Computational cost of IGA-FEM direct solvers over *h* refined grids with T-splines and B-splines

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Marcin Skotniczny
Grzegorz Gurgul

Outline

1. Motivation:
   Direct solvers performance on h adaptive grids for standard FEM

2. Computational complexities of IGA-FEM using T-splines
   over grids refined towards singularities

3. Computational complexities of IGA-FEM using
   B-splines with C0 separators (refined Isogemetirc Analysis (**rIGA**))
   over grids refined towards singularities

4. Conclusions

Mesh based solvers and element partition trees

M. Paszyński, **Fast Solvers for mesh-based computations,** Taylor & Franics, CRC Press, 2016

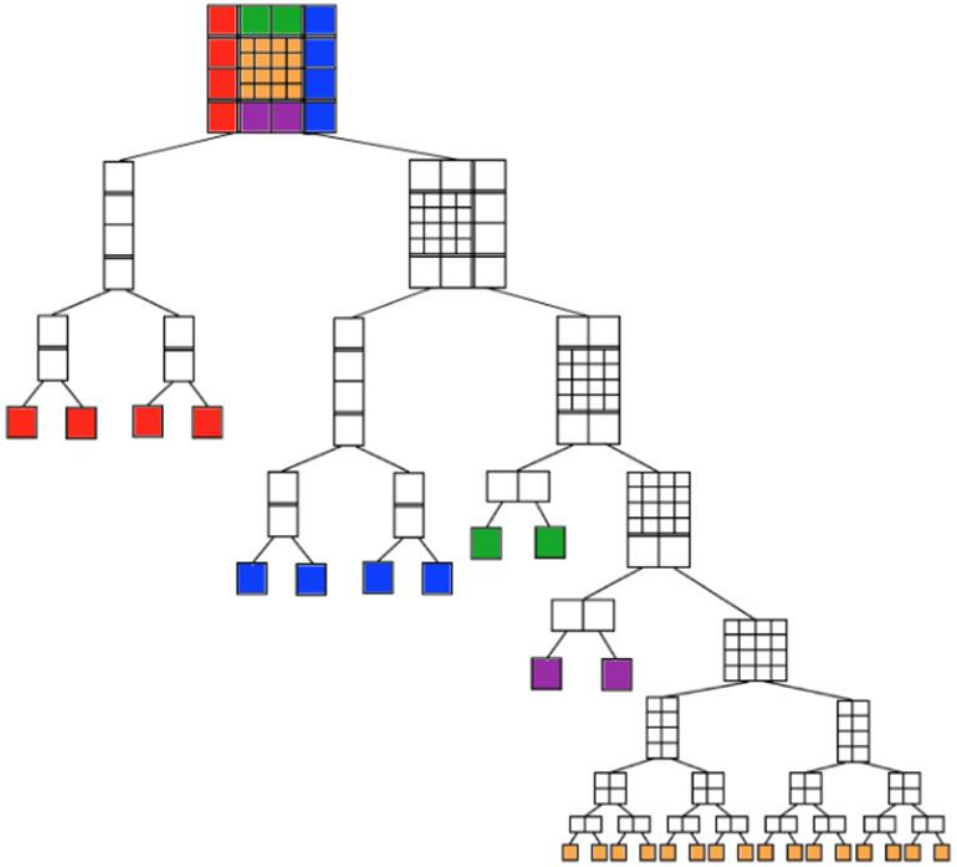
Direct solver

can be controlled by

Element partition tree

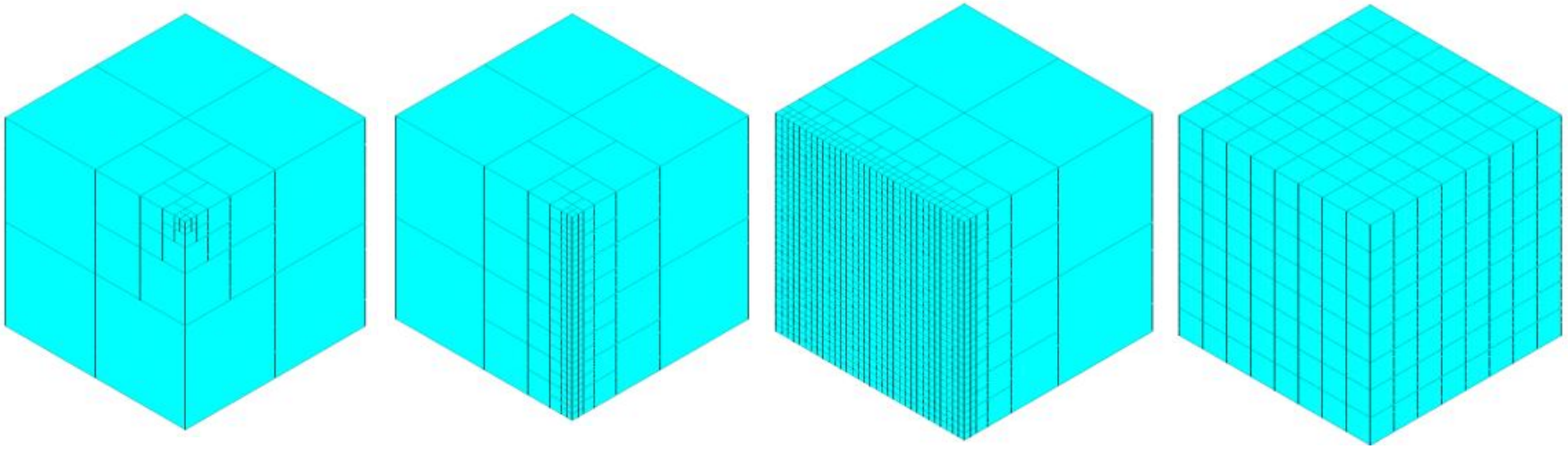
generates

Ordering
$P^{-1}AP$



Motivation

Maciej Paszynski, David Pardo, Victor Calo, **Direct solvers performance on h-adapted grids**, *Computers & Mathematics with Applications*, 70(3) 2015, 282–295

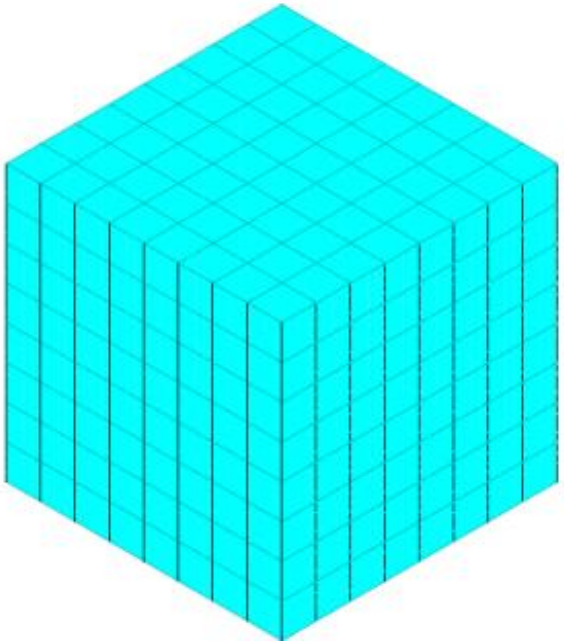


$N = N_e p^3$

hp3d	$O(N_e p^6)$	$O(N_e p^6)$	$O(N_e^{1.5} p^{4.5})$	$O(N_e^2 p^6)$
	p^6 per element	p^6 per element	$N_e^{0.5} p^{4.5}$ per element	$N_e p^6$ per element

