Parallel isogeometric simulations and inversion of hazardous environmental effects during oil/gas extraction

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Agenda

- Isogeometric L2 projections algorithm

Proposed by prof. Victor Calo: L. Gao, V.M. Calo, *Fast Isogeometric Solvers for Explicit Dynamics*, Computer Methods in Applied Mechanics and Engineering, (2014).

- Non-linear flow in heterogenous media

Obtained from prof. Victor Calo: M. Alotaibi, V.M. Calo, Y. Efendiev, J. Galvis, M. Ghommem, *Global-Local Nonlinear Model Reduction for Flows in Heterogeneous Porous Media*, Computer Methods in Applied Mechanics and Engineering, (2015).

- **L**iquid **F**ossil **F**uel **E**xtraction **P**roblem (**LFFEP**)
- **E**volutionary **M**ulti-**A**gent **S**ystems **(EMAS)**
- Numerical results
- Conclusions
- Further research

Isogeometric L2 projections

Proposed by prof. Victor Manuel Calo
L. Gao, V.M. Calo, *Fast Isogeometric Solvers for Explicit Dynamics*, **Computer Methods in Applied Mechanics and Engineering**, (2014).

Parallel version for distributed memory machines (MPI) (collaboration with prof. Calo at KAUST)
M. Woźniak, M. Łoś, M. Paszyński, L. Dalcin, V. M. Calo, *Parallel fast isogeometric solvers for explicit dynamcs*, **Computing and Informatics** (2016)

Parallel version for shared memory parallel machines (GALOIS) (collaboration with prof. Pingali from ICES)
Paper under construction

Isogeometric L2 projections

In general: non-stationary problem of the form

$$\partial_t u - \mathcal{L}(u) = f(x, t)$$

with some initial state u_0 and boundary conditions

$\mathcal{L}$ – well-posed linear spatial partial differential operator

Discretization:

- spatial discretization: isogeometric FEM

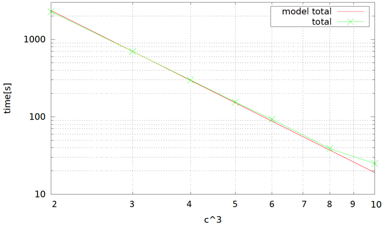
  Basis functions: $\phi_1, \ldots, \phi_n$ (tensor product B-splines)

- time discretization with explicit method
- implies isogeometric L2 projections in every time step

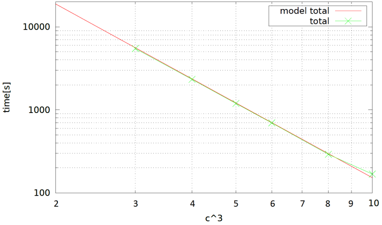
Isogeometric L2 projections

Linear computational cost for 3D problems

Perfect scalability on distributed memory parallel machines



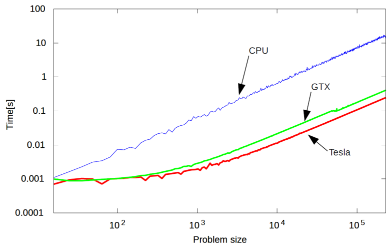
Execution time for
$N = 512^3 = 134,217,728$
cubic B-splines
up to 1024 processors



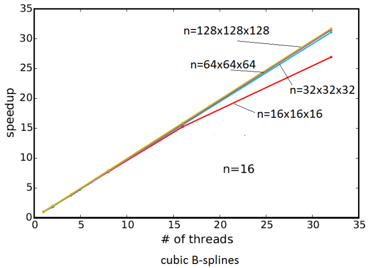
Execution time for
$N = 1024^3 = 1,073,741,824$
cubic B-splines
up to 1024 processors

Isogeometric L2 projections

Expensive isogeometric integration that can be speeded-up on GPGPU or multi-core machines

