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THE POLYNOMIAL METHOD FOR VARIATIONS OF THOMASSEN PROOF ON LIST COLOURING

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It is well known that planar graphs are 5-choosable. A brilliant proof-from-the-book for this fact was given by Thomassen. Actually, to proceed with inductive argument, he proved the stronger result.

Theorem 1 *Let G be any plane near-triangulation (every face except the outer one is a triangle) with outer cycle C. Let x, y be two consecutive vertices on C. Then G can be coloured from any list of colours such that the length of lists assigned to x, y, any other vertex on C and any inner vertex is 1, 2, 3, and 5, respectively.*

It is straightforward that the above theorem implies also 3-choosability of outerplanar graphs. Moreover, lists of 2 neighbouring vertices may have some deficiency. Hutchinson characterised the possibility of this deficiency in case that the selected pair of vertices (x and y) of outerplanar graph is not adjacent. Postle and Thomas reverted this problem back to planar graphs showing the existence of Thomassen-like colouring for plane near-triangulation in which vertices x and y are not adjacent and have lists of length 2 and providing additional conditions to have list of length 1 for one vertex. There are also another approaches to Thomassen proof showing eg. 5-choosability of graphs with small crossing number.

Recently, Zhu gave a proof analogous to the one of Thomassen using the polynomial method. Under assumption of Theorem 1 he proved that the graph polynomial of G contains non-vanishing monomial of the form $x^0y^1\prod_{i=1}^{k} v_i^{\alpha_i}\prod_{i=1}^{m} u_i^{\beta_i}$ where $\alpha_i \leq 2$, $\beta_i \leq 4$, cycle C consists of vertices x, y, and $v_1, \dots v_k$ while $u_1, \dots, u_m$ are inner vertices. By Combinatorial Nullstellensatz this implies the existence of a suitable list colouring but also the analogous result for paintability.

During the talk we will discuss an application of the polynomial method for the variations of Thomassen approach described above. In particular, we show how to simplify the proof of Hutchinson obtaining a more general result in terms of non-vanishing monomials in a graph polynomial.

This is joint work with Paweł Twardowski (Lodz University of Technology).