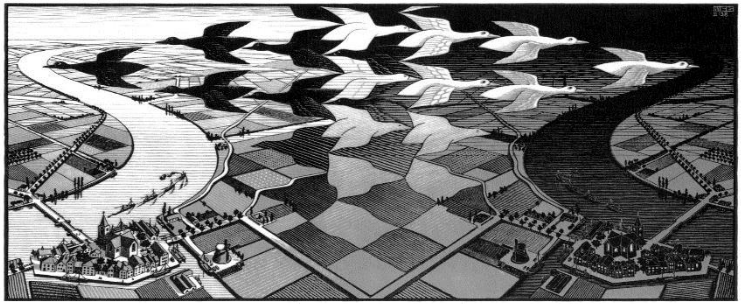

CP-Violation in Heavy Flavour Physics
Lecture 5

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Outline

- Some detour for the starters: currents, amplitudes and all of these
- How many weak coupling do we need? Or why Cabbibo theory is nice
- We need more quarks! Story on the GIM mechanism
- The CKM matrix, i.e., mix it up!
- Unitary triangles – astonishing way the Nature works…

A brief intro: currents, amplitudes, ...

- This is mainly a vocabulary – if you want more just come to my lecture „Introduction to the SM” in summer semester...
- Will need that to place the CKM matrix elements, so...

- Creating a coherent mathematical framework for the weak int. (WI) is not easy
  - Need to incorporate neutrinos, leptons and quarks (hadrons)
  - Also, need to convey what is left and right
- This is done by introducing interacting „currents”, which specify the flow of particles
  - For instance, we say, using this formalism that β decay can be seen as one current converting a neutron into proton and the other creating an electron and the appropriate neutrino
- The tricky part is to come up whit a general form of such currents...

A brief intro: currents, amplitudes, …

- Firstly we need to write the currents such all of the experimental facts (read conservation rules) are observed – let's focus on leptonic prcesses first
- For instance we observe that whenever an electron-neutrino is absorbed an electron is created or whenever an electron-neutrino is created a positron must be created as well
- So, our lepton wave functions must always come in **pairs**
- Also, we need to add some dynamic factor, that takes into account parity, charge-parity and CP-violation accordingly

$$j^w = \bar{\psi}_l \Lambda \psi_{\nu_l}, \qquad l = \{e, \mu\}$$

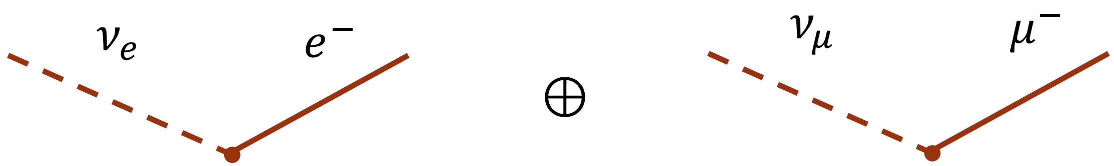
Leptonic current

Dynamic „coupling" factor

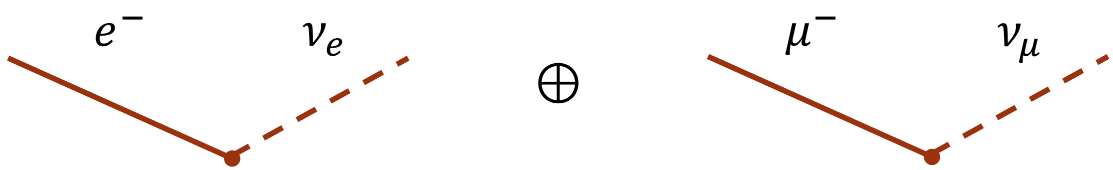
$$\bar{j}^w = \bar{\psi}_{\nu_l} \Lambda \psi_l, \qquad l = \{e, \mu\}$$

