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ON THE MINIMUM SIZE OF HAMILTONIAN SATURATED HYPERGRAPHS

Andrzej Żak (joint work with A. Ruciński)

AGH University of Science and Technology, Cracow, Poland

For $1 \leq \ell < k$, an *ℓ-overlapping k-cycle* is a k-uniform hypergraph in which, for some cyclic vertex ordering, every edge consists of k consecutive vertices and every two consecutive edges share exactly ℓ vertices. A k-uniform hypergraph H is *ℓ-hamiltonian saturated* if H does not contain an ℓ-overlapping hamiltonian k-cycle but every hypergraph obtained from H by adding one edge does contain such a cycle. Let $\mathrm{sat}(n, k, \ell)$ be the smallest number of edges in an ℓ-hamiltonian saturated k-uniform hypergraph on n vertices. In the case of graphs Clark and Entringer showed in 1983 that $\mathrm{sat}(n, 2, 1) = \lceil \frac{3n}{2} \rceil$. The present authors proved that for $k \geq 3$ and $\ell = 1$, as well as for all $0.8k \leq \ell \leq k-1$, $\mathrm{sat}(n, k, \ell) = \Theta(n^\ell)$. Thus the smallest open case is $k = 4$ and $\ell = 2$. In the talk I will show our recent result that states that $\mathrm{sat}(n, 2\ell, \ell) = \Theta\left(n^\ell\right)$.