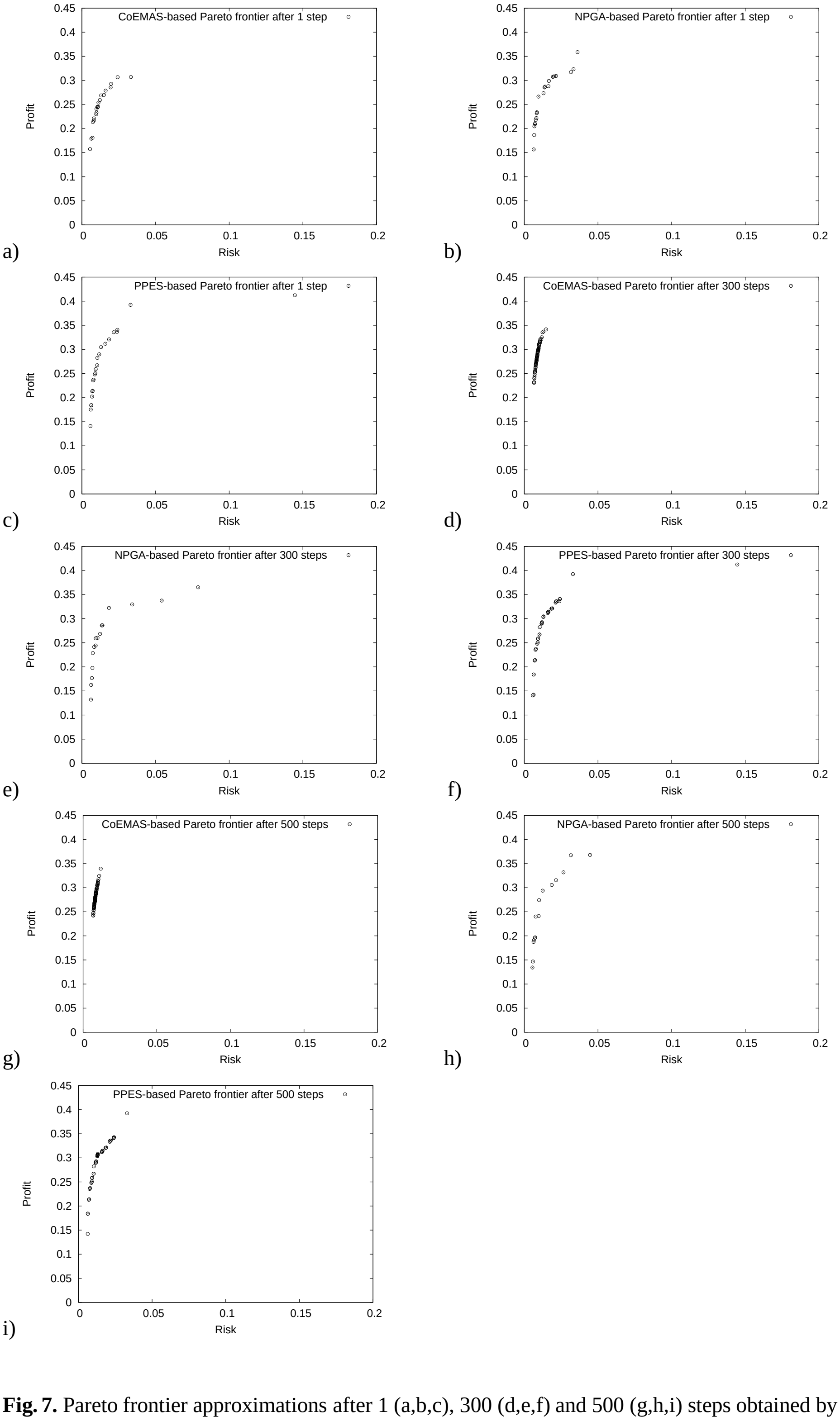


**Fig. 7.** Pareto frontier approximations after 1 (a,b,c), 300 (d,e,f) and 500 (g,h,i) steps obtained by CoEMAS, PPES, and NPGA for building effective portfolio consisting of 17 stocks

