Neutral meson mixing parameters

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1 The short introduction

Neutral mesons, such as K^0 and $\bar{K^0}$, due to their ability to being changeable by weak interactions, have the possibility to oscillate between each other. It would have seemed a little bit strange, because in the quantum mechanics any particles are able to cross only between their eigenstates, and meson and anti-meson are not the eigenstates of the same particle. However, in this short report we will show a solution why the oscillatons are possible. If we combine the states of those particles using linear combination with equivalent parameters, the new particles consist of states of both former particles, and since when the new particles can be actually measured in experiment, the oscillation between that two former different states (K^0 and $\bar{K^0}$) must be possible. The following (2) chapter provides a mathematical description of oscillations described above. The third chapter in turn presents the results from numerical simulation, which has been performed by us for the purpose of that study. The last chapter which ends this document presents the conclusion of that study and summarize the results.

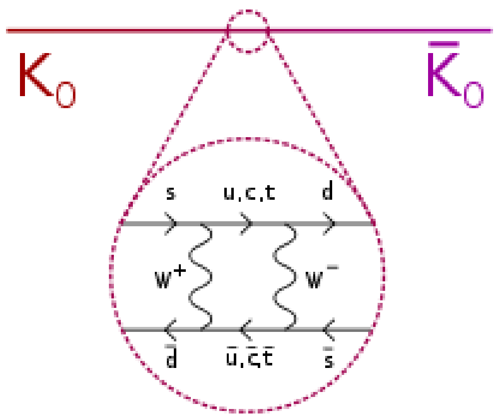

Figure 1: Two different neutral K mesons, carrying different strangeness, can turn from one into another through the weak interactions, since these interactions do not conserve strangeness..[1]

2 Derivation of required equations

At the beginning we can assume that we dispose a pair of particles: meson and anti-meson P^0 i $\bar{P}^0$. By their linear combinations we can create two another states, which can be directly measured in experiment (what was already mentioned in the intruduction):

$$\begin{aligned} \big|P_1\big\rangle &= p\big|P^0\big\rangle + q\big|\bar{P}^0\big\rangle \\ \big|P_2\big\rangle &= p\big|P^0\big\rangle - q\big|\bar{P}^0\big\rangle \end{aligned} \tag{1}$$

where: $|p|^2 + |q|^2 = 1$.

Although mesons P^0 i $\bar{P}^0$ are produced by the strong interactions, they are propagating in time as the eigenstates of the operator $\mathcal{CP}$, in other words they are able to oscillate between each other due to sensivity to being subject to weak interactions. The proper particles (called the former ones in the introduction) are also linear combination of the new ones:

$$\begin{aligned} \big|P^0(t)\big\rangle &= \frac{1}{2p}\left(\big|P_1(t)\big\rangle + \big|P_2(t)\big\rangle\right) \\ \big|\bar{P}^0(t)\big\rangle &= \frac{1}{2q}\left(\big|P_1(t)\big\rangle - \big|P_2(t)\big\rangle\right) \end{aligned} \tag{2}$$

Using the time-depending Schroedinger equation, which provides the information how those states behave through time, we can obtain the time evolution of those states. That evolution is given below:

$$\begin{aligned} \big|P_1(t)\big\rangle &= \big|P_1\big\rangle e^{-i\left(m_1-\frac{i\Gamma_1}{2}\right)t} \\ \big|P_2(t)\big\rangle &= \big|P_2\big\rangle e^{-i\left(m_2-\frac{i\Gamma_2}{2}\right)t} \end{aligned} \tag{3}$$

Putting (3) to the first one of (2), we obtain the time evolution of the meson P^0 in the base of states $\big|P_1\big\rangle$ i $\big|P_2\big\rangle$:

$$\big|P^0(t)\big\rangle = \frac{1}{2p}\left(\big|P_1\big\rangle e^{-i\left(m_1-\frac{i\Gamma_1}{2}\right)t} + \big|P_2\big\rangle e^{-i\left(m_2-\frac{i\Gamma_2}{2}\right)t}\right) \tag{4}$$

