Zagadnienia na egzamin z DMRF

3 pytania, po jednym oznaczonym D (max. 1 pkt), T (max. 2 pkt.), O (max. 2 pkt),
liczba punktów = ocena z egzaminu

SINGLE STEP BINOMIAL MARKET MODEL

1.(O1) A general description (the main assumptions, modelling stock and bond prices).

2.(D1) Definition of an arbitrage (in the primary market). No Arbitrage Principle.

3.(T1) NAP in the single step binomial model - theorem + proof.

4.(D2) Definition of a derivative security and its replicating portfolio.

5.(D3) Definition of an arbitrage (in the extended market). No Arbitrage Principle.

6.(T2) Computation of a fair price of a given derivative security using replicating portfolio: theorem + proof.

7.(D4) Definition of a risk-neutral measure. Formula for risk-neutral probability in binomial model.

8.(T3) Computation of a fair price of a given derivative security using risk neutral measure: theorem + proof.

SINGLE STEP TRINOMIAL MARKET MODEL

9.(O2) Problem of a replication in trinomial model. Is it always possible? Give some example and counterexample.

10.(D5) Definition of a super- and a sub-replicating portfolio and super- and sub-replication price of a given derivative security.

11.(D6) No-arbitrage interval: definition.

12.(O3) No-arbitrage interval: properties (choose one and give a proof).

13.(O4) Risk-neutral measure in trinomial model - the question of existence and uniqness. The sufficient and necessary condition for the existence of a risk-neutral measure in trinomial model.

14.(D7) Definition of the interval I_H.

15.(T4) The theorem about the price taken from the interval I_H + proof.

16.(O5) Relation between no-arbitrage interval and I_H (without the proof).

17.(O6) The description of two methods of computing a sub- and a super-replicating portfolio.

A GENERAL FINITE SINGLE STEP MARKET MODEL

18.(D8) The definition of a risk-neutral measure.

19.(T5) The First Fundamental Theorem of Asset Pricing (+ proof of easier implication).

20.(D9) The definition of a complete market model + example and counterexample.

21.(T6) The Second Fundamental Theorem of Asset Pricing (+ proof of easier implication).

22.(O7) Why we aim to work within the setup of a complete model? Explain what are the methods of pricing derivative securites in such a model.

GENERAL PROPERTIES OF DERIVATIVE PRICES

23.(T7) Law of One Price + proof.

24.(T8) Put-call parity + proof.

MULTI-STEP BINOMIAL MODEL

25.(O8) A general description (the main assumptions, modelling stock and bond prices).

26.(O9) Construction of a filtration in a multi-step model.

27.(D10) A conditional expectation - definition.

28.(T9) A conditional expectation w.r.t. a sigma-field generated by a partition - formula + proof.

29.(O10) A conditional expectation - properties.

30.(D11) Definition of a martingale.

31.(T10) The expected value of a martingale: property + proof.

32.(D12) Definition of a strategy and its value in a multi-step model.

33.(D13) Defintion of a self-financing strategy.

34.(D14) Definition of an arbitrage (on the primary market).

35.(D15) Definition of a martingale measure.

36.(O11) What property is satisfied for the process of discounted values of a self financing strategy under the risk neutral measure? Give a proof.

37.(D16) Definition of a derivative security (path-dependent and path-independent).

38.(D17) Definition of a strategy on the extended market and its value.

39.(D18) Definition of an arbitrage on the extended market.

40.(D19) Definition of a process of fair prices of a derivative security.

41.(T11) The Law of One Price in multi-step model + proof.

42.(O12) Definition of a replicating strategy and its relation to the fair price of derivative security which it replicates.

43.(O13) The consequences of the existence of a martingale measure for attainable derivative securities + proof.

44.(T12) CRR formula for the call option + proof.

45.(D20) Fundamental Theorems of Asset Pricing in general multi-step model.

46.(O14) Why we aim to work within the setup of a complete model? Explain what are the methods of pricing derivative securites in such a model.

47.(O15) Explain the idea of a delta hedging.

DERIVATIVE SECURITIES OF AMERICAN TYPE

48.(D21) Definition of a derivative security of an American type.

49.(D22) Definition of an exercise strategy.

50.(D23) Definition of a payoff relative to a given exercise strategy.

51.(D24) Definition of an arbitrage on the market extened by a given American option.

52.(D25) Defitnion of a process of fair prices of a given American option.

53.(O16) Pricing American options: idea + formula (without proof).

54.(D26) Definition of a supermartingale.

55.(D27) Definition of a Snell envelope + example from finance.

56.(O17) The property of a Snell envelope + proof.

57.(T13) Doob's decomposition theorem + proof.

58.(D28) Definition of a compensator of a stochastic process.

59.(D29) Doob's Optional Smapling Theorem.

60.(D30) Definition of an optimal exercise strategy.

61.(D31) Definition of the exercise strategy τ_H.

62.(O18) Main properties of τ_H (choose one and give a proof).

63.(O19) The question of replication of an American option (what does it mean to "replicate an American option"? Is it always possible to replicate it?)

64.(O20) How to hedge an American option?