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3-COLORABILITY AND 3-CHOOSABILITY OF PLANAR GRAPHS

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The question which planar graphs are 3-colorable is well investigated. Starting with Heawood, who showed that a plane triangulation is 3-colorable if and only if all its vertices have even degrees, it continued by Grötzsch's result showing that every triangle-free planar graph is 3-colorable. Allowing some triangles in a graph, but still retaining 3-colorability yielded two intriguing conjectures. First, Havel conjectured that a 3-colorable planar graph may contain many triangles as long as they are sufficiently far apart. This conjecture was recently proved by Dvořák, Kráľ, and Thomas. The second conjecture is due to Steinberg. It allows arbitrary many triangles but it forbids short cycles. Namely, Steinberg conjectured that every planar graph without cycles of length 4 and 5 is 3-colorable. The conjecture was disproved by Cohen-Addad et al.

In our talk, we present a result showing that a 4-regular planar graph obtained as the medial graph of a bipartite plane graph is 3-choosable. Note that we do assume a special structure of a graph, but we do not particularly bound the number of triangles, they can even have common vertices (so the distance between them can be as small as one), and the graph can contain cycles of any length.