STUDY OF NEUTRAL MESON OSCILLATIONS MIXING PARAMETERS

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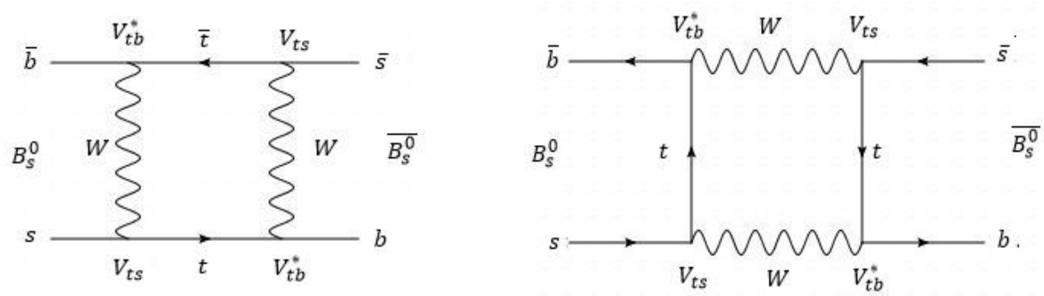
Oscillations of neutral meson such as $K^0 - \overline{K^0}$, $B^0 - \overline{B^0}$, $D^0 - \overline{D^0}$ or $B_s^0 - \overline{B_s^0}$, are the source of CP (charge-parity) violation in the universe. The mathematical description of these oscillations and different methods of measurements of indirect CP violation have been presented in this paper.

I INTRODUCTION

Heavy neutral particles are produced in strong interaction and are able to oscillate into another neutral particle through an interaction that does not conserve quantum numbers. An example of this type processes is second-order weak interactions. This process is represented by Feynman diagram type box which has been shown in diagram 1 and 2.



. Diag. 1,2: Feynaman's diagram of oscillation of B_s^0 meson.

II THEORY

Oscillations of heavy neutral meson are classified into two types: particle-antiparticle oscillation (for example $K^0 - \overline{K^0}$ oscillations, $B^0 - \overline{B^0}$ oscillations, $D^0 - \overline{D^0}$ oscillations or $B_s^0 - \overline{B_s^0}$ oscillations) and flavor oscillation (for example, ν_e-ν_μ oscillations). There are 3 types of CP violation: in decay (direct), in mixing (indirect) and in interference between mixing and decay. Mathematical description of the third type is presented below.

Mathematical apparatus describing the oscillations of neutral mesons involve basic of quantum physics. Neutral mesons are produced in strong interaction but propagate as a mixture of two states with a different mass. First, of them - high state $|P_H\rangle$ and the second one – low state - $|P_L\rangle$ is described by eq. 1,2.

$$|P_L\rangle = p|P\rangle + q|\bar{P}\rangle \qquad (1)$$

$$|P_H\rangle = p|P\rangle - q|\bar{P}\rangle \qquad (2)$$

$p, q \epsilon \mathbb{C}$, $|p|^2 + |q|^2 = 1$

Γ is a distribution width .

$$\Delta m_s = m_H - m_L \;\; \Delta\Gamma_s = \Gamma_H - \Gamma_L \;\;\; \bar{\Gamma} = \frac{\Gamma_H - \Gamma_L}{2} \qquad (3)(4)$$

Neutral meson propagates as a mixture of two states (eq. 5,6).

$$|P\rangle = \frac{1}{2p}(|P_L\rangle + |P_H\rangle) \qquad (5)$$

$$|\bar{P}\rangle = \frac{1}{2p}(|P_L\rangle - |P_H\rangle) \qquad (6)$$

Equation 7 and 8 describe time evolution high and low states.

$$|P_L(t)\rangle = P_L * \; e^{-im_L t - \frac{1}{2}\Gamma_L} \qquad (7)$$

$$|P_H(t)\rangle = P_H * \; e^{-im_H t - \frac{1}{2}\Gamma_H} \qquad (8)$$

Time evolution of state can be derived from equation 5 and 6 using equation 1 and 2.