A brief intro: currents, amplitudes, …

$j^w \equiv$

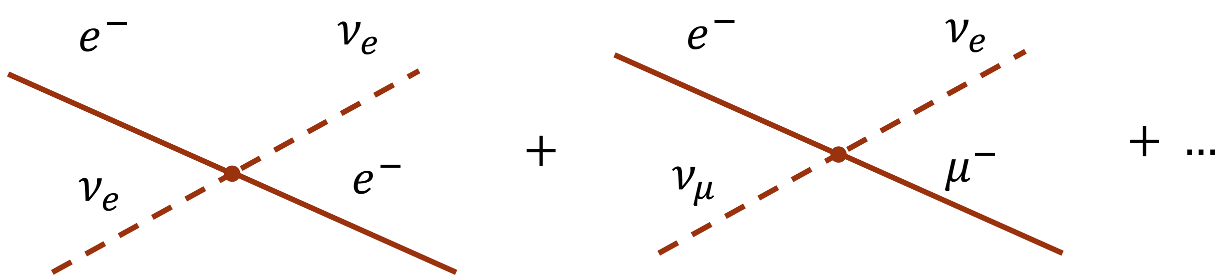


$\bar{j}^w \equiv$



- ❑ Now comes the sweet part – all first order amplitudes observed in nature can be generated by simple **product** of these **currents**!

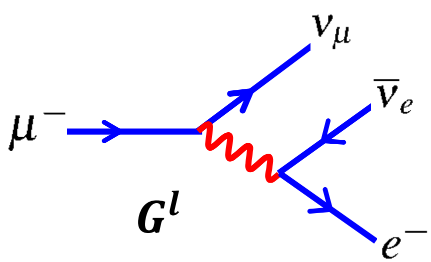
$A^{(1)} \sim G_F \bar{j}^w j^w \equiv$

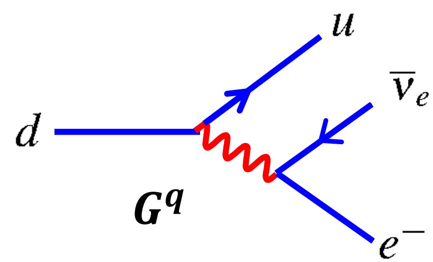


- ❑ Now, these are space time diagrams, so, we could use the same one to describe scattering and decay

Cabbibo picture

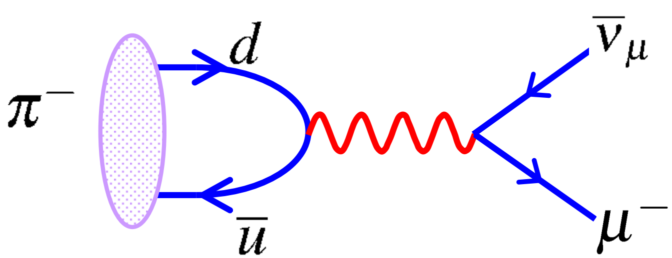
- It seemed there is something awkward with the WI (what's new…)

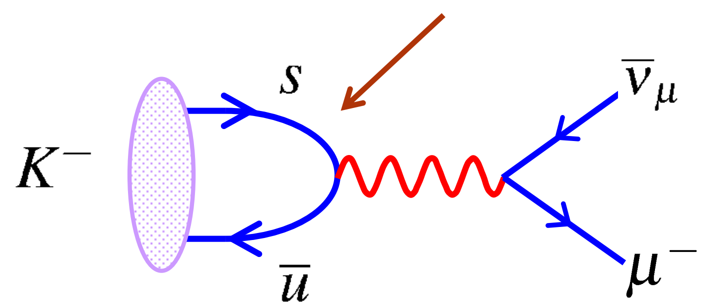




- In order to describe correctly the observed processes we need two different „coupling constants"
  - Shame…, would be nice to have leptonic and hadronic currents share the same coupling – weak universality
- It is even worse…