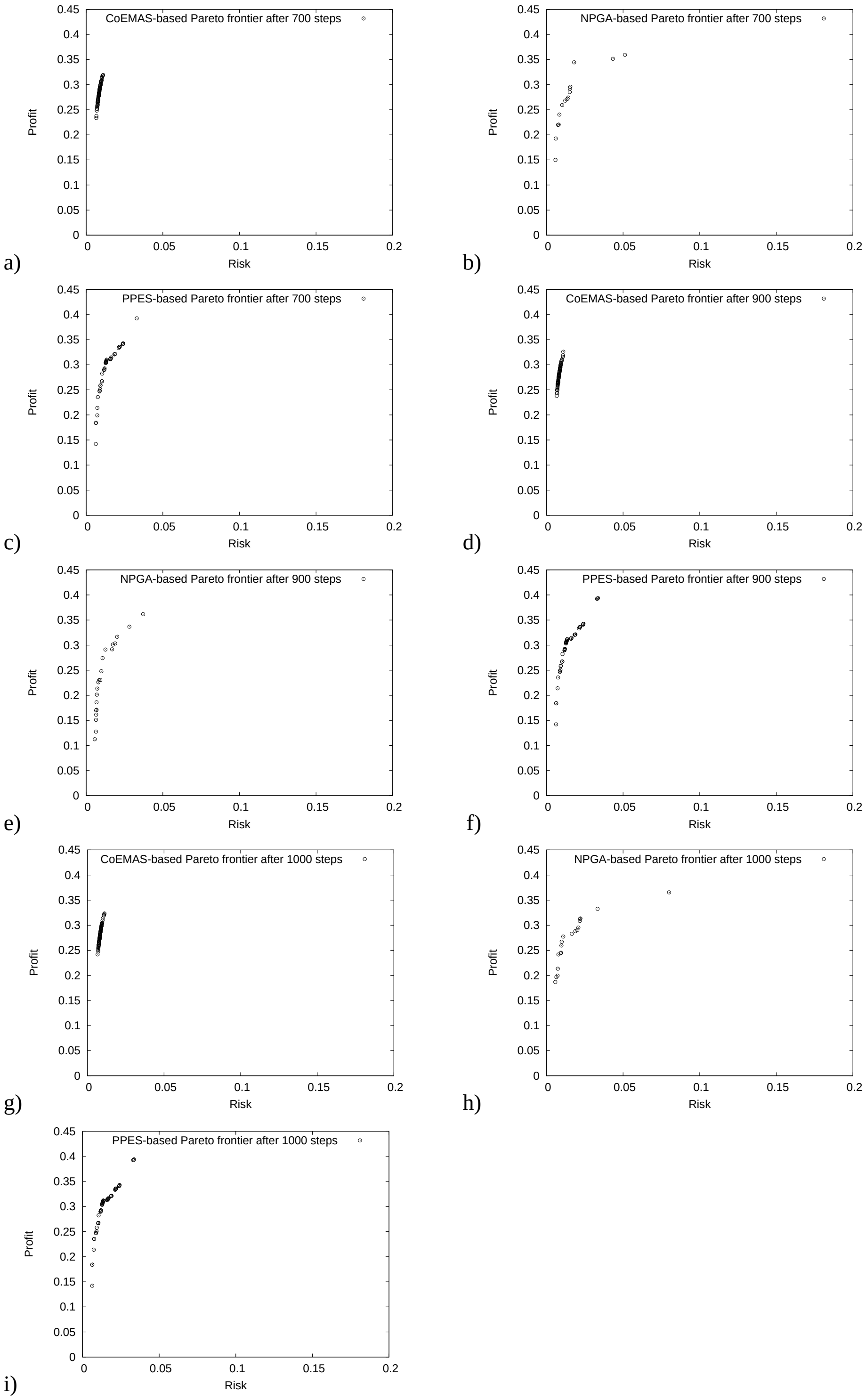


**Fig. 8.** Pareto frontier approximations after 700 (a,b,c), 900 (d,e,f), 1000 (g,h,i) steps obtained by CoEMAS, PPES, and NPGA for building effective portfolio consisting of 17 stocks

