Wstęp do oddziaływań hadronów

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Wykład 11

Quantum Chromodynamics (QCD) appears to be very similar to QED:

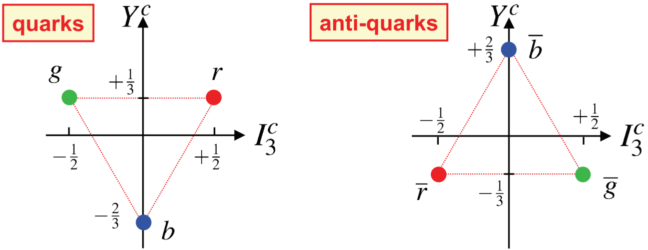
In QED	In QCD
Interactions are mediated by:	
a single massless photon corresponding to the single generator of the U(1) local gauge symmetry,	8 massless gluons corresponding to the 8 generators of the SU(3) local gauge symmetry,
Charge:	
electron carries unit of charge $-e$ anti-electron carries unit of anti-charge $+e$	quarks carry colour charges: r, g, b anti-quarks carry anti-colour charges: $\bar{r}, \bar{g}, \bar{b}$

Remarks:

- The colour charges, similarly as the electric charge, are conserved.
- The colour symmetry SU(3) is an exact symmetry, unlike the approximate SU(3) *uds* flavour symmetry.
- QCD is invariant under unitary transformations in colour space, what means that the strength of the strong interaction is independent of the colour charge.

Colour and QCD

By analogy to the *uds* flavour symmetry SU(3), the colour states of quarks and anti-quarks can be labeled by two additive quantum numbers: the third component of colour isospin I_3^c and colour hypercharge Y^c:



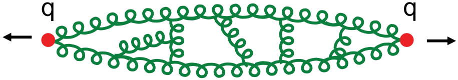
Experimental evidence: only colourless objects are observed in nature (and not e.g. free quarks).

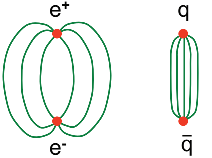
Colour confinement hypothesis:

Coloured objects are always confined to colour singlet states, what means that no objects with non-zero colour charge can propagate as free particles.

Although not yet proven analyticaly, it is believed to originate from the gluon-gluon self-interactions that arise because the gluons carry colour charge.

Interaction between quarks can be thought of in terms of exchange of virtual gluons. The energy stored in the field is proportional to the separation of the quarks ($V(r) \sim \kappa r, \;\; \kappa \approx 1$ GeV/fm).



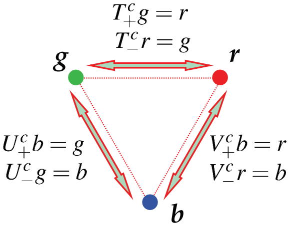


Colour confinement implies that quarks are always observed to be confined in bound colourless states.

- SU(3) colour singlet states are are colourless combinations with zero colour quantum numbers:

$$I_3^c = Y_3^c = 0$$

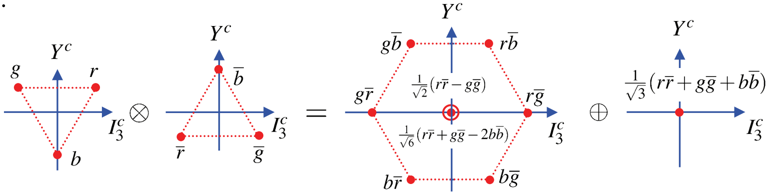
- In addition the action of any of the SU(3) colour ladder operators on a colour singlet state must yield 0.



- The confinement hypothesis implies that only combinations of quarks and anti-quarks which can form a colour singlet states are allowed.
- To construct colour wave functions for hadrons one can apply results of the uds SU(3) flavour symmetry with replacements: $u, d, s \to r, g, b$, respectively.

Meson colour wave function

The combination of a colour with an anti-color is mathematically identical to the construction of meson flavour wavefunctions in SU(3) flavour symmetry. The resulting colour multiplets are a colour are coloured octet and colourless singlet:



Colour confinemnet implies that mesons exist only in colour singlet states, so the colour wave function for mesons is:

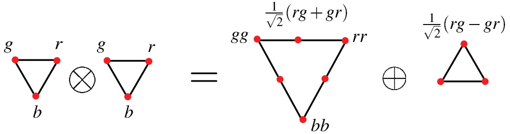
$$\psi^c(q\bar{q}) = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

The states with $I_3^c = Y^c = 0$ in the coloured octet do not transform as singlets when acted upon by the ladder operators, e.g.

$$T_+^c(r\bar{r} - g\bar{g}) = 0 - r\bar{g} - r\bar{g} + 0 \propto r\bar{g}$$

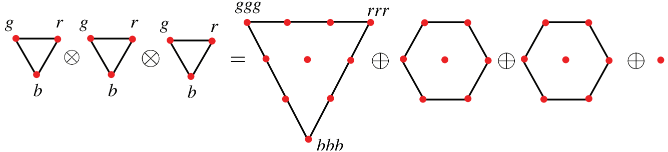
Barion colour wave function

The combination of two colour triplets yields a colour sextet and triplet ($\bar{3}$):



The absence of a colour singlet state implies that bound states of two quarks do not exist in nature.