$$|P(t)\rangle = \frac{1}{2p}(P_L * \; e^{-im_L t - \frac{1}{2}\Gamma_L} + P_H * \; e^{-im_H t - \frac{1}{2}\Gamma_H}) \qquad (9)$$

$$|P(t)\rangle = f_+(t)|P\rangle + \frac{q}{p} f_-(t)|\bar{P}\rangle \qquad (10)$$

$$f_\pm(t) = \frac{1}{2}\,(e^{-im_L t - \frac{1}{2}\Gamma_L} \pm \; e^{-im_H t - \frac{1}{2}\Gamma_H}) \qquad (11)$$

$$f_\pm(t) = \frac{1}{2}\,(e^{-\frac{1}{2}\Gamma_L} \pm \; e^{-\frac{1}{2}\Gamma_H} \pm 2e^{-\bar{\Gamma} t} \,\mathbf{cos(\Delta m t))} \qquad (12)$$

Time evolution of probabilities of mixing i.e the probability of observation of the same neutral meson P in time t (eq. 13).

$$P(P \to P; t) = |f_+(t)|^2 \qquad (13)$$

$$P(P \to \bar{P}; t) = \left|\frac{q}{p} f_-(t)\right|^2 \qquad (14)$$

$$P_{\pm}(P \to P; t) = e^{-\frac{\bar{\Gamma}t}{2}}\left(\cosh\left(\frac{1}{2}\Delta\Gamma t\right) \pm \cos(\Delta m t)\right) \tag{15}$$

Following parameterization has been assumed:

$$x = \frac{\Delta m}{\bar{\Gamma}} \tag{16}$$

$$y = \frac{2\Delta\Gamma}{\bar{\Gamma}}$$

Oscillation of neutral meson may be used in indirect CP violation measurement:

$$a_{CP}(t) = \frac{\Gamma_f - \bar{\Gamma}_f}{\Gamma_f + \bar{\Gamma}_f} \tag{17}$$

Γ_f is a decay rate:

$$\Gamma_{B_s^0 \to f}(t) \sim |\langle f|T|B_s^0(t)\rangle|^2 \tag{18}$$

$$\Gamma_{B_s^0 \to f}(t) = |A_f|^2\left(1 + |\lambda_f|^2\right)\frac{e^{-\Gamma_s t}}{2} * \tag{19}$$

$$* \left(cosh\frac{\Delta\Gamma_s t}{2} + D_f\, sinh\frac{\Delta\Gamma_s t}{2} + C_f\, cos\,\Delta m_s t - S_f\, sin\,\Delta m_s t\right)$$

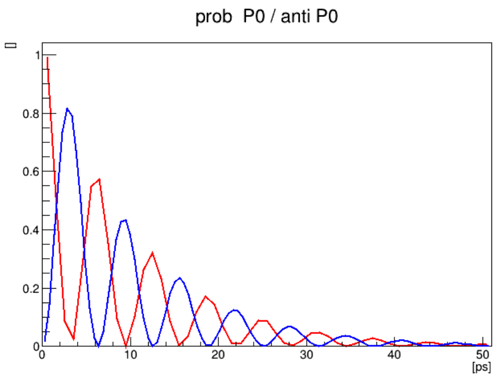
$$A_f = \langle f|T|B_s^0\rangle \;,\; \lambda_f \equiv \frac{1}{\bar{\lambda}_f} = \frac{q}{p}\frac{\bar{A}_f}{A_f},\;\; D_f = \frac{2Re\lambda_f}{1+|\lambda_f|^2}, C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, S_f = \frac{2Im\lambda_f}{1+|\lambda_f|^2} \tag{20}$$

III ANALYSIS

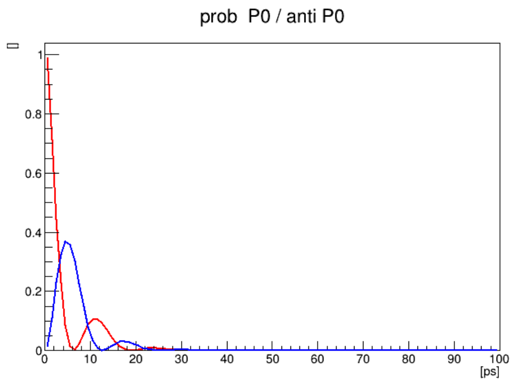
The sensitivity of the frequency of the oscillation in function of the oscillations parameters has been shown in equation 14 and 15. Time is expressed in ps (typical for neutral meson oscillation), the Δm and the rest of parameter are expressed in ps^{-1}.