Suppressed ~ 20 times

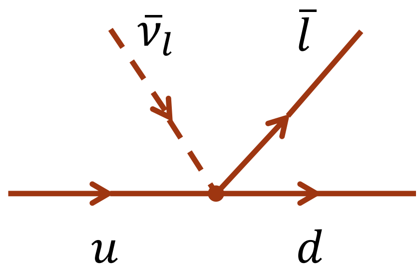




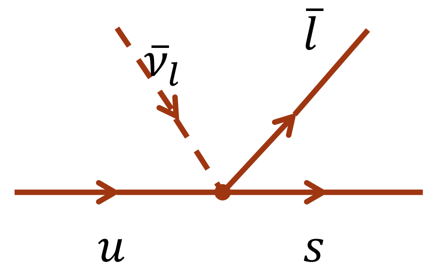
$$G^{ud} \neq G^{us}$$

Cabbibo picture

- ❑ This is bad! Quark currents are not universal w.r.t. the WI either…?
- ❑ Shall we introduce a number of coupling constants? Not very nice…



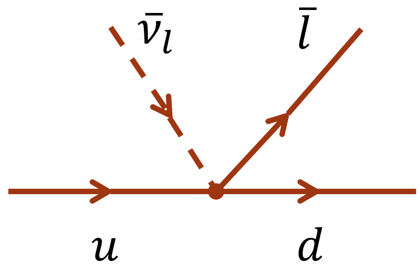
$\boxed{\propto \boldsymbol{G^{ud}}}$



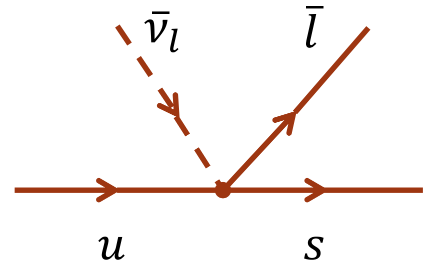
$\boxed{\propto \boldsymbol{G^{us}}}$

- ❑ Cabbibo found much more elegant way, which brought back simplicity to the WI
  - ❑ weak e-states (flavour) are different than the mass ones
  - ❑ we already seen the same effect for kaons!
  - ❑ some of quarks are **mixed** (have not specified flavour) – this way we can show that there is just one universal coupling for leptons and quarks! Awesome!

Cabbibo picture



$\boxed{\propto \ cos(\theta_C)}$

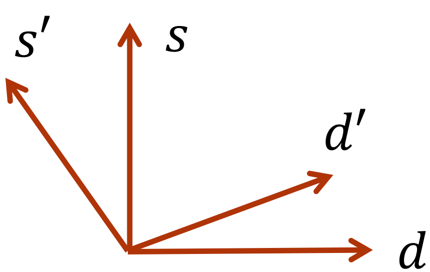


$\boxed{\propto sin(\theta_C)}$

- In Cabbibo theory both **d** and **s** quarks are mixed, so we can come up with the following mixing matrix

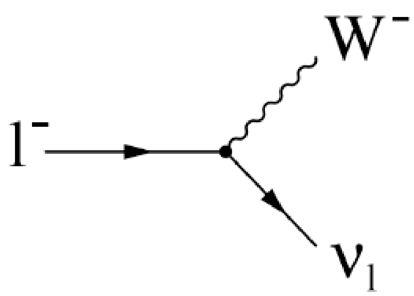
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Weak e-states **Mixing matrix** **Mass e-states**

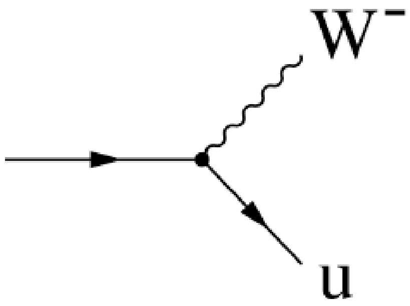


$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \cdot cos(\theta_c) + s \cdot sin(\theta_c) \end{pmatrix}$$

Cabbibo picture



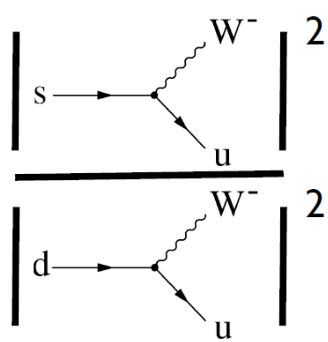
$d\cos\theta_C + s\sin\theta_C$



$$\boxed{g^l = g^{ud'} = g_W}$$

- Mixing (Cabbibo) angle is a parameter of, so called, flavour sector of the SM – cannot be predicted only measured!