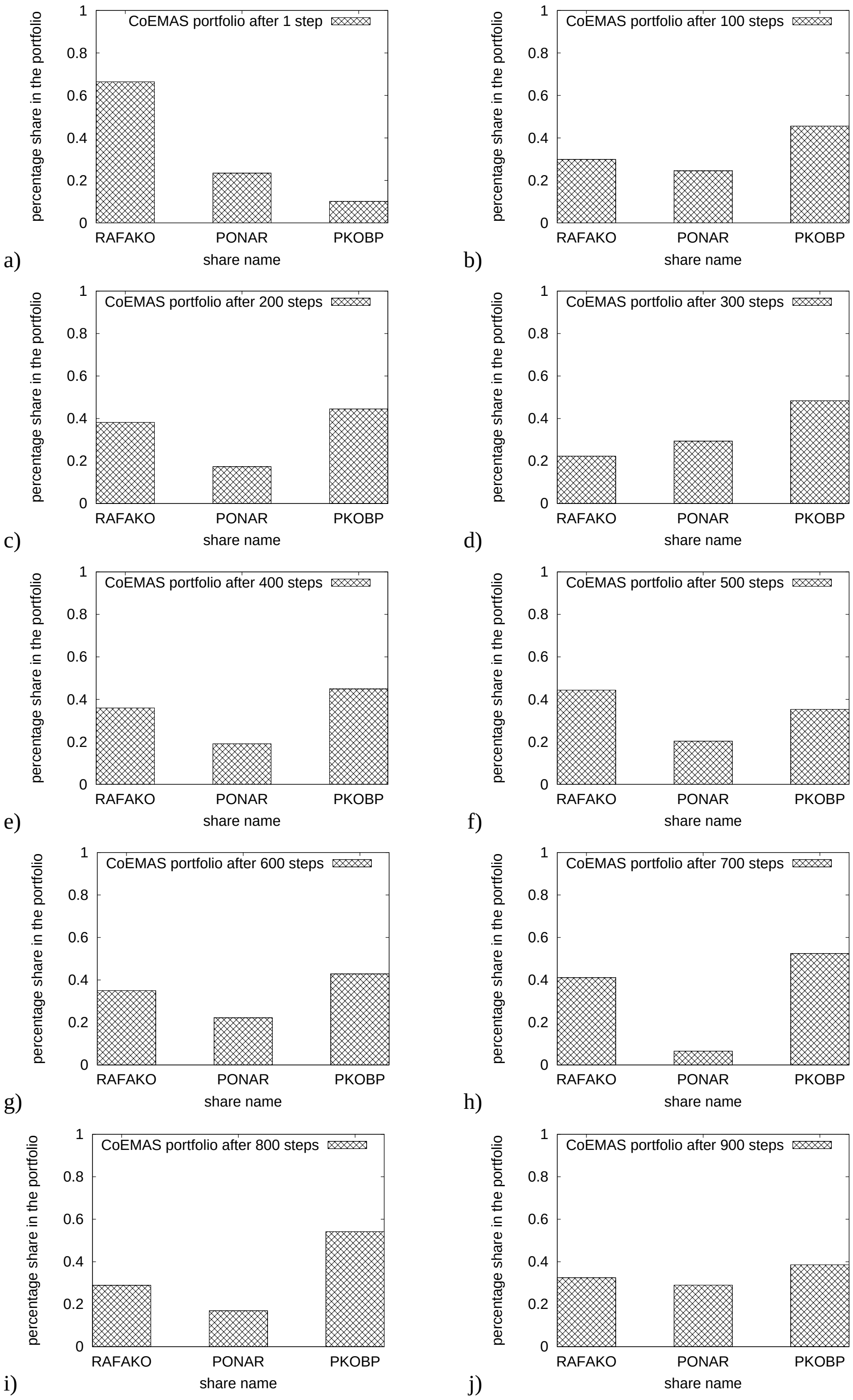


**Fig. 9.** Effective portfolio consisting of three stocks proposed by CoEMAS in consecutive steps