A) Δm

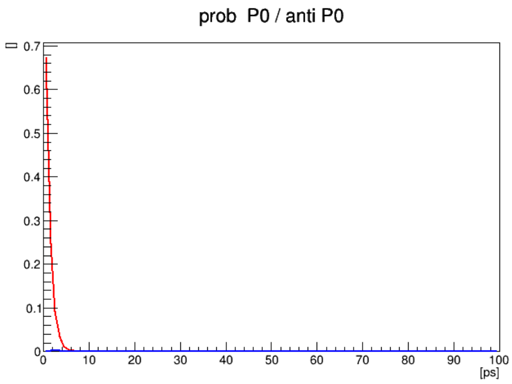
The following diagram presents three cases of time evolution of a state with a different value of Δm. (x,y =0.01).
Case 1: Δm = 1, case 2: Δm = 0.5, case 3: Δm = 0.1. Red line – probability of observing P meson, blue- $\bar{P}$.



Diag. 2: case 1: Δm = 1



Diag. 3: case 2 Δm = 0.5

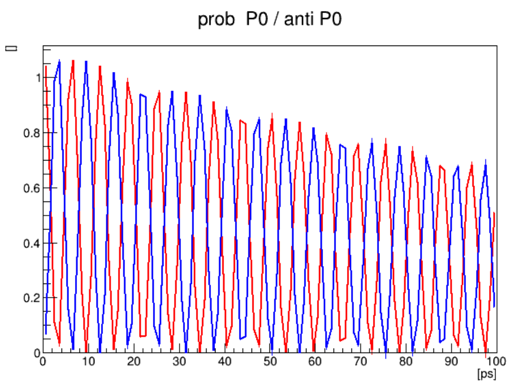


Diag. 4: case 3 Δm = 0.1

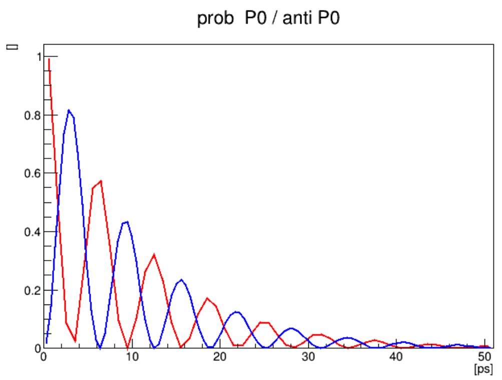
The frequency of oscillation is sensitive to a difference of high and low states mass. Bigger mass resulted in higher frequency. For a low difference of mass, oscillation disappear.

B) x

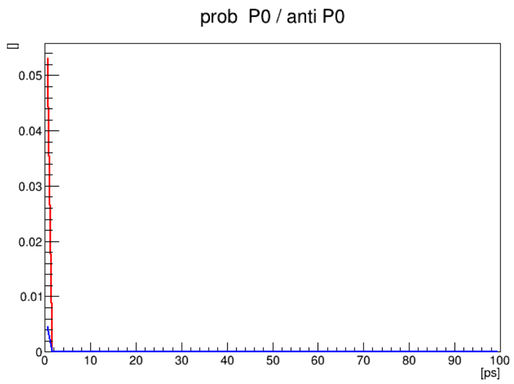
The following diagram presents three cases of time evolution of a state for different value of x. ($\Delta m =$ 1, y = 0.01).