Using (1) we can change the base with states $\big|P^0\big\rangle$ and $\big|\bar{P}^0\big\rangle$:

$$\big|P^0(t)\big\rangle = \frac{1}{2p}\left[\left(p\big|P^0\big\rangle + q\big|\bar{P}^0\big\rangle\right) e^{-i\left(m_1-\frac{i\Gamma_1}{2}\right)t} + \left(p\big|P^0\big\rangle - q\big|\bar{P}^0\big\rangle\right) e^{-i\left(m_2-\frac{i\Gamma_2}{2}\right)t}\right] \tag{5}$$

Finally, the formula which describes the time evolution looks as follows:

$$\begin{aligned} \big|P^0(t)\big\rangle &= f_+(t)\big|P^0\big\rangle + \tfrac{q}{p} f_-(t)\big|\bar{P^0}\big\rangle \\ \big|\bar{P^0}(t)\big\rangle &= f_+(t)\big|\bar{P^0}\big\rangle + \tfrac{p}{q} f_-(t)\big|P^0\big\rangle \end{aligned} \tag{6}$$

where functions $f_\pm(t)$ are defined as:

$$f_\pm(t) = \frac{1}{2}\left[e^{-i\left(m_1-\frac{i\Gamma_1}{2}\right)t} \pm e^{-i\left(m_2-\frac{i\Gamma_2}{2}\right)t}\right] \tag{7}$$

To obtain the probability of finding the following states: $\big|P^0\big\rangle$ and $\big|\bar{P^0}\big\rangle$ in the beam containing at the beginning only mesons $\big|P^0\big\rangle$, is correct to use definition given below:

$$\begin{aligned} \mathrm{P}\left(P^0 \to P^0; t\right) = \left|\big\langle P^0\big|P^0(t)\big\rangle\right|^2 = \left|f_+(t)\big\langle P^0\big|P^0\big\rangle + \tfrac{q}{p} f_-(t)\big\langle P^0\big|\bar{P^0}\big\rangle\right|^2 &= \left|f_+(t)\right|^2 \\ \mathrm{P}\left(P^0 \to \bar{P^0}; t\right) = \left|\big\langle \bar{P^0}\big|P^0(t)\big\rangle\right|^2 = \left|f_+(t)\big\langle \bar{P^0}\big|P^0\big\rangle + \tfrac{q}{p} f_-(t)\big\langle \bar{P^0}\big|\bar{P^0}\big\rangle\right|^2 &= \left|\tfrac{q}{p}\right|^2 \left|f_-(t)\right|^2 \end{aligned} \tag{8}$$

Now we can exploit that $\big|P^0\big\rangle$ and $\big|\bar{P^0}\big\rangle$ are orthonormal:

$$\begin{array}{ll} \big\langle P^0\big|P^0\big\rangle = 1 & \big\langle P^0\big|\bar{P^0}\big\rangle = 0 \\ \big\langle \bar{P^0}\big|P^0\big\rangle = 0 & \big\langle \bar{P^0}\big|\bar{P^0}\big\rangle = 1 \end{array}$$

what is not applicable to $\big|P_1\big\rangle$ and $\big|P_2\big\rangle$.

For the further analysis we need to obtain the expression which stands for $|f_\pm(t)|^2$, which will be used to calculate equivalent probabilities (8):

$$\begin{aligned} |f_\pm(t)|^2 &= \frac{1}{4}\left[e^{-i\left(m_1-\frac{i\Gamma_1}{2}\right)t} \pm e^{-i\left(m_2-\frac{i\Gamma_2}{2}\right)t}\right] * \left[e^{+i\left(m_1+\frac{i\Gamma_1}{2}\right)t} \pm e^{+i\left(m_2+\frac{i\Gamma_2}{2}\right)t}\right] = \\ &= \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} \pm \left(e^{-i(m_1-m_2)t} + e^{-i(m_1-m_2)t}\right) e^{-\frac{\Gamma_1+\Gamma_2}{2}t}\right] = \\ &= \frac{1}{2} e^{-\frac{\Gamma_1+\Gamma_2}{2}t}\left[\cosh\left(\tfrac{\Gamma_1-\Gamma_2}{2}t\right) \pm \cos\left(\left(m_1-m_2\right)t\right)\right] = \end{aligned} \tag{9}$$

