Zestaw 2 - granice ciągów

1. Oblicz (jeżeli istnieje) granicę ciągu $(a_n)_{n\in\mathbb{N}}$, jeśli:

   (a) $a_n = \frac{n}{n+1}$,

   (b) $a_n = \frac{4n-3}{6-5n}$,

   (c) $a_n = \frac{n^2-1}{3-n^3}$,

   (d) $a_n = \frac{2n^3-4n-1}{6n+3n^2-n^3}$,

   (e) $a_n = \frac{(n-1)(n+3)}{3n^2+5}$,

   (f) $a_n = \frac{(2n-1)^2}{(4n-1)(3n+2)}$,

   (g) $a_n = \frac{(2n-1)^3}{(4n-1)^2(1-5n)}$,

   (h) $a_n = \frac{3}{n} - \frac{10}{\sqrt{n}}$,

   (i) $a_n = \frac{(-1)^n}{2n-1}$,

   (j) $a_n = \left(\frac{2n-3}{3n+1}\right)^2$,

   (k) $a_n = \left(\frac{5n-2}{3n-1}\right)^3$,

   (l) $a_n = \frac{(\sqrt{n}+3)^2}{n+1}$,

   (m) $a_n = \frac{\sqrt{n}-2}{3n+5}$,

   (n) $a_n = \frac{n-10}{3}$,

   (o) $a_n = \frac{(-0,8)^n}{2n-5}$,

   (p) $a_n = \frac{2-5n-10n^2}{3n+15}$,

   (q) $a_n = \frac{2n+(-1)^n}{n}$,

   (r) $a_n = \frac{\sqrt{1+2n^2}-\sqrt{1+4n^2}}{n}$,

   (s) $a_n = \sqrt{\frac{3n-2}{n+10}}$,

   (t) $a_n = \sqrt[3]{\frac{n-1}{8n+10}}$,

   (u) $a_n = \frac{\sqrt{n^2+4}}{3n-2}$,

   (v) $a_n = \frac{\sqrt{n^2-1}}{\sqrt[3]{n^3+1}}$,

   (w) $a_n = \frac{n}{\sqrt[3]{8n^3-n}-n}$,

   (x) $a_n = \frac{1}{\sqrt{4n^2+7n}-2n}$.

2. Oblicz (jeżeli istnieje) granicę ciągu $(a_n)_{n\in\mathbb{N}}$, jeśli:

   (a) $a_n = \sqrt{n+2} - \sqrt{n}$,

   (b) $a_n = \sqrt{n^2+n} - n$,

   (c) $a_n = n - \sqrt{n^2+5}$,

(d) $a_n = \sqrt{3n^2 + 2n - 5} - n\sqrt{3}$,

(e) $a_n = 3n - \sqrt{9n^2 + 6n - 15}$,

(f) $a_n = \sqrt[3]{n^3 + 4n^2} - n$,

(g) $a_n = n\sqrt[3]{2} - \sqrt[3]{2n^3 + 5n^2 - 7}$.

3. Oblicz (jeżeli istnieje) granicę ciągu $(a_n)_{n\in\mathbb{N}}$, jeśli:

(a) $a_n = \frac{4^{n-1}-5}{2^{2n}-7}$,

(b) $a_n = \frac{5\cdot 3^{2n}-1}{4\cdot 9^n+7}$,

(c) $a_n = \frac{3\cdot 2^{2n+2}-10}{5\cdot 4^{n-1}+3}$,

(d) $a_n = \frac{-8^{n-1}}{7^{n+1}}$,

(e) $a_n = \frac{2^{n+1}-3^{n+2}}{3^{n+2}}$,

(f) $a_n = \left(\frac{3}{2}\right)^n \frac{2^{n+1}-1}{3^{n+1}-1}$.

4. Oblicz (jeżeli istnieje) granicę ciągu $(a_n)_{n\in\mathbb{N}}$, jeśli:

(a) $a_n = \sqrt[n]{3^n + 2^n}$,

(b) $a_n = \sqrt[n]{10^n + 9^n + 8^n}$,

(c) $a_n = \sqrt[n]{10^100} - \sqrt[n]{\frac{1}{10^100}}$,

(d) $a_n = \sqrt[n]{(\frac{2}{3})^n + (\frac{3}{4})^n}$.