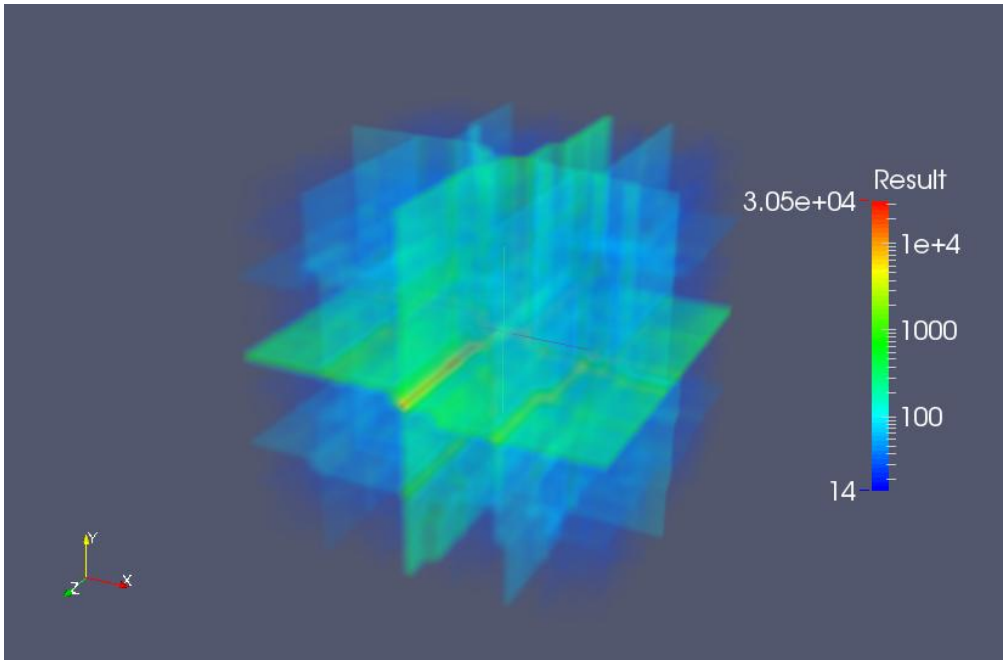
Motivation

Average cost per dof

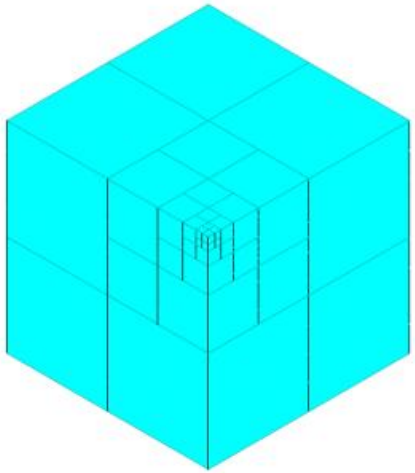


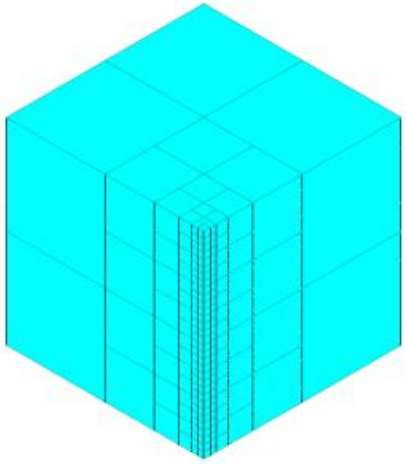
$N_e p^6$ per dof

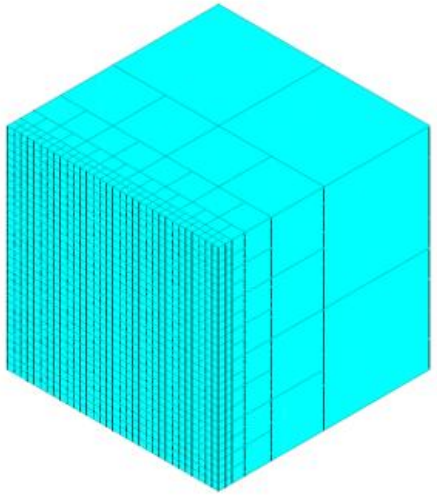
Exact cost per dof

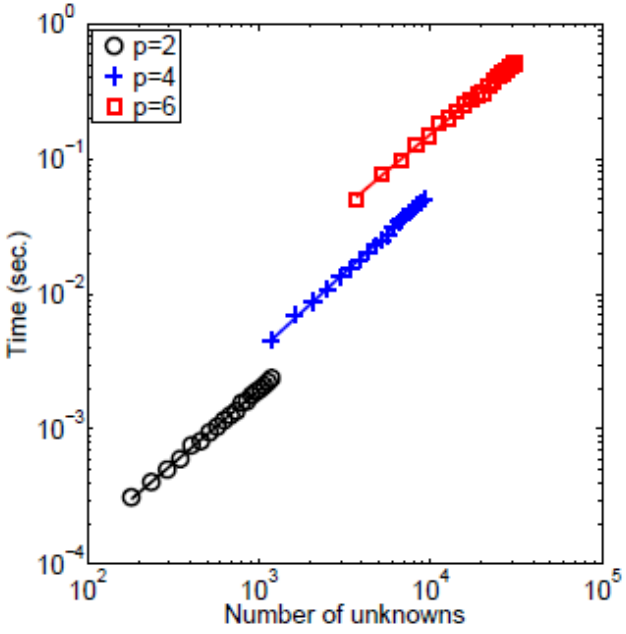


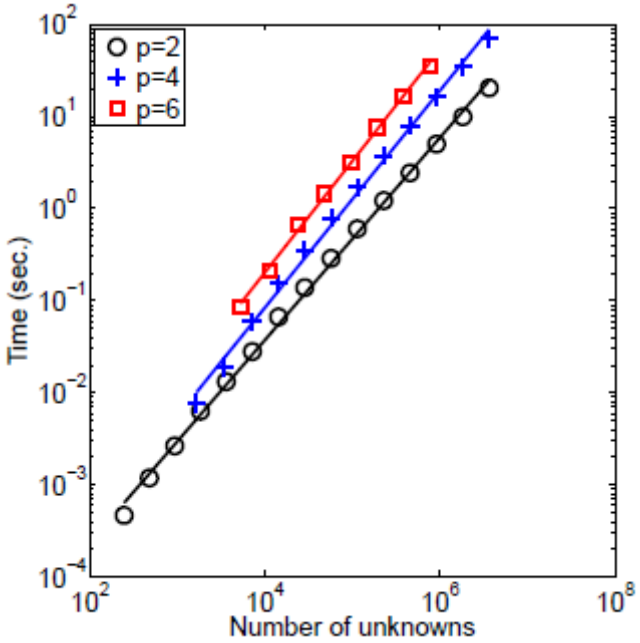
Motivation

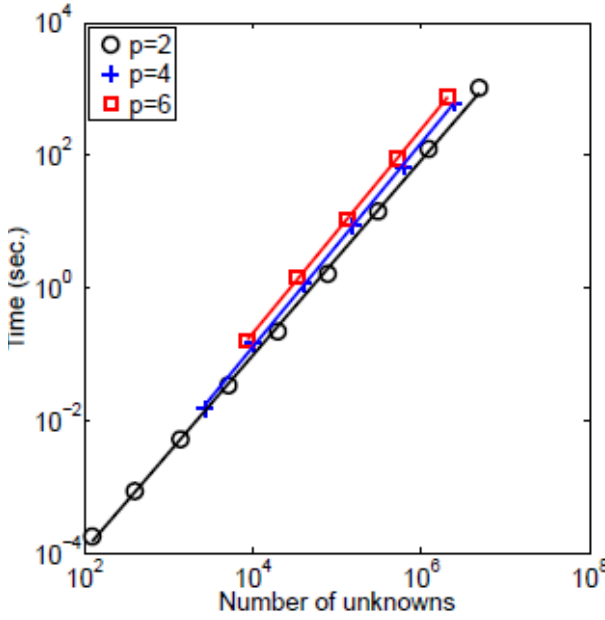












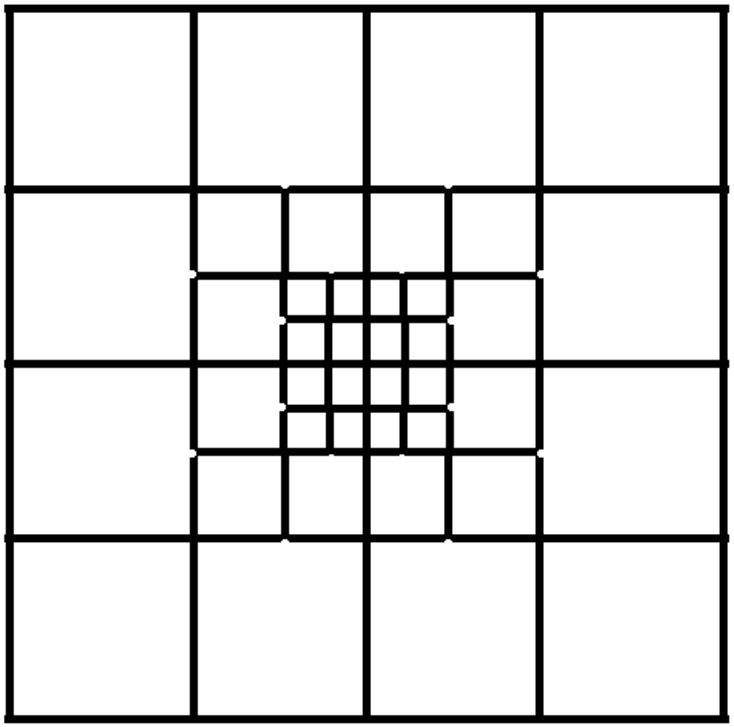
Seconds

Minutes

Hours

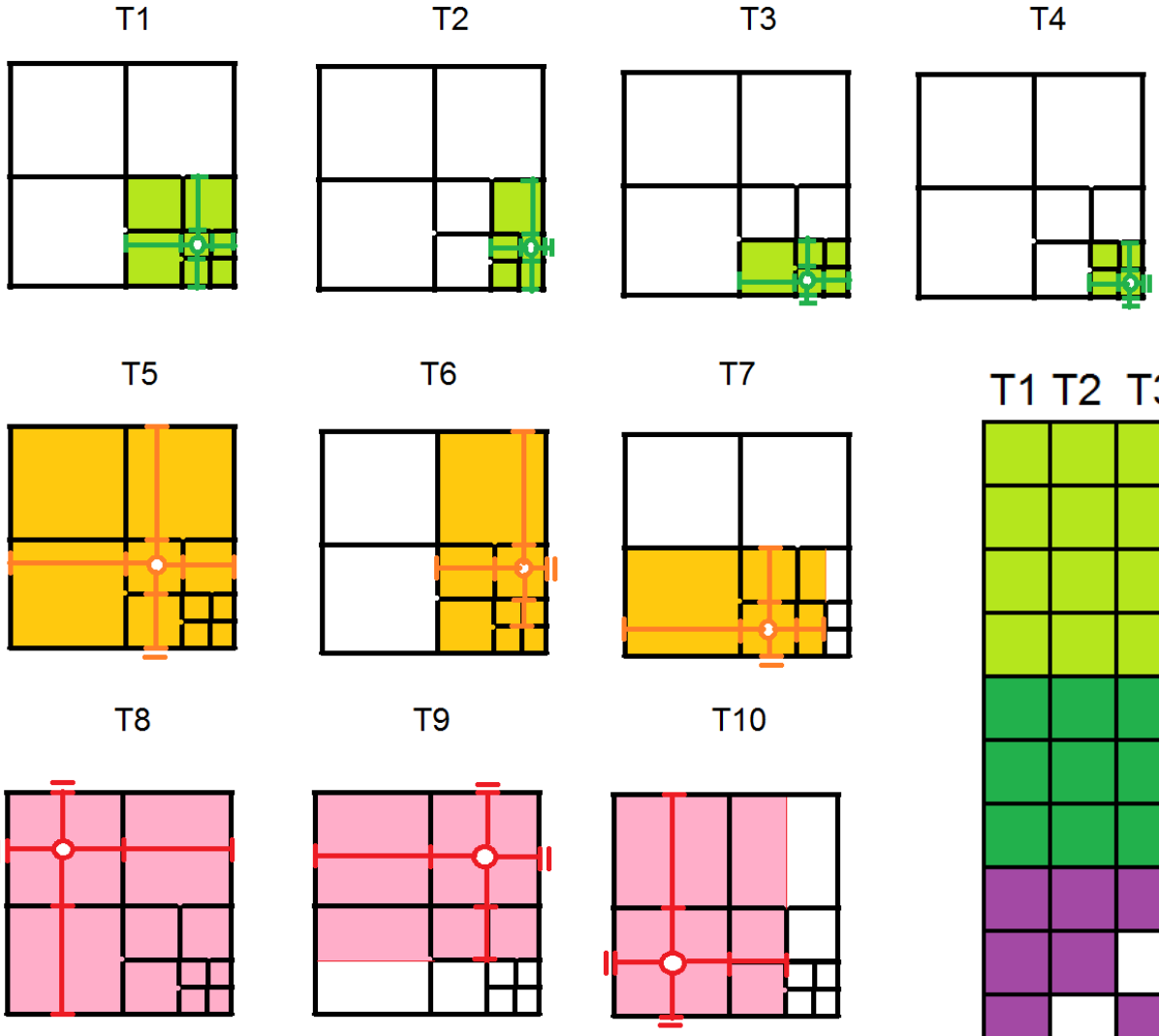