After using following subsitutions:

$$\Delta m = m_1 - m_2 \qquad \Delta\Gamma = \Gamma_1 - \Gamma_2 \qquad \bar{\Gamma} = \frac{\Gamma_1 + \Gamma_2}{2} \tag{10}$$

Finally we get:

$$|f_{\pm}(t)|^2 = \frac{1}{2}e^{-\bar{\Gamma}t}\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) \pm \cos\left(\Delta m t\right)\right] \tag{11}$$

3 The results

After derivation of equations, now we are able to make their numerical simulation. For the purposes of simulation we have used following substitutions:

$$x = \frac{\Delta m}{\Gamma} \tag{12}$$

$$y = \frac{\Delta\Gamma}{\bar{\Gamma}} \tag{13}$$

Using root package we have created the program with graphical user interface (which is enclosed to this report), in which we can change specified parameters and understand what is their role in the neutral meson oscillations. There are 3 independent, different parameters: x, y, Δm.

At the beginning, let's look how the oscillations depends on the value of Δm parameter. Three plots for different Δm, with fixed x and y parameters, are depicted in the Figure 2. The equivalent parameters are shown in the legend. We can see that increasing the mass width decreases the period of oscillations, thus also lifetime of the mesons. The numbers of consecutive peaks in different plots are constant, what in turn causes that smaller Δm parameter enlarges the time mesons are oscillating into each other through.

The x parameter is the part of exponent function, so we expect that lower values of that parameter will suppress the oscillations. And it is actually true, what could be seen in the Figure 3. Higher x enhances the lifetime of kaons and anti-kaons, which leads to possibility of longer oscillations. Respectively, the lower values of x causes the oscillations vanish.

Changing of third parameter y does not entail a lot. For higher values of that parameter the "tail" of oscillations grows up, however owing to decays of mesons and anti-mesons, that tail is decreasing across the time, which is shown in the Figure 4.