The combination of three colours yields (among others) a single singlet:



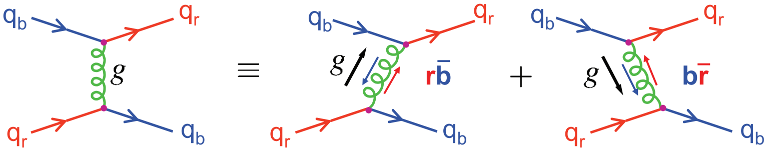
The wave function of the singlet state is fully anti-symmetric:

$$\psi^c(qqq) = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr)$$

Possible exotic colour singlet states like $q\bar{q}q\bar{q}$ or $qqq\bar{q}\bar{q}$ (pentaquarks), have not been confirmed experimentally do date.

Gluons

Interactions between coloured objects proceed via exchange of virtual gluons:

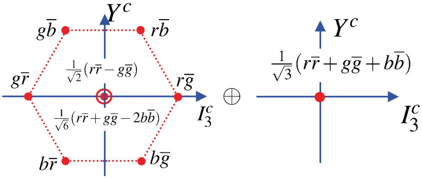


Gluons carry both colour and anti-colour:

Gluon colour wave functions are described by the same octet and singlet as obtained for mesons (also colour + anti-colour).

Colour singlet gluon would be unconfined and does not exist in nature (strong interaction would have infinite range).

This is because strong interaction arises from SU(3) symmetry, and the gluons correspond to the 8 generators of the group. The 9th gluon would exist if the underlying symmetry was U(3).



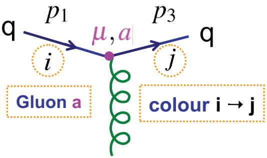
The quark-gluon interaction

The colour part of the quark wave function can be represented by orthogonal states:

$$c_1 = r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad c_2 = g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad c_3 = b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

This colour degree of freedom can be included in the quark wave function replacing the Dirac spinor $u(p)$ by $c_i u(p)$.

Taking into account the QCD vertex factor, the quark current associated with the QCD vertex can be written as:



$$j_q^\mu = \overline{u}(p_3) c_j^\dagger \left\{ -\frac{1}{2} i g_S \lambda^a \gamma^\mu \right\} c_i u(p_1) = -\frac{1}{2} i g_S \left[c_j^\dagger \lambda^a c_i \right] \times [\overline{u}(p_3) \gamma^\mu u(p_1)] =$$

$$= -\frac{1}{2} i g_S \lambda^a_{ji} \left[\overline{u}(p_3) \gamma^\mu u(p_1) \right]$$

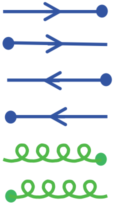
Remarks:

- 3×3 Gell-Mann matricies λ^a act only on colour wavefunction.
- γ^μ marices act only on the Dirac spinor.

Feynman rules for QCD

External lines:

- spin 1/2:
  - incoming quark $u(p)$
  - outgoing quark $\bar{u}(p)$
  - incoming anti-quark $\bar{v}(p)$
  - outgoing anti-quark $v(p)$
- spin 1:
  - incoming gluon $\varepsilon^{\mu}(p)$
  - outgoing gluon $\varepsilon^{\mu}(p)^{*}$



Internal lines (propagators):

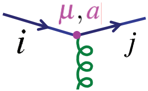
- spin 1: gluon

$$\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}$$

Vertex factor:

- spin 1/2: quark

$$-ig_s\frac{1}{2}\lambda^{a}_{ji}\gamma^{\mu}$$



Remarks:

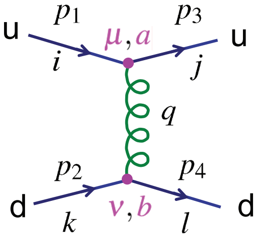
- $a, b = 1, ..., 8, \quad i, j = 1, 2, 3$ are gluon and quarks colour indicies, respectively.
- Not listed above are 3-gluon and 4-gluon vertices.