$$\frac{\Gamma(K^+ \to \mu\nu_\mu)}{\Gamma(\pi^+ \to \mu\nu_\mu)} \sim tan^2(\theta_c)$$



$= \tan^2\theta_C$

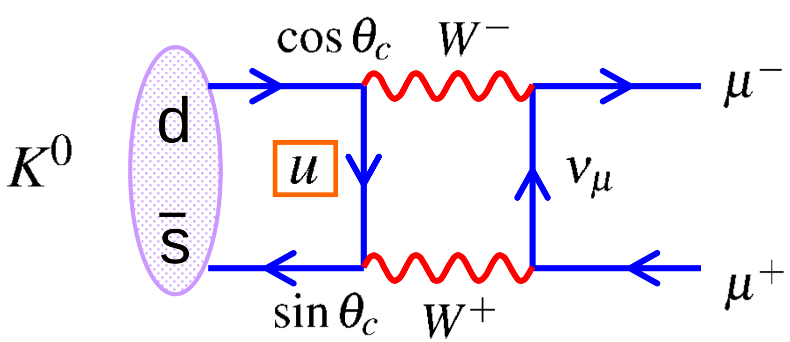
$$\boxed{\theta_c \approx 13.1^o}$$

We need more quarks!

- Hm, let's have a look at quark families…, they look strange

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \cdot cos(\theta_c) + s \cdot sin(\theta_c) \end{pmatrix}, (s') = (-d \cdot sin(\theta_c) + s \cdot cos(\theta_c))$$

- What is wrong with this picture? Is there something missing maybe…?
- Some clues were offered by a missing decay…

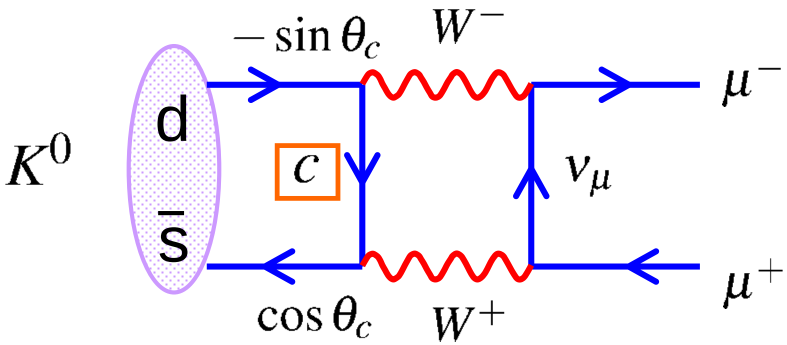


$$A_u^{K\to\mu\mu} \sim g_w^4 \cdot sin(\theta_c) \cdot cos(\theta_c)$$

- This is a legitimate decay channel of neutral kaon, the observed decay rate much much smaller than the predicted

We need more quarks!

- Can we account for this and fix the quark family structure? Yes! Just need some charm…



$$A_c^{K\to\mu\mu} \sim -g_w^4 \cdot sin(\theta_c)\cdot cos(\theta_c)$$

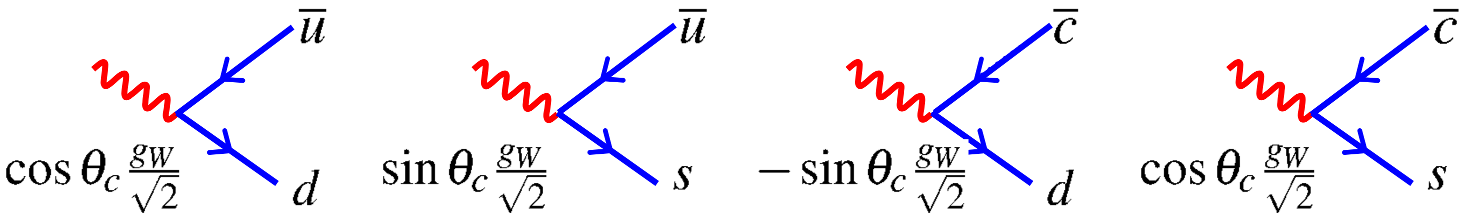
- So, we have the same final state, so, to calculate observable we need to add amplitudes

$$\left|A^{K\to\mu\mu}\right|^2 = \left|A_u^{K\to\mu\mu} + A_c^{K\to\mu\mu}\right|^2 \approx 0$$