References

[1] *https://en.wikipedia.org/wiki/Kaon*

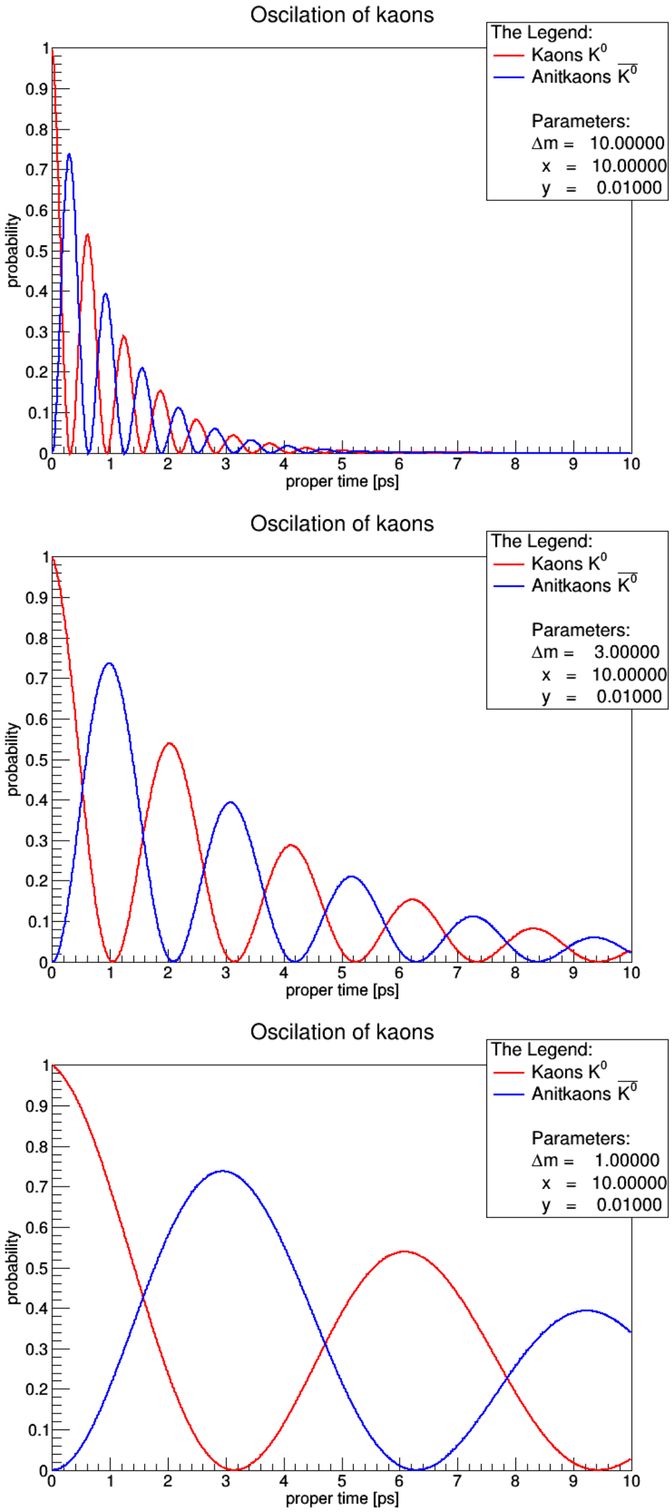

Figure 2: Oscillation of kaons for the different Δm, with x and y parameters fixed at constant values, which has been shown in the legend of the pictures. The red ones represent kaons, the lines drawn in blue correspond to anti-kaons.

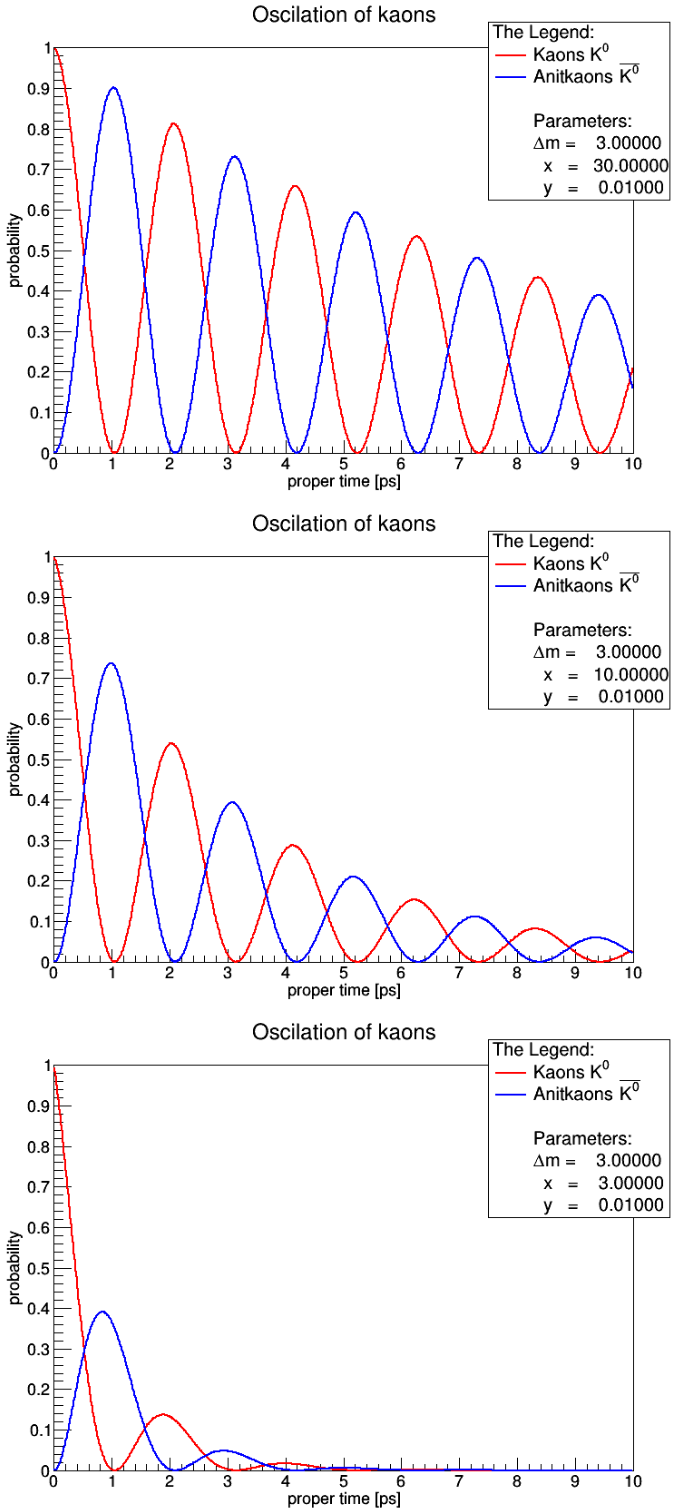

Figure 3: Oscillation of kaons for the different x, with y and Δm parameters fixed at constant values, which has been shown in the legend of the pictures. The red ones represent kaons, the lines drawn in blue correspond to anti-kaons.