Matrix element for quark-quark scattering

Let us consider QCD scattering (gluon exchange) of an up and a down quark.

- In terms of quark colours this scattering is

$$ik \to jl, \quad i,j,k,l = 1,2,3$$



- The 8 different gluons are accounted for by the colour indices $a, b = 1, ..., 8$ (the δ-function in the propagator ensures $a = b$, i.e. the gluon "emitted" at a is the same as that "absorbed" at b).

- Applying the Feynman rules we obtain the matrix element:

$$-iM = \left[\overline{u}(p_3)\left\{-\frac{1}{2}ig_S\lambda^a_{ji}\gamma^\mu\right\}u(p_1)\right]\ \frac{-ig_{\mu\nu}}{q^2}\delta^{ab}\ \left[\overline{u}(p_4)\left\{-\frac{1}{2}ig_S\lambda^b_{lk}\gamma^\mu\right\}u(p_2)\right]$$

Summing over a and b we obtain:

$$M = -\frac{g_S^2}{4}\lambda^a_{ji}\lambda^a_{lk}\frac{1}{q^2}\left[\overline{u}(p_3)\gamma^\mu u(p_1)\right]\left[\overline{u}(p_4)\gamma^\mu u(p_2)\right]$$

- QCD matrix element can be obtained from the QED matrix element by making replacement $\alpha \to \alpha_S = g_S^2/4\pi$ and including additional colour factor:

$$C(ik \to jl) \equiv \frac{1}{4}\sum_{a=1}^{8}\lambda^a_{ji}\lambda^a_{lk}$$

Colour factors

QCD colour factors can be evaluated using the Gell-Mann matrices, which reflect the gluon states that are involved:

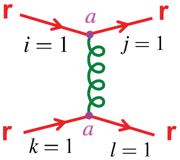
$r\bar{g}, g\bar{r}$ | $r\bar{b}, b\bar{r}$ | $g\bar{b}, b\bar{g}$ | $\frac{1}{2}(r\bar{r} - g\bar{g}), \frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

There are four different classes of the colour factors:

- All four colours are the same, e.g. $rr \to rr$:



Only matrices with non-zero entries in 11 position are involved:

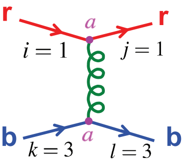
$$C(rr \to rr) = \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{11} \lambda^a_{11} = \frac{1}{4}(\lambda^3_{11}\lambda^3_{11} + \lambda^8_{11}\lambda^8_{11}) = \frac{1}{3}$$

Similarly we find (or just use exact colour SU(3) symmetry):

$$C(rr \to rr) = C(gg \to gg) = C(bb \to bb) = \frac{1}{3}$$

Colour factors

- Other configurations where quarks do not change colour, e.g. $rb \to rb$:



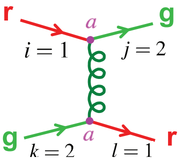
Only matrices with non-zero entries in 11 and 33 positions are involved:

$$C(rb \to rb) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{11}\lambda^a_{33} = \frac{1}{4}(\lambda^8_{11}\lambda^8_{33}) = -\frac{1}{6}$$

Similarly: $C(rb \to rb) = C(rg \to rg) = C(gr \to gr) =$

$$= C(gb \to gb) = C(br \to br) = C(bg \to bg) = -\frac{1}{6}$$

- Configuration where quarks swap colours, e.g. $rg \to gr$:



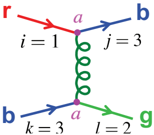
Only matrices with non-zero entries in 12 and 21 positions are involved:

$$C(rg \to gr) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{21}\lambda^a_{12} = \frac{1}{4}(\lambda^1_{21}\lambda^1_{12} + \lambda^2_{21}\lambda^2_{12}) = \frac{1}{2}$$ Similarly:

$$C(rb \to br) = C(rg \to gr) = C(gb \to bg) =$$

$$= C(br \to rb) = C(gr \to rg) = C(bg \to gb) = \frac{1}{2}$$

- Configurations involving three colours, e.g. $rb \to bg$:



Only matrices with non-zero entries in 13 and 32 positions are involved. However, there are not such Gell-Mann matrices, so the colour factor is zero.

It is so, because colour is conserved quantity.