Computational costs with T-splines

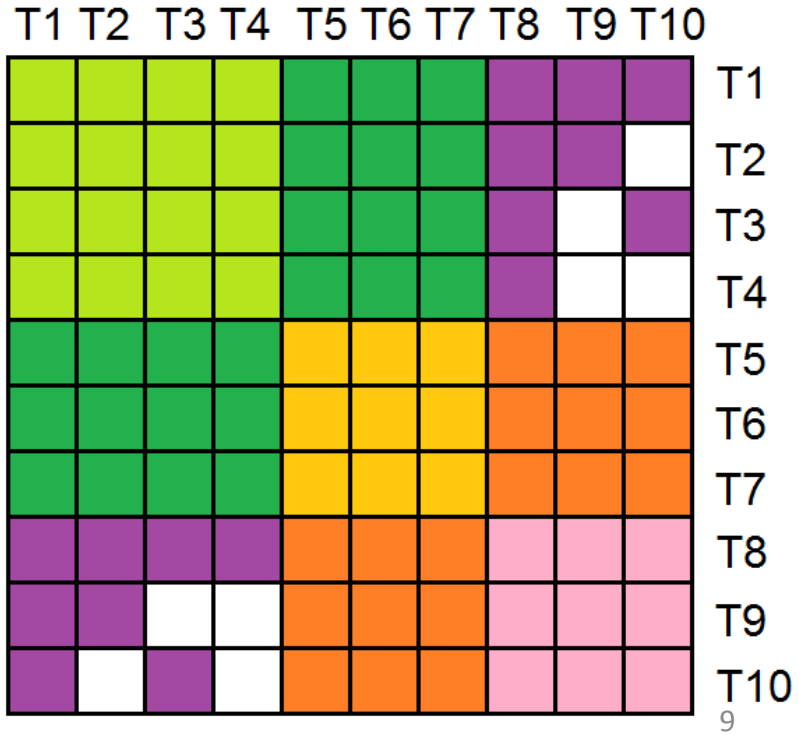
Naive definition of T-splines over the grid without T-junction extensions



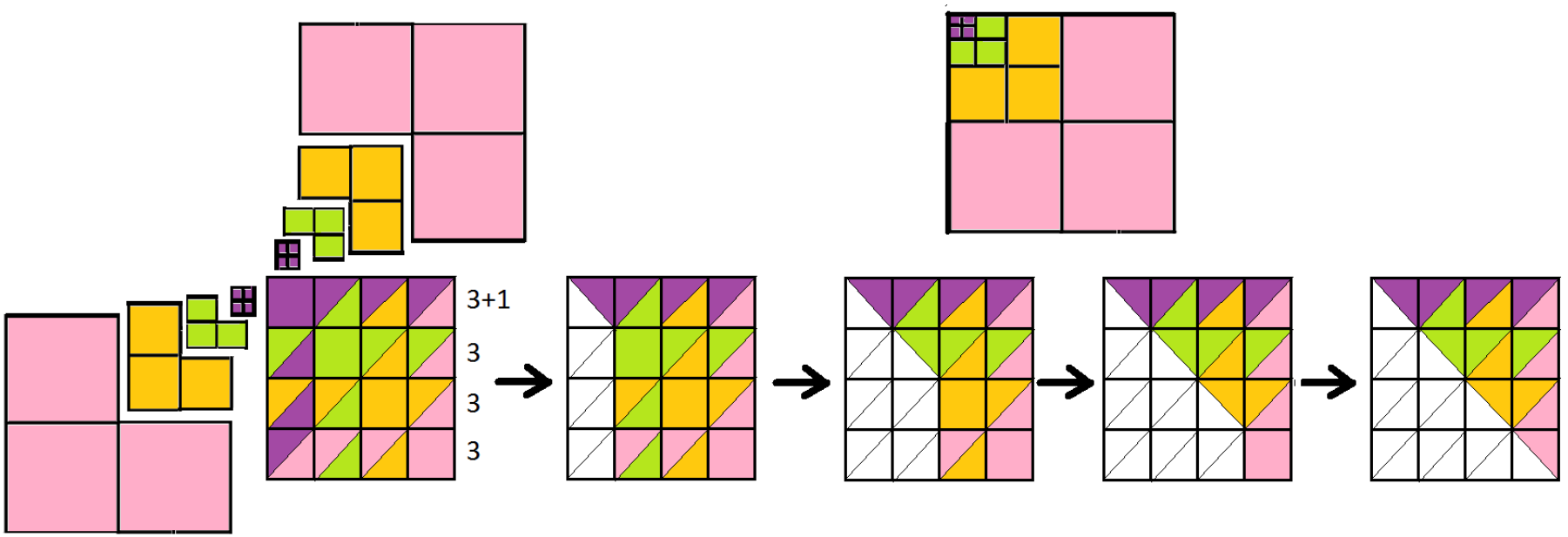
T-splines over 2D grid with point singularity

T-splines over 2D grid: Matrix





Elimination with T-splines over 2D grid



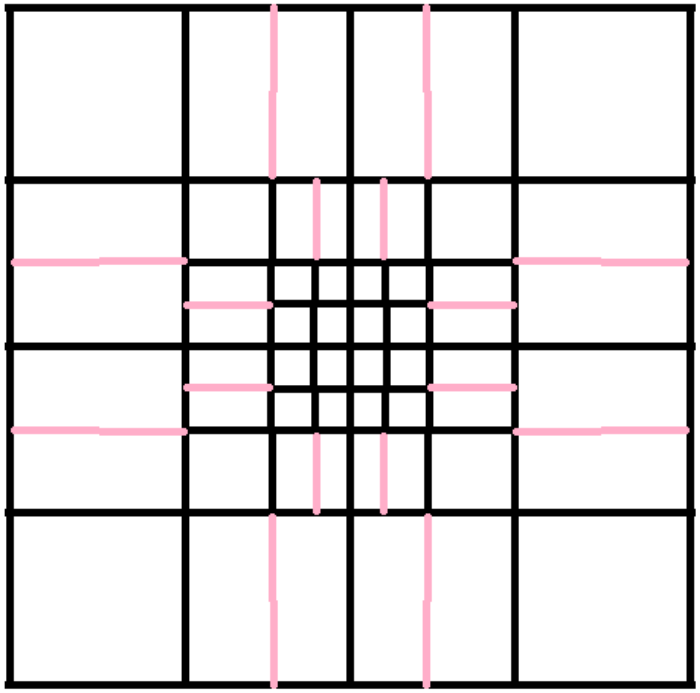
Computational complexity $O(N^3)$

Computational cost with T-splines over analysis suitable mesh (AS T-mesh)