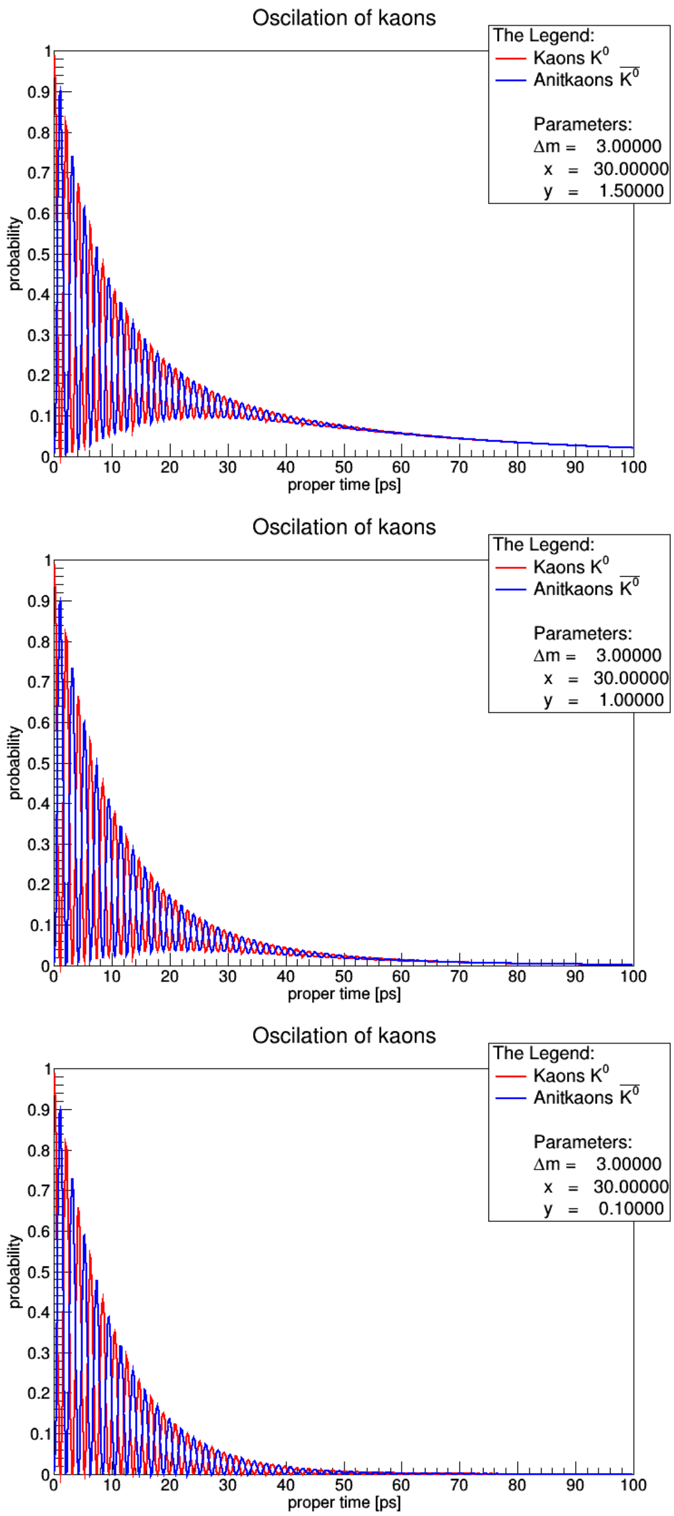

Figure 4: Oscillation of kaons for the different y, with x and Δm parameters fixed at constant values, which has been shown in the legend of the pictures. The red ones represent kaons, the lines drawn in blue correspond to anti-kaons.