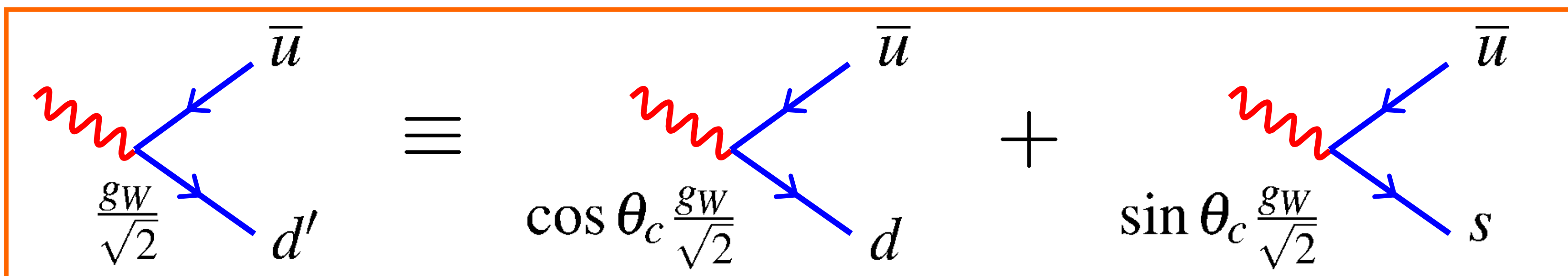
- It is almost canceled out – the non zero value is due to mass difference (BEH mechanism enters the scenes!)

We need more quarks!

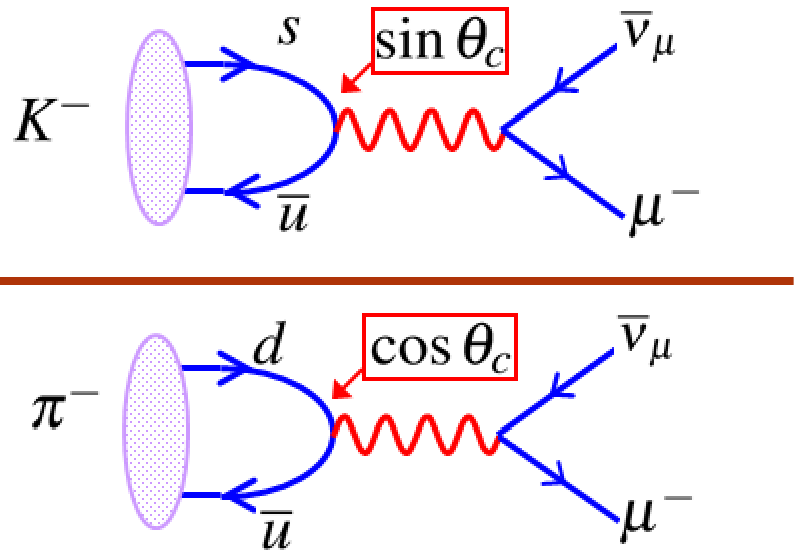
- The small decay rate of kaons to muons prompted an idea of adding another quark – charm
- This was summed up in Glashow-Weinberg-Salam model (GIM)
  - GIM is of course much more than that – intermediate bosons, weak isospin structure of quark and lepton families, symmetry breaking (BEH mechanism)



- Flavour changing charged current weak interactions – can couple different quark generations!