Following paper

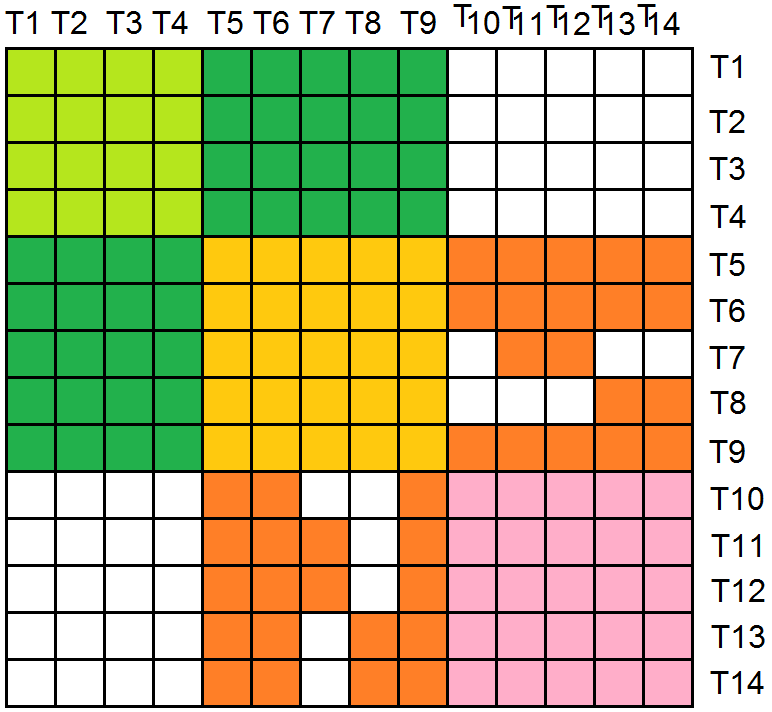
L. Beirao da Veiga, A. Buffa, G. Sangalli, R. Vazquez,
Analysis-suitable T-splines of arbitrary degree: definition and properties,
Mathematical Models and Methods in Applied Sciences, 23(11) 2013