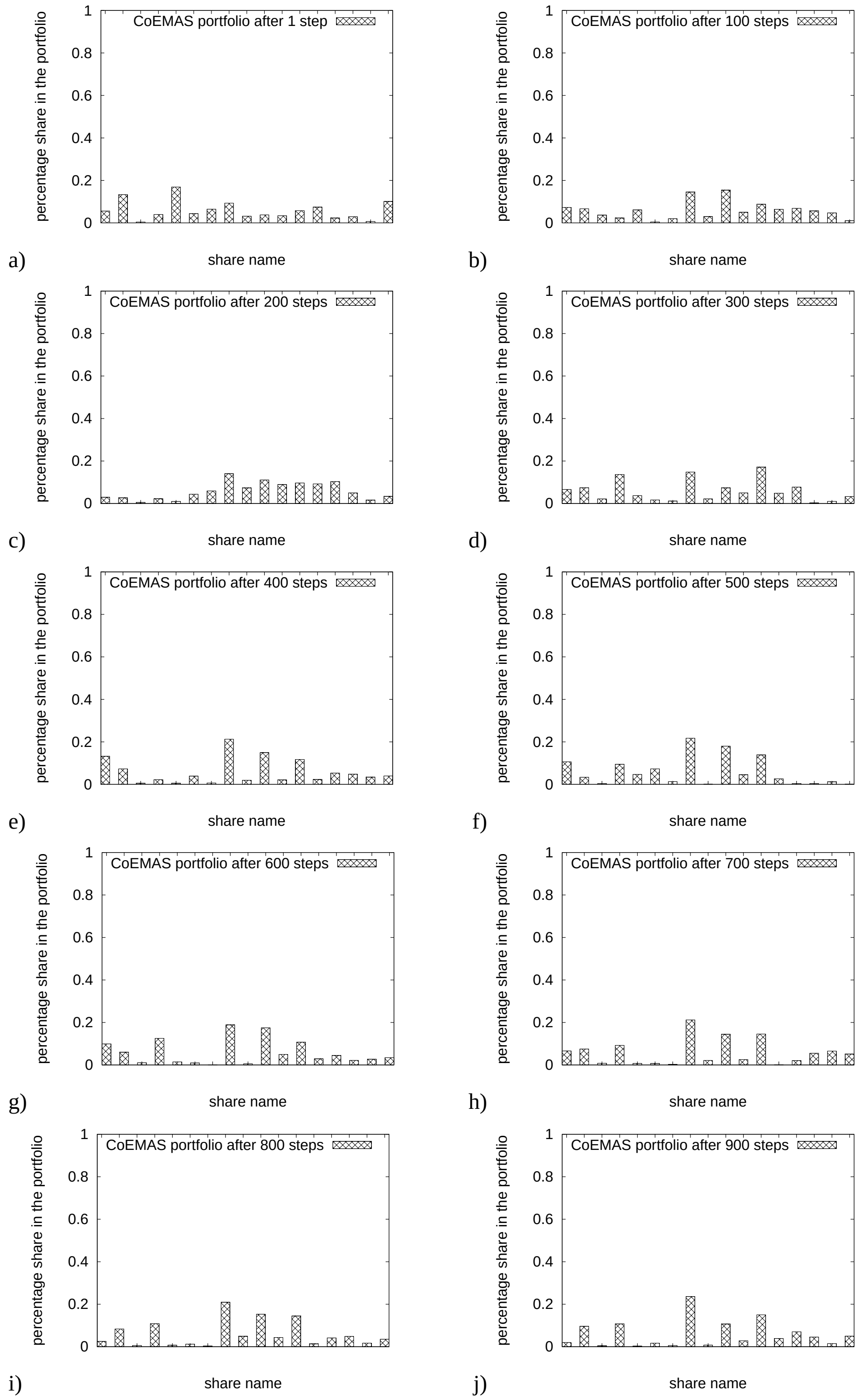


**Fig. 10.** Effective portfolio consisting of seventeen stocks proposed by CoEMAS in consecutive steps