5. Obliczyć granicę ciągu o wyrazie ogólnym:

(a) $a_n = \frac{1+2+...+n}{n^2}$,

(b) $a_n = \frac{1+\frac{1}{2}+\frac{1}{4}+...+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+...+\frac{1}{3^n}}$.

6. Obliczyć granicę ciągu o wyrazie ogólnym:

(a) $a_n = \left(1 + \frac{2}{n}\right)^n$,

(b) $a_n = \left(1 - \frac{1}{n^2}\right)^n$,

(c) $a_n = \left(\frac{n+5}{n}\right)^n$,

(d) $a_n = \left(1 - \frac{3}{n}\right)^n$,

(e) $a_n = \left(1 - \frac{4}{n}\right)^{-n+3}$,

(f) $a_n = \left(\frac{n^2+6}{n^2}\right)^{n^2}$,

(g) $a_n = \left(\frac{n^2+2}{2n^2+1}\right)^{n^2}$.

7. Obliczyć granicę ciągu o wyrazie ogólnym:

(a) $a_n = \sqrt{n + \sqrt{n}} - \sqrt{n - \sqrt{n}}$,

(b) $a_n = \sqrt{n(n-\sqrt{n^2-1})}$,

(c) $a_n = n(\sqrt{2n^2+1} - \sqrt{2n^2-1})$,

(d) $a_n = \sqrt[n]{2n^3-3n^2+15}$,

(e) $a_n = \sqrt{n^10 - 2n^2 + 2}$,

(f) $a_n = \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}$,

(g) $a_n = \frac{1}{2n}\cos n^3 - \frac{3n}{6n+1}$,

(h) $a_n = 2^{-n} a \cos n\pi$,

(i) $a_n = n(\ln(n+1) - \ln n)$,

(j) $a_n = \frac{\ln\left(1+\frac{3}{n}\right)}{\frac{1}{n}}$,

(k) $a_n = \frac{\log_2 n^5}{\log_8 n}$,

(l) $a_n = \frac{9^{\log_3 n}}{4^{\log_2 n}}$,

(m) $a_n = \frac{8^{\log_2 n}}{2^n}$.

8. Oblicz (jeżeli istnieje) granicę ciągu $(a_n)_{n\in\mathbb{N}}$, jeśli:

(a) $a_n = \frac{3^n-2^n}{4^n-3^n}$,

(b) $a_n = \frac{1-2+3-4+\ldots-2n}{\sqrt{n^2+1}}$,

(c) $a_n = \frac{(n+1)\cdot\cos(n!)}{n^3+1}$,

(d) $a_n = \left(\frac{n+1}{2n+3}\right)^n$,

(e) $a_n = \frac{(2n+1)^6-(n-1)^6}{(2n+1)^6+(n-1)^6}$,

(f) $a_n = \left(\frac{4n-1}{4n+1}\right)^{n+4}$,

(g) $a_n = \left(\frac{n^2+2n}{n^2+2n+2}\right)^n$,

(h) $a_n = n\cdot\sqrt[3]{2} - \sqrt[3]{2n^3+5n^2-7}$,

(i) $a_n = n[\ln(n+3) - \ln n]$,

(j) $a_n = \sqrt[n]{\left(\frac{1}{2}\right)^n + \left(\frac{2}{3}\right)^n + \left(\frac{1}{5}\right)^n}$,

(k) $a_n = \frac{2^n}{n!}$,

(l) $a_n = 9^n - 8^n + 1$,

(m) $a_n = \frac{9^{\log_3 n}}{4^{\log_2 n}}$,

(n) $a_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}}$,

(o) $a_n = \arctan\left(\frac{n^2+1}{n}\right)$,

(p) $a_n = \frac{1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\ldots+\frac{1}{3^n}}$,

(q) $a_n = \sqrt[n]{2^n+4^n+1}$,

(r) $a_n = \frac{(n+1)!-n!}{(n+1)!+n!}$,

(s) $a_n = \sqrt[3]{n^3+n^2} - n$,

(t) $a_n = \frac{1+3+5+...+(2n-1)}{n+1} - n$,

(u) $a_n = \frac{(2n)!}{n^{2n}}$,

(v) $a_n = \frac{n^{3n}}{(3n)!} \cdot \sin n!$.