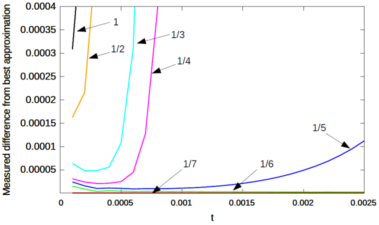


GPGPU integration
cubics, 2D problem
different mesh sizes

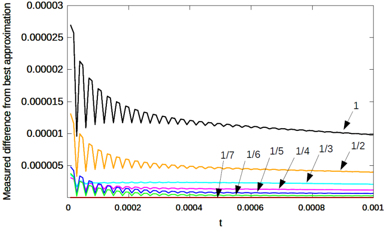


GALOIS multi-thread integration
cubics, 3D problem
different mesh sizes

Time step size limited by Courrant-Fourrier-Levy (CFL) condition



Relative errors for the time step $Dt = 10^{-4}$



Relative errors for the time step $Dt = 10^{-5}$

Lack of convergence for $Dt = 10^{-4}, \frac{10^{-4}}{2}, \ldots, \frac{10^{-4}}{5}$

Convergence for $Dt = 10^{-5}$ and smaller time steps

Oil extraction problem

Fracking (hydraulic fracturing) – oil/gas extraction technique consisting in high-pressure fluid injection into the deposit

- highly efficient
- can lead to contamination of ground waters

Two conflicting objectives:

- maximize resource extraction
- minimize ground water contamination

$\Longrightarrow$ **Multiobjective optimization**

Model

Formulation: non-stationary flow in heterogeneous media

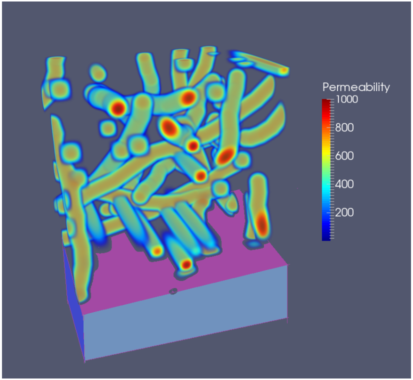
Spatial domain – $\Omega = [0,1]^3$

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nabla \cdot (\kappa(\mathbf{x}, u)\, \nabla u) = h(\mathbf{x}, t) & \text{in } \Omega \times [0, T] \\ \nabla u \cdot \hat{n} = 0 & \text{on } \partial\Omega \times [0, T] \\ u(x, 0) = u_0 & \text{in } \Omega \end{cases}$$

- u – pressure
- zero Neumann boundary conditions
- initial state u_0
- κ – permeability
- h – **forcing** (induced by extraction method)

$$\kappa(\mathbf{x}, u) = K_q(x)\, b(u)$$
$$b(u) = e^{\mu u}$$

$K_q(\mathbf{x})$ – property of the terrain (example below)



Below the formation we assume the presence of groundwaters

Pumps and sinks

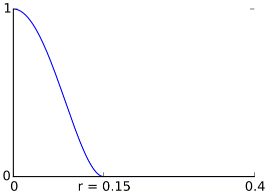
Extraction process modeled by **pumps** and **sinks**

- pump/sink has a location $\mathbf{x} \in \Omega$
- pumps locally increase the pressure u
- sinks locally decrease u (the higher, the faster)

$$h(x,t) = \sum_{p \in P} \phi\left(\|x_p - x\|\right) - \sum_{s \in S} u(x,t)\phi\left(\|x_s - x\|\right)$$

- P, S – sets of pump and sinks
- x_p, x_s – location of pump p/sink s
- ϕ – cut-off function ($r = 0.15$)

$$\phi(t) = \begin{cases} \left(\frac{t}{r} - 1\right)^2 \left(\frac{t}{r} + 1\right)^2 & \text{for } t \leq r \\ 0 & \text{for } t > r \end{cases}$$



Quantities of interest

Drained liquid

$$D = \sum_{s \in S} \int_0^T u(x,t)\, \theta\left(\|x - x_s\|\right)\, \mathrm{d}t$$

Groundwater contamination

$$C = \int_{\Omega_G} u(x, T)\, dx$$

$$\Omega_G = \{\mathbf{x} = (x, y, z) \colon z < 0.2\}$$

(groundwater region, 0.2 – arbitrary constant)

Optimization problem – definition

Liquid **F**ossil **F**uel **E**xtraction **P**roblem (**LFFEP**)