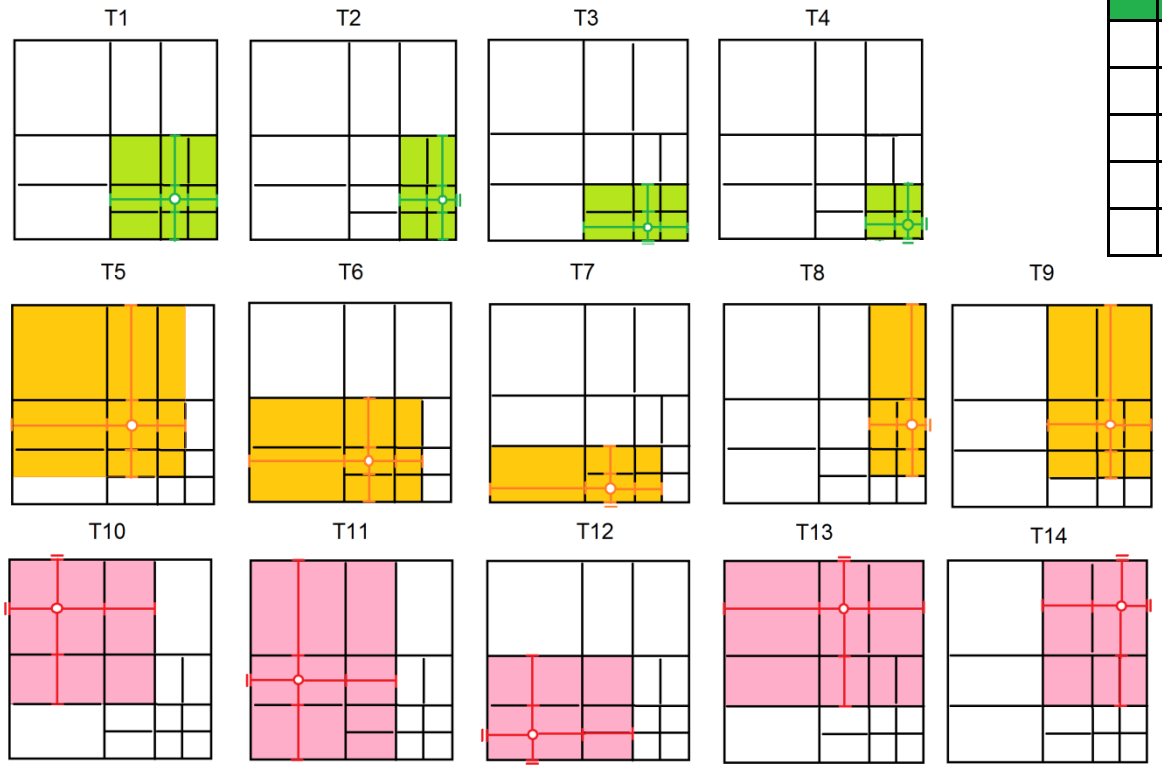
we add T-junction extensions



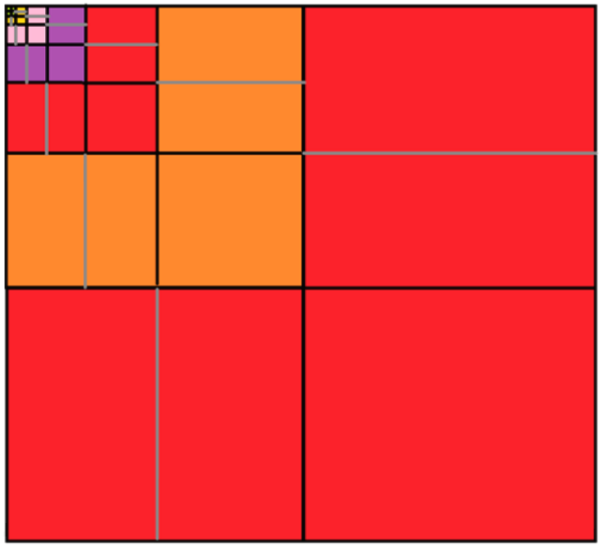
T-splines over 2D AS T-mesh

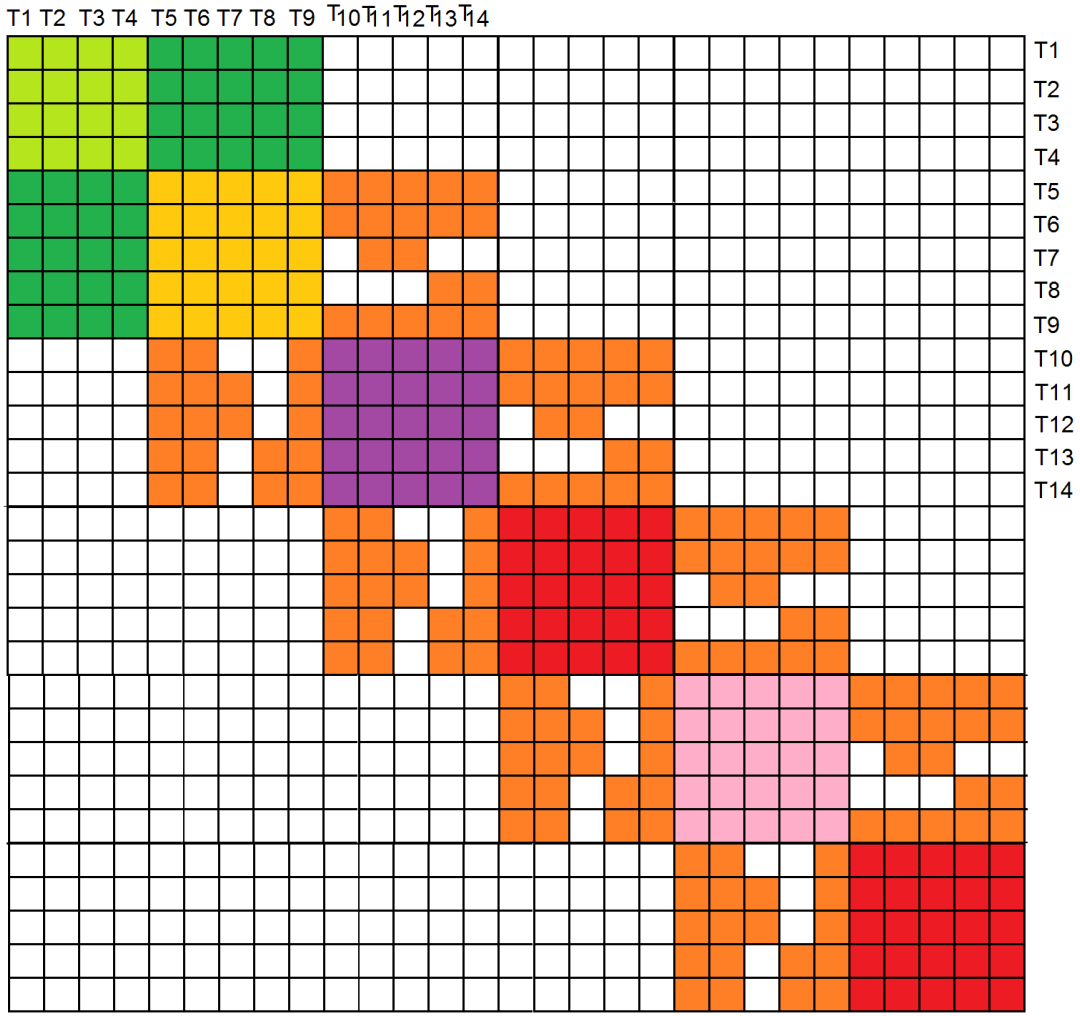
T-splines over 2D AS T-mesh: Matrix



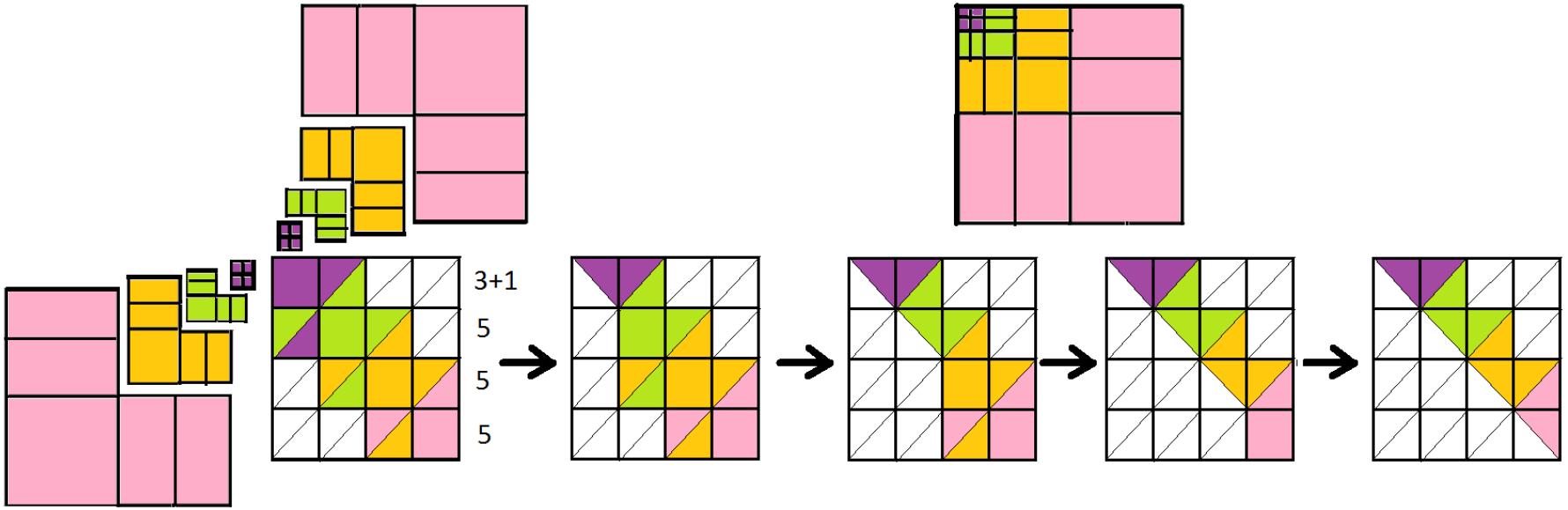


T-splines over 2D AS T-mesh: Matrix



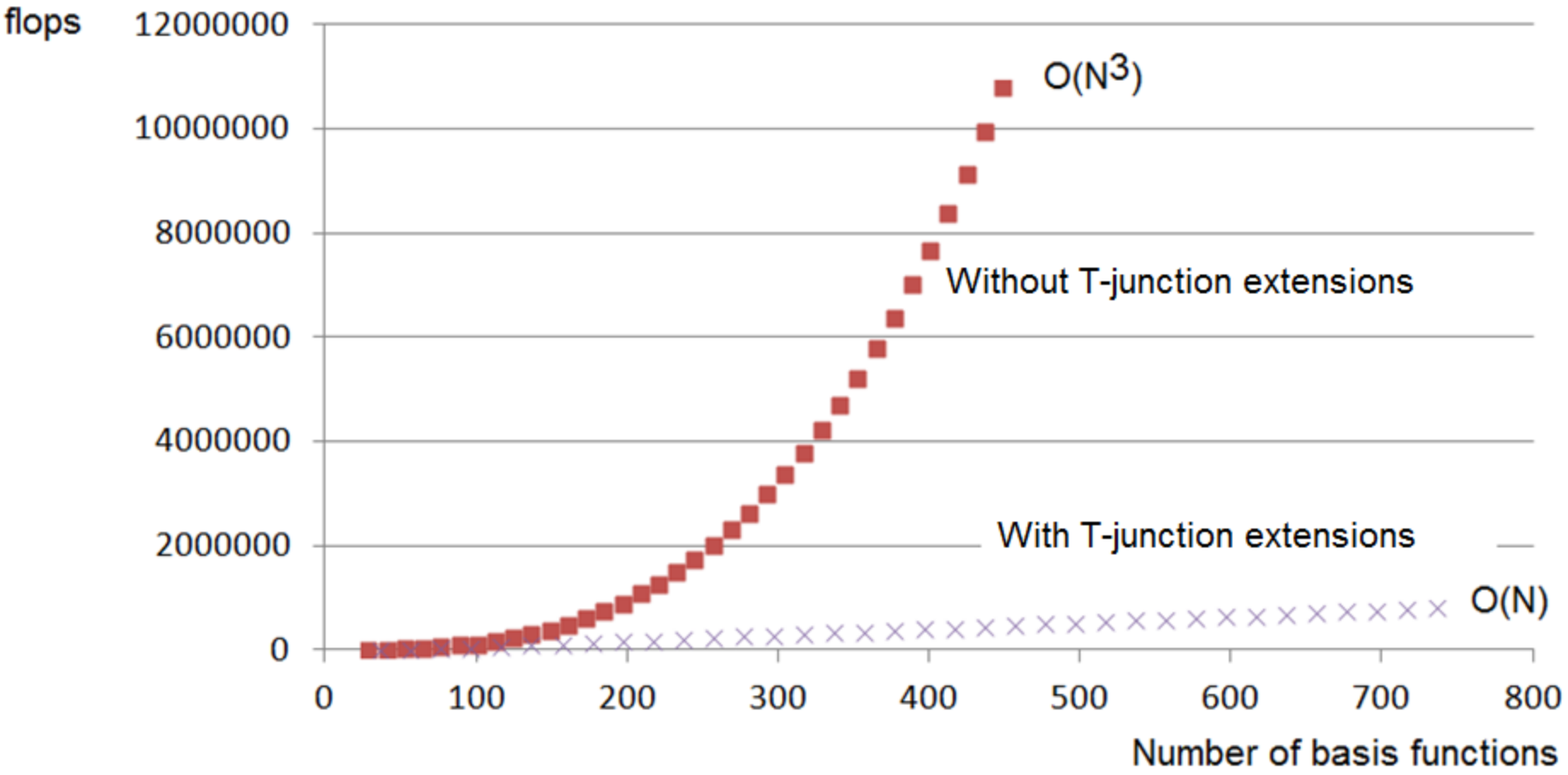


Elimination with T-splines over 2D AS T-mesh

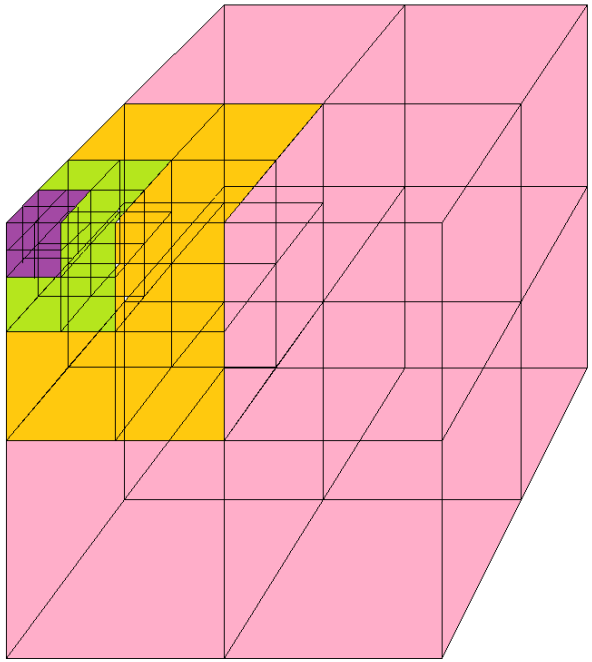


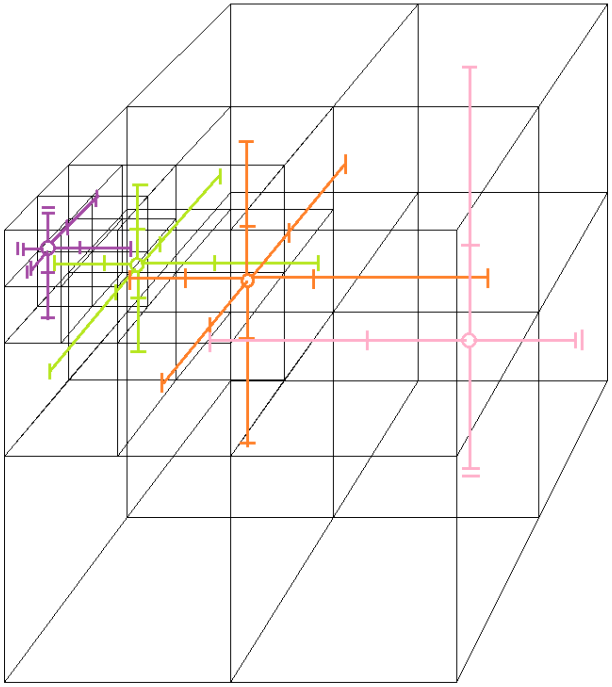
T-spline basis functions over analysis suitable T-mesh

Comparison of methods for T-splines



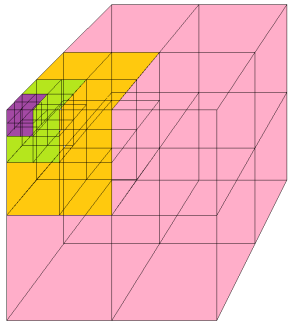
T-splines over 3D mesh with point singularity