We need more quarks!



$$\rightarrow \frac{\Gamma\left(K^- \rightarrow \mu^- \bar{\nu}_\mu\right)}{\Gamma\left(\pi^- \rightarrow \mu^- \bar{\nu}_\mu\right)} \sim \frac{sin^2(\theta_c)}{cos^2(\theta_c)}$$

$$\boxed{tan^2(\theta_c) \approx 0.05}$$

- ❑ Very nice! But – there is no room for CP violation here
- ❑ Cabbibo mixing matrix is described by a single parameter that is real number!
- ❑ Any idea how to make a progress?
- ❑ Yes! More quarks!

Mix it up!

- In order to accommodate CP-violation effects in the SM K & M came up with the idea of third generation of quarks
- In this picture up-type quarks decay into mixed (weak e-state) down-type ones
- Remember – this is just a convention, we could build a theory with mixed up-type quarks with the same observables!

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak e-states

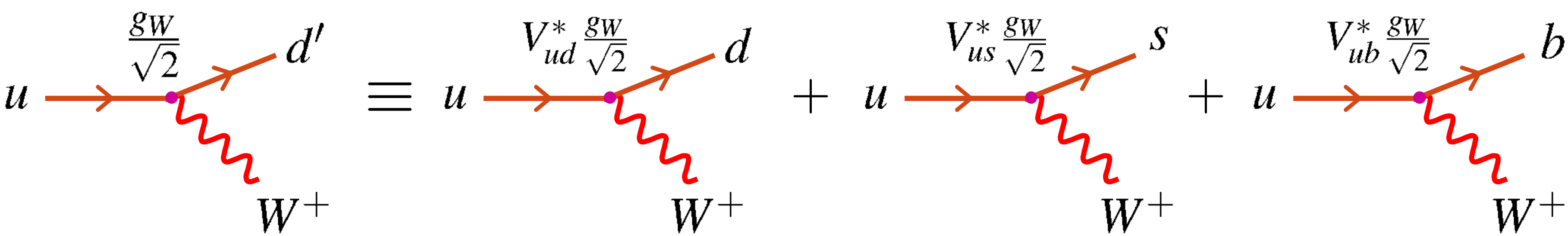
Mixing matrix
Cabbibo-Cobayashi-Maskawa

Mass e-states

- Elements, V_{ij} of the CKM matrix are **complex numbers**
- The CKM matrix is **unitary** (probability conservation)
- The elements V_{ij} cannot be predicted – constants of the flavour sector

Mix it up!

- So, in general we have the following transitions



- Depending on the direction of transition we will have either V_{ij} or its conjugate partner V_{ud}^*
- Would be nice to write down the quark current explicitely to see how the CKM matrix fit in
- For this we are going to take another short detour…

Chiral notation

- When solving Dirac equation we realised that there are in principle four different solutions to it (spin, energy), we write:

$$\boxed{\psi(x) = \binom{u}{v} = \begin{pmatrix} particle \uparrow\downarrow \\ antiparticle \uparrow\downarrow \end{pmatrix}}$$

- We could write this also in a very peculiar way using, so called, helicity operator

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}, \Sigma_k = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \; \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Helicity operator

$$\boxed{h = \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|} = \vec{\sigma} \cdot \vec{p}}$$

$$\gamma^0 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} -I_2 & 0 \\ 0 & I_2 \end{pmatrix}$$

„Gamma" matrices

$$\not{p} = \gamma^\mu p_\mu = \gamma^0 p_0 - \gamma^1 p_1 - \gamma^2 p_2 - \gamma^3 p_3$$

Shlashed notation

Chiral notation

$$\not{p} = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix} p_0 - \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix} p_1 - \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix} p_2 - \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix} p_3 =$$

$$= \begin{pmatrix} 0 & E - \vec{p}\cdot\vec{\sigma} \\ E + \vec{p}\cdot\vec{\sigma} & 0 \end{pmatrix}$$