N_P, N_S – fixed number of pumps and sinks

$$\textbf{LFFEP} = \begin{cases} \max D & \textit{(extracted fuel)} \\ \min C & \textit{(contamination)} \\ \text{variables :} & x_{P_1}, x_{P_2}, \ldots, x_{P_{N_P}} \\ & x_{S_1}, x_{S_2}, \ldots, x_{S_{N_S}} \\ & \textit{(pumps/sinks location)} \\ \text{constraints :} & x_{P_1}, x_{P_2}, \ldots, x_{P_{N_P}} \in \Omega \\ & x_{S_1}, x_{S_2}, \ldots, x_{S_{N_S}} \in \Omega \end{cases}$$

Variational formulation:

$$(\partial_t u, v)_\Omega + b(u, v) = (f, v)_\Omega$$

where $b(u, v) = (\mathcal{L}u, v)_\Omega$

Semidiscretization:

$$(\partial_t u_h, v_h)_\Omega + b(u_h, v_h) = (f, v_h)_\Omega$$
$$v_h \in \mathcal{V}_h = \text{span}\{\phi_1, \ldots, \phi_n\}$$
$$u_h(x, t) = u_1(t)\phi_1(x) + \ldots + u_n(t)\phi_n(x)$$

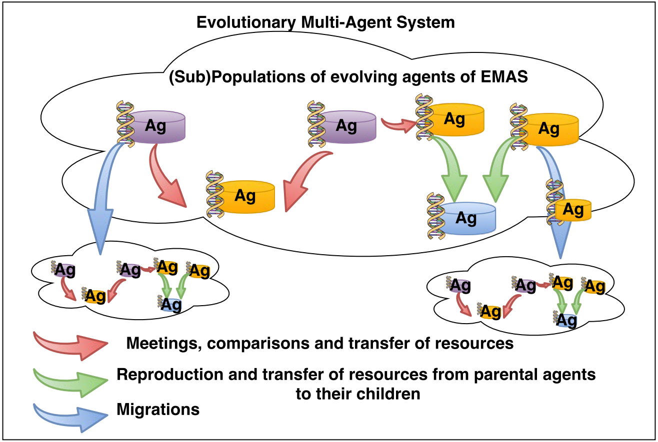
$\Rightarrow$ ODE system wrt. time

$$\mathcal{M} u_h' = \mathcal{B}\, u_h + \mathcal{F}$$

Forward Euler method:

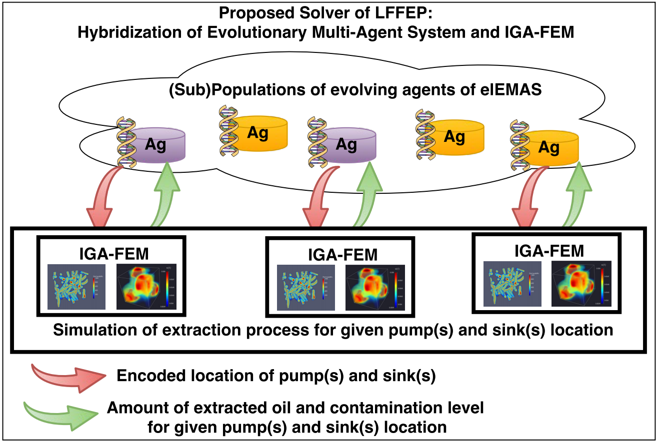
$$\mathcal{M} u_h^{(t+1)} = \mathcal{M} u_h^{(t)} + \Delta t \left(\mathcal{B} u_h^{(t)} + \mathcal{F} \right)$$

EMAS



Distributed agent-based platform for optimization using genetic algorithms

EMAS – hybrid



Genetic code in agents models location of pumps and sinks

Evaluation returns amount of extracted oil and contamination

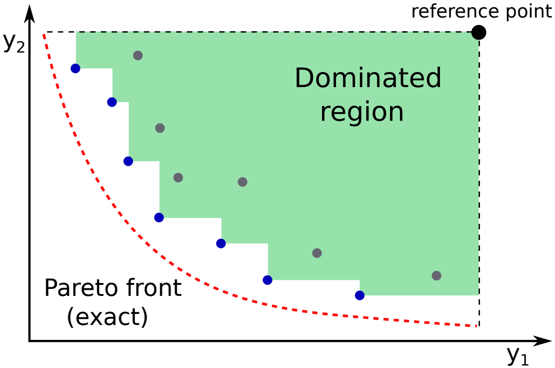
Experiment

Parameters

- 3 pumps, 1 sink ($3 \times (3+1) = 12$ variables)
- 2 objectives – maximize D, minimize C
- quality measure – hypervolume (HV)
- 5 runs for each algorithm
- simulation:
  - $32 \times 32 \times 32$ mesh
  - timestep $\Delta t = 10^{-7}$ (stability)
  - 10,000 iterations