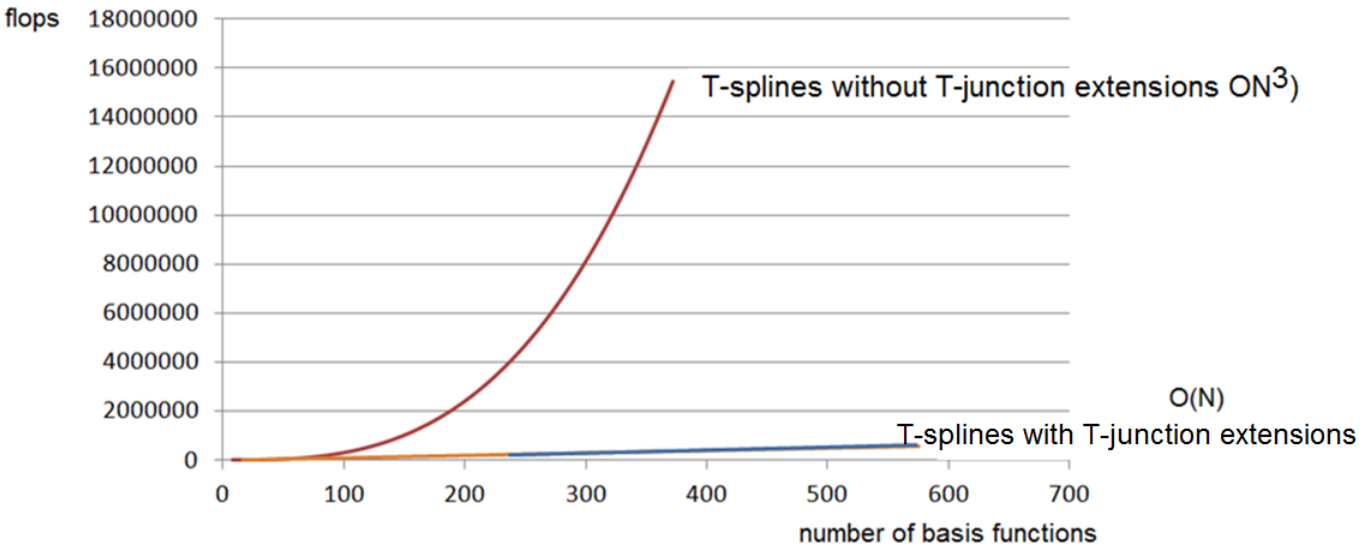


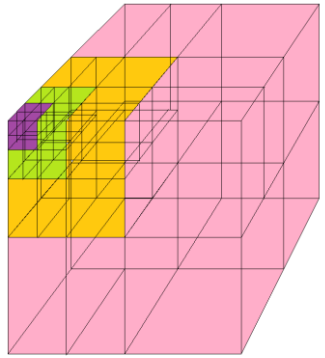


The diagonal T-splines overlap

Comparison of methods

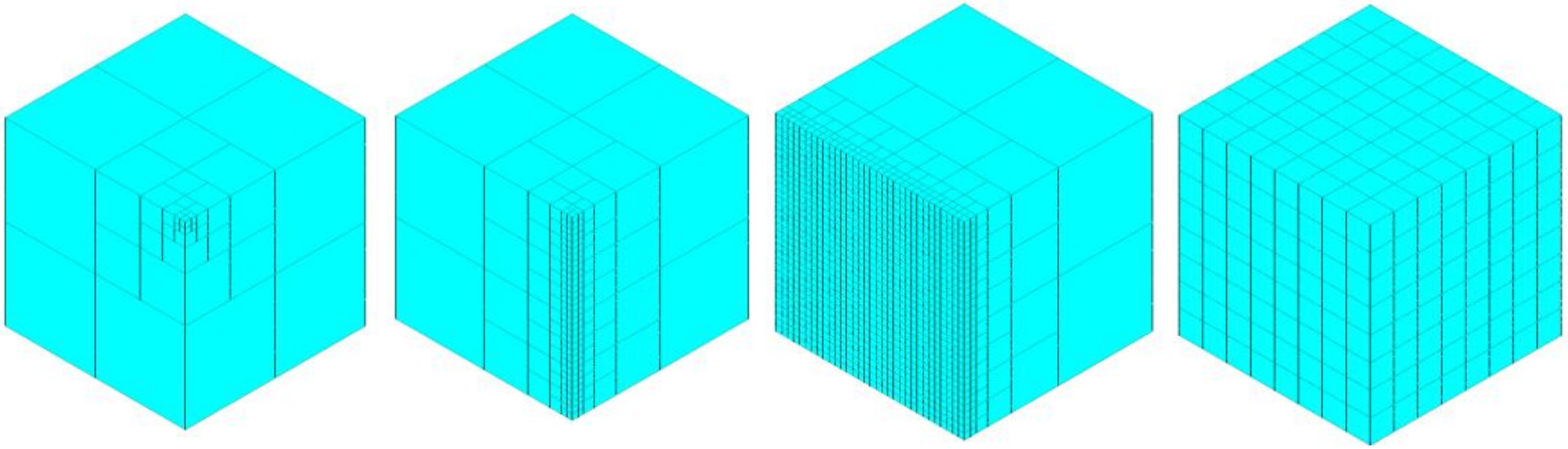






Computational complexities for T-splines

L. Beirao da Veiga, A. Buffa, G. Sangalli, R. Vazquez,
Analysis-suitable T-splines of arbitrary degree: definition and properties,
Mathematical Models and Methods in Applied Sciences, 23(11) 2013

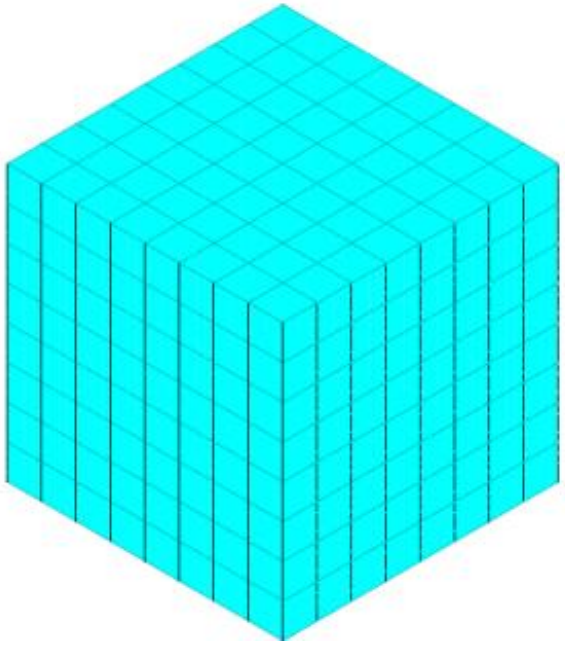


hp3d	$O(N_e p^6)$	$O(N_e p^6)$	$O(N_e^{1.5} p^{4.5})$	$O(N_e^2 p^6)$
	p^6 per element	p^6 per element	$N_e^{0.5} p^{4.5}$ per element	$N_e p^6$ per element
T-splines AS T-mesh	$O(N_e p^3)$	$O(N_e p^3)$	$O(N_e^{1.5} p^3)$	$O(N_e^2 p^3)$
	p^3 per element	p^3 per element	$N_e^{0.5} p^3$ per element	$N_e p^3$ per element