Diag. 5: case 1: x = 0.005



Diag. 6: case 2: x = 0.1

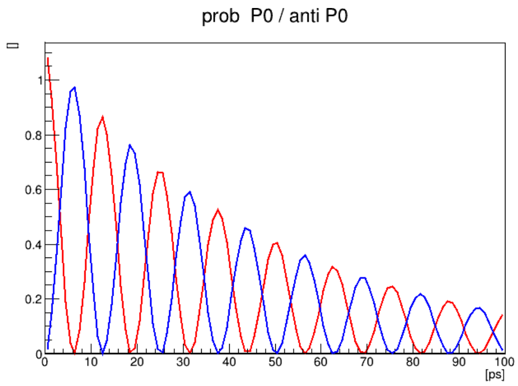


Diag. 7: case 3: x= 0.1

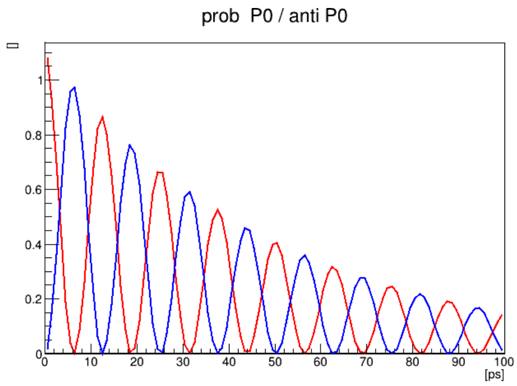
A number of oscillation depends on the difference of decay width of the high and low state. Increasing value of $\bar{\Gamma}$ resulted in lower number of oscillation in the same time. $\bar{\Gamma}$ depend on the lifetime of. The long-lived particle may oscillate more times.

C) *y*

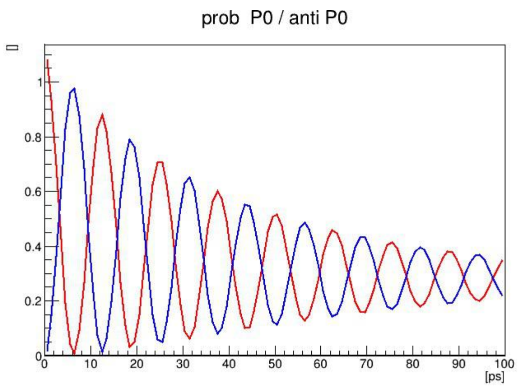
The following diagram presents three cases of the time evolution of a state with a different value of y. (Δm=1, x = 0.01).



Diag 8: case 1: y = 0.001



Diag 9: case 2: y = 0.1



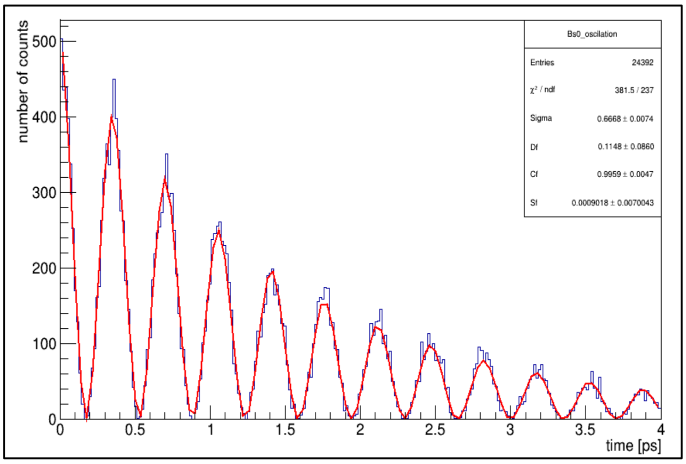
Diag10: case 3: y= 1

The lower value of y does not affect the number of oscillation. A higher value of y resulted on a higher number of oscillation in the same time.

The case of neutral meson B_s^0 has been presented below. Meson parameters have been shown in table 1. Δm parameter is relatively large so high oscillation frequency is expected. The date comes from generator level study of the $\mathrm{B_s^0} \rightarrow \mathrm{D_s^*}\,\mathrm{K^*}$ decay.

Table 1: B_s^0 meson oscillation parameter.

	$\Delta m\ [ps^{-1}]$	$\bar{\Gamma}\ [ps^{-1}]$
B_s^0	17.7	0.67



Diag 8: B_s^0 meson oscillation.

	Measurement	
Γ	0.6668 ± 0.0074	
D_f	0.1148 ± 0.0668	
C_f	0.9959 ± 0.0047	
S_f	0.0009 ± 0.0070	
	Measurement	**Theory**
$\Delta\Gamma_s$	0,0667 +- 0.001 $\frac{1}{ps}$	0,081 $\frac{1}{ps}$

Table 2: Results.

The frequency of the $\mathrm{B_s^0}$– anti- $\mathrm{B_s^0}$ oscillations is very high. The $\mathrm{B_s^0}$ meson oscillates more often than the other neutral meson.

IV SUMMARY

In this paper, the mathematical description of the oscillations of heavy neutral meson has been presented. The sensitivity of a number of oscillation to difference of a mass and width of low and high state has been proved. The last chapter present, the results of analysis of $\mathrm{B_s^0}$ meson oscillations from generated data.