Averaged colour factor

- In the real scattering process, $u + d \to u + d$, both initial and final quarks can have one of three colours. Therefore the matrix element must be sumed over all possible colour states and averaged over colors combinations of initial state quarks:

$$\langle |M|^2 \rangle = \frac{1}{9} \sum_{i,j,k,l=1}^{3} |M(ij \to kl)|^2$$

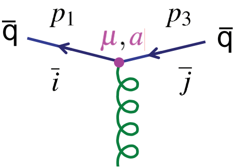
- The colour part of the above matrix element can be evaluated using the individual colour factors, and for the $u + d \to u + d$ scattering is:

$$\langle |C|^2 \rangle = \frac{1}{9} \sum_{i,j,k,l=1}^{3} |C(ij \to kl)|^2 = \frac{1}{9} \left[3 \times \left(\frac{1}{3} \right)^2 + 6 \times \left(-\frac{1}{6} \right)^2 + 6 \times \left(\frac{1}{2} \right)^2 \right] = \frac{2}{9}$$

- The entire effect of the $8 \times 3^4 = 648$ possible combinations of quark colours and types of gluons is encompassed in the above single number.
- The QCD cross section for the scattering process, $u + d \to u + d$, can be obtained from QED cross section for the process $e^- q \to e^- q$ replacing α by α_S and myltiplying by the averaged colour factor:

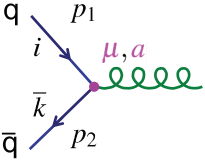
$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2} = \frac{4\pi\alpha_S}{9q^4} \left[1 + \left(1 + \frac{q^2}{\hat{s}} \right)^2 \right]$$

Colour factors in processes involving anti-quarks



The antiquark current associated with the $\bar{q}\bar{q}g$ vertex:

$$j^{\mu}_{\bar{q}} = \overline{v}(p_1)c_i^{\dagger}\left\{-\frac{1}{2}ig_S\lambda^a\gamma^{\mu}\right\}c_j v(p_3) =$$
$$= -\frac{1}{2}ig_S\lambda^a_{ij}[\overline{v}(p_1)\gamma^{\mu}v(p_3)]$$



Quark-antiquark anihilation vertex $\bar{q}\bar{q}g$:

$$\overline{v}(p_2)c_k^{\dagger}\left\{-\frac{1}{2}ig_S\lambda^a\gamma^{\mu}\right\}c_i u(p_1) = -\frac{1}{2}ig_S\lambda^a_{ki}[\overline{v}(p_2)\gamma^{\mu}u(p_1)]$$

Colour factors for the $q\bar{q}$ and $\bar{q}\bar{q}$ scattering processes:

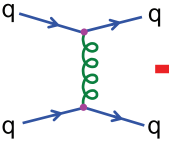
$$C(i\bar{k}\to j\bar{l}) = \frac{1}{4}\sum_{a=1}^{8}\lambda^a_{ji}\lambda^a_{kl} \quad \text{and} \quad C(\bar{i}\bar{k}\to \bar{j}\bar{l}) = \frac{1}{4}\sum_{a=1}^{8}\lambda^a_{ij}\lambda^a_{kl}$$

Color factor for the $q\bar{q}\to q\bar{q}$ anihilation process:

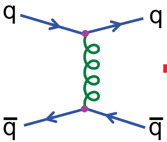
$$C(i\bar{k}\to j\bar{l}) = \frac{1}{4}\sum_{a=1}^{8}\lambda^a_{ki}\lambda^a_{jl}$$

QCD color potential and heavy mesons

- By analogy with QED we expect potential between $q\bar{q}$ of the form:



$$V(r) = +C\frac{\alpha_S}{r}$$



$$V(r) = -C\frac{\alpha_S}{r}$$

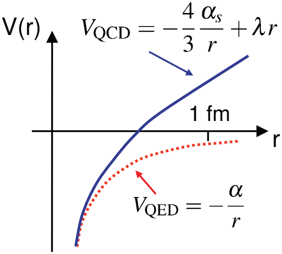
Whether it is attractive or repulsive depends on the sign of the colour factor C.

- Consider the $q\bar{q}$ system:

$$\langle V_{q\bar{q}} \rangle = \langle \psi | V | \psi \rangle \quad \text{with} \quad \psi = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

$$\langle V_{q\bar{q}} \rangle = -\frac{1}{3}\frac{\alpha_S}{r}\left[3 \times C(r\bar{r} \to r\bar{r}) + 6 \times C(r\bar{r} \to g\bar{g})\right] =$$

$$= -\frac{4}{3}\frac{\alpha_S}{r}$$



- As expected $\langle V_{q\bar{q}} \rangle$ is negative $\Leftrightarrow$ attractive.
- Similar calculation for the octet states, e.g. $r\bar{g}$ gives a positive repulsive potential, $C(r\bar{g} \to r\bar{g}) = -\frac{1}{6}$.
- V_{QCD} is found to give a good description of the observed charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) bound states (c and b are heavy and non-relativistic).