Lower cost per element of IGA-FEM vs FEM on adaptive girds

Motivation

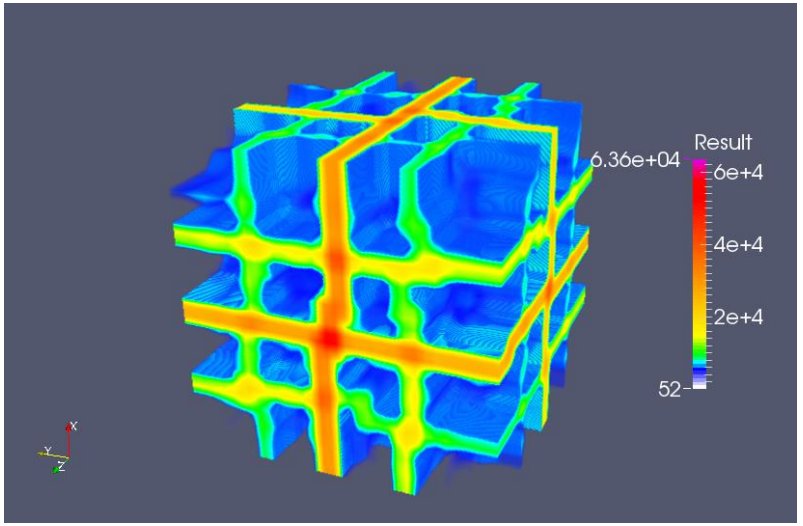
Average cost per dof



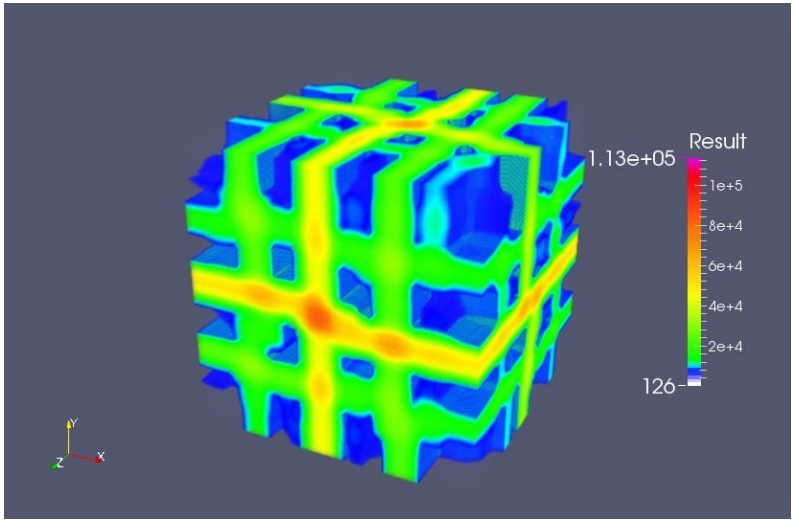
$N_e p^3$ per dof

Exact cost per dof

p=2

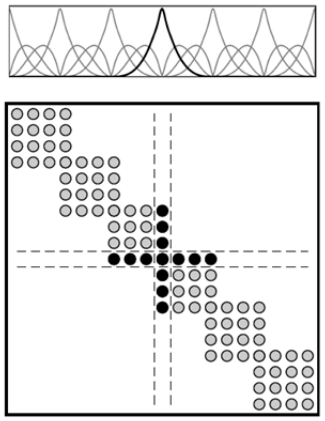


p=3

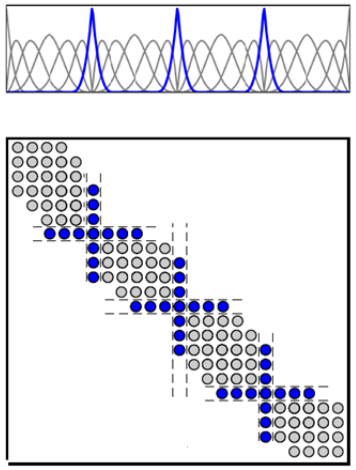


B-splines with C^0 separators (refined Isogeometric Analysis (**rIGA**))

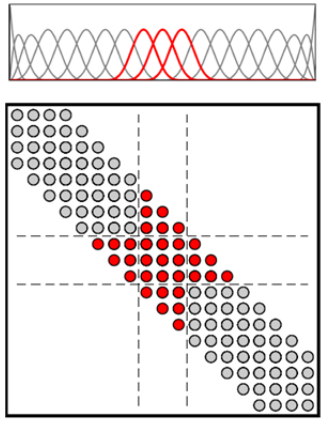
Daniel Garcia, David Pardo, Lisandro Dalcin, Maciej Paszynski, Victor M. Calo,
Refined Isogeometric Analysis (rIGA): Fast Direct Solvers by Controlling Continuity,
submitted to *Computer Methods in Applied Mechanics and Engineering,* 2016