Hypervolume metric

HV (hypervolume) – popular solution quality metric

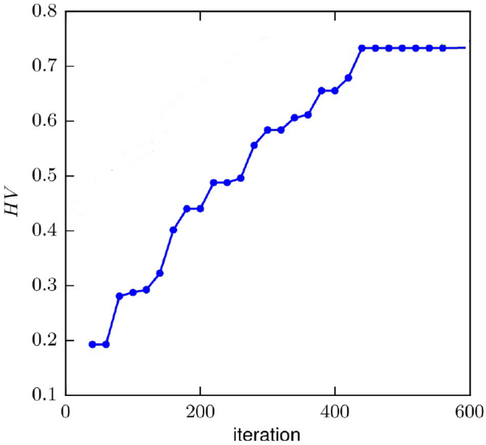


HV = volume of the dominated region

Simulations

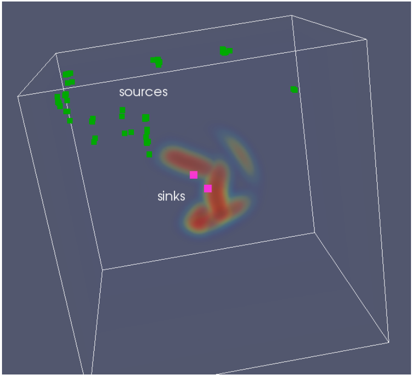
- 20 EMAS agents in population
- 25 steps of EMAS
- Single evaluation by IGA-FEM includes 10,000 time steps (around 10 minutes)
- Total sequential time 25*20=500 evaluations * 10 minutes = **3,5 days**
  using 1 sequential IGA solver
  (Intel(R) Xeon(R) CPU @ 2.40GHz)
- Total parallel time 25*10 minutes = **4 hours**
  using 20 processors and sequential IGA solvers
- Total parallel time 25*40 seconds = **16 minutes**
  using 20 processors with 16 working cores
  (one parallel multi-thread IGA solver per 1 processor 16 cores)

Results – solution quality



	Best final HV	Mean HV	Std. Dev.
EMAS	0.745	0.664	0.081

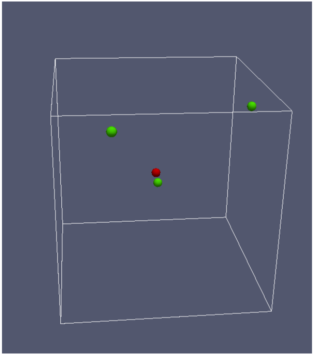
Pareto-optimal pump/sink locations



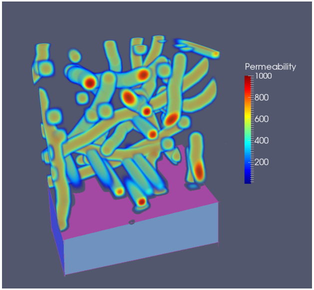
EMAS

- sinks in the center
- pumps – uniform distribution of three pumps above the sinks (sum of different solutions presented in green)
- no pumps near the groundwater area

Exemplary IGA-FEM simulation (1/2)



Locations of 3 pumps and 1 sink



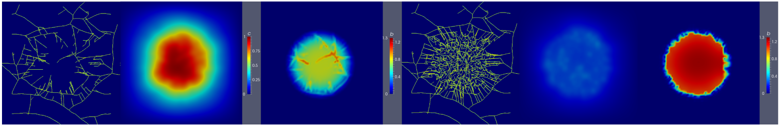
Formation map

Exemplary IGA-FEM simulation (2/2)

Please click in the middle

Conclusions and future research

- Multiobjective EMAS optimization applied to fracking problem to find a compromise between profit and damage
- Isogeometric L2 projections as efficient primal problem solver
- Future research
  - propose better PDE modeling the problem (*better physics*)
  - use more accurate formation map (*to see the rocks*)
  - adaptively increase the mesh size when HV is small (*better accuracy and resolution*)
  - massive parallel IGA + EMAS computations
  - applications for other problems (*ongoing project: tumor growth*)



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Thank you for attention

Questions...?