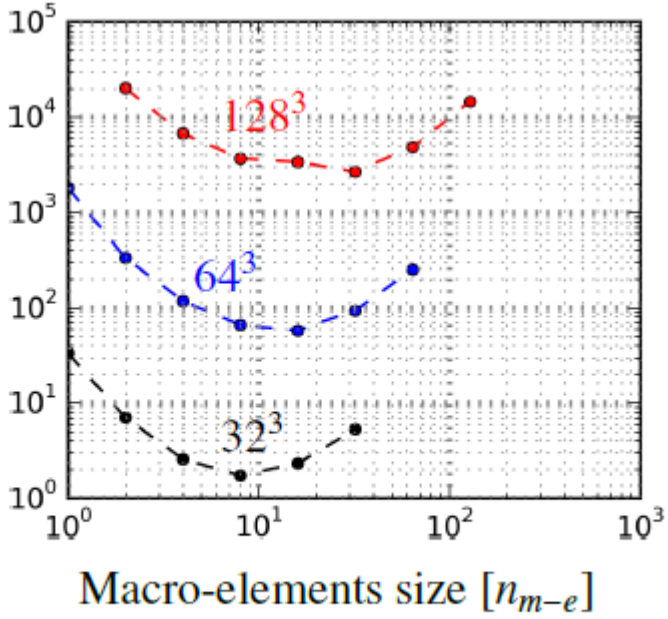
FEM



*r*IGA

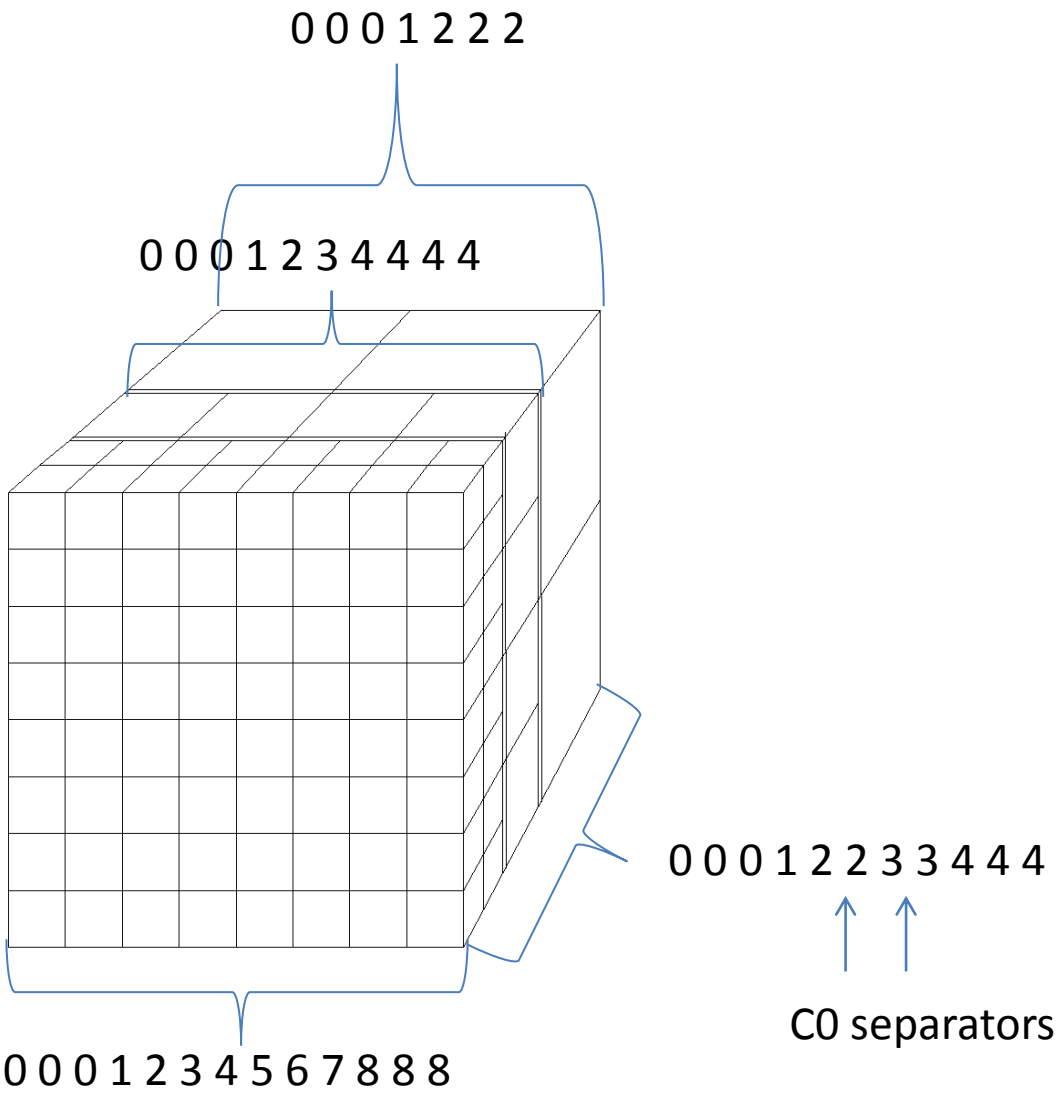


IGA-FEM

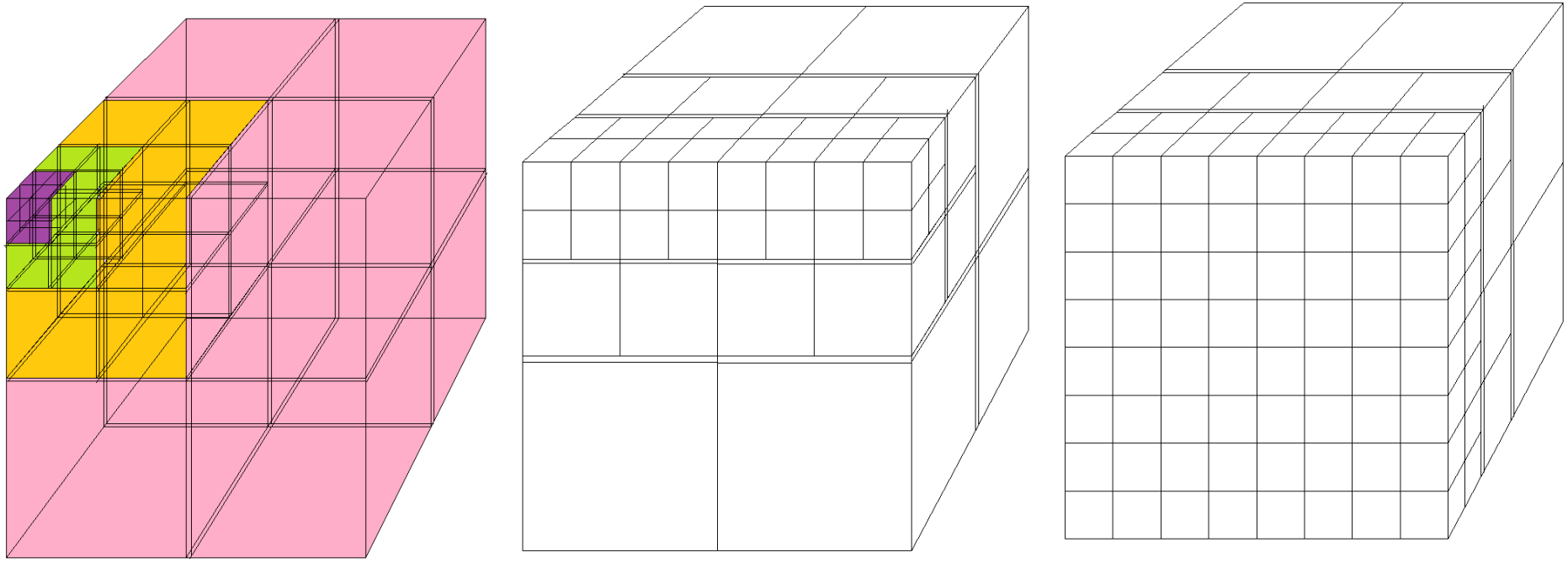


Polynomial order $p = 2$

B-splines with C^0 separators (refined Isogeometric Analysis (**rIGA**))

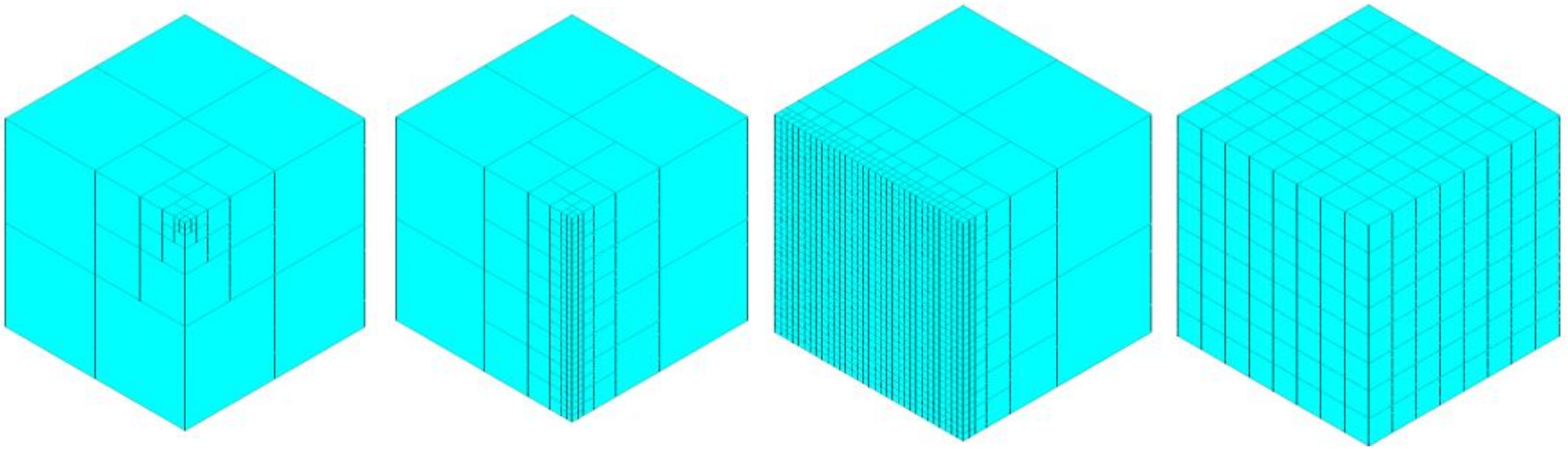


B-splines with C^0 separators (refined Isogeometric Analysis (**rIGA**))



Point singularity here
it is equivallent to hp3d

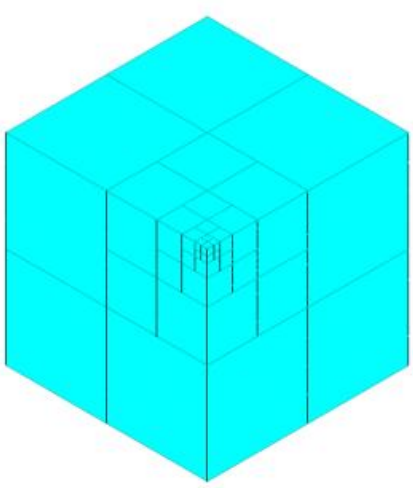
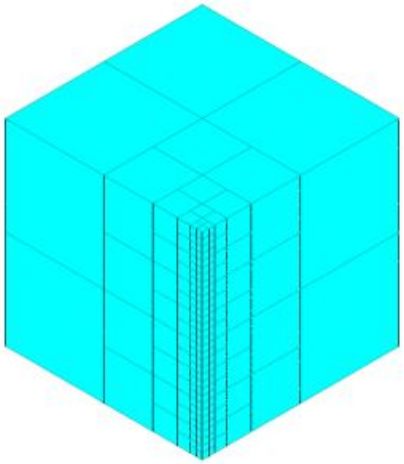
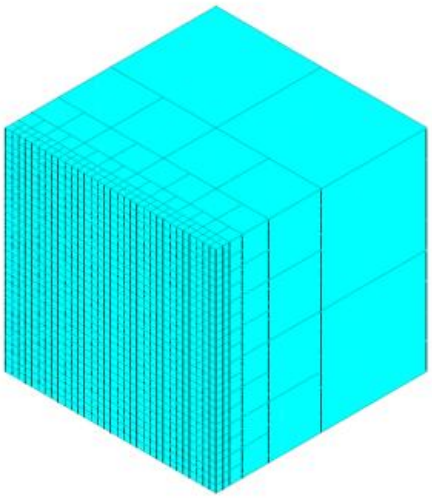
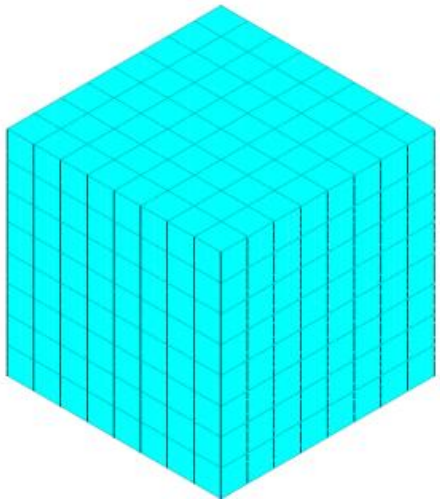
Computational complexity summary



hp3d	$O(N_e p^6)$	$O(N_e p^6)$	$O(N_e^{1.5} p^{4.5})$	$O(N_e^2 p^6)$
T-splines AS T-mesh	$O(N_e p^3)$	$O(N_e p^3)$	$O(N_e^{1.5} p^3)$	$O(N_e^2 p^3)$
B-splines C0 separators	$O(N_e p^6)$	$O(N_e p^6)$	$O(N_e^{1.5} p^{4.5})$	$O(N_e^2 p^6)$

Identical computational complexities on adaptive grids of FEM and rIGA

Computational cost summary

hp3d	$C_{hp3d}N_e p^6$)	$C_{hp3d}N_e p^6$	$C_{hp3d}N_e^{1.5}p^{4.5}$	$C_{hp3d}N_e^{2}p^6$
B-splines C0 separators	$C_{rIGA}N_e p^6$	$C_{rIGA}N_e p^6$	$C_{rIGA}N_e^{1.5}p^{4.5}$	$C_{rIGA}N_e^{2}p^6$
Ratio	$C_{hp3d} / C_{rIGA} = 1$	$C_{hp3d} / C_{rIGA} \approx 2$	$C_{hp3d} / C_{rIGA} \approx 10$	$C_{hp3d} / C_{rIGA} \approx 100$

Conclusions

Computational complexity per element
of FEM and IGA-FEM with analysis suitable T-splines

a) p^6 (FEM) → p^3 (IGA-FEM) point and edge singularities
b) $N_e^{0.5}p^{4.5}$ (FEM) → $N_e^{0.5}p^3$ (IGA-FEM) face singularity
c) $N_e p^6$ (FEM) → $N_e p^3$ (IGA-FEM) uniform grid (IGA-FEM)

IGA-FEM per element always wins

Computational costs of **B-splines with C0 separators (rIGA)** vs **FEM**

a) Identical for point singularities
b) ≈ 2 times faster for edge singularity
c) ≈ 10 times faster for face singularity
d) ≈ 100 times faster for uniform grids

B-splines with C0 separators (rIGA) always wins

Future work

- Developing estimates for parallel direct solvers for adaptive grids
- Incorporating automatic algorithms for adding T-junction extensions into automatic hp adaptive finite element method in two- and three- dimensions
- Incorporating B-splines with C0 separators into automatic hp adaptive finite element methods in two- and three- dimensions
- Implementation of the T-spline and rIGA adaptive packages working on the element partition tree based workflow in the cloud environment (collaboration with Marin Bubak, AGH)

This work has been suported by National Science Centre, Poland grant no. DEC-2012/07/B/ST6/01229.