Right-hand spinor

$$\boxed{\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}}$$

Left-hand spinor

Dirac equation

$$\not{p}\psi = 0 \;\rightarrow\; \begin{pmatrix} 0 & E - \vec{p}\cdot\vec{\sigma} \\ E + \vec{p}\cdot\vec{\sigma} & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

$$\boxed{\hat{h}\psi_L = (\vec{p}\cdot\vec{\sigma})\psi_L = -E\psi_L}$$

$$\boxed{\hat{h}\psi_R = (\vec{p}\cdot\vec{\sigma})\psi_R = +E\psi_R}$$

$$\boxed{\psi_L = \frac{1}{2}(I_4 - \gamma_5)\psi}$$

$$\boxed{\psi_R = \frac{1}{2}(I_4 + \gamma_5)\psi}$$

Chiral notation

- This notation can be used to write down in very elegant form the quark current

$$\gamma_5\psi = \begin{pmatrix} -I_2 & 0 \\ 0 & I_2 \end{pmatrix}\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} -\psi_L \\ \psi_R \end{pmatrix}$$

$$\boxed{(I_4 - \gamma_5)\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} - \begin{pmatrix} -\psi_L \\ \psi_R \end{pmatrix} = 2\begin{pmatrix} \psi_L \\ 0 \end{pmatrix}}$$

We say, we projecting „out" right-hand part

$$\gamma_5\psi_L = \frac{1}{2}(\gamma_5 I_4 - \gamma_5\gamma_5)\,\psi = \frac{1}{2}(\gamma_5 - I_4)\,\psi = -\frac{1}{2}(I_4 - \gamma_5)\,\psi = -\psi_L$$

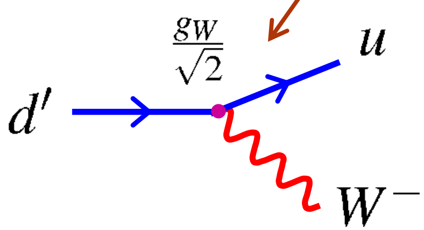
$$\gamma_5\psi_R = \frac{1}{2}(\gamma_5 I_4 + \gamma_5\gamma_5)\,\psi = \frac{1}{2}(\gamma_5 + I_4)\,\psi = +\frac{1}{2}(I_4 + \gamma_5)\,\psi = +\psi_R$$

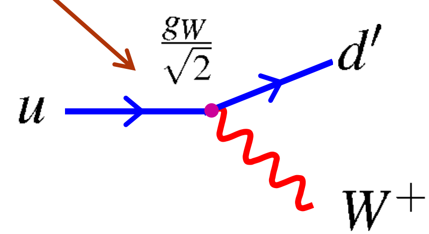
- Now, go back to the CKM matrix…

CKM matrix

- Now quark currents can be written out as

$$j^{w}_{d'u} = \bar{u}\Lambda d', \qquad j^{w}_{ud'} = \bar{d'}\Lambda' u$$





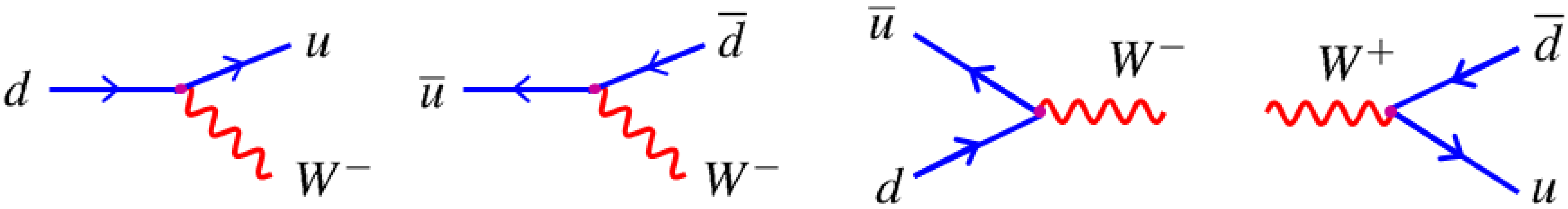
We did sth similar when introduced Cabbibo matrix!

$$j^{w}_{d'u} = \bar{u}\left[-i\frac{g_w}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right]d' \rightarrow j^{w}_{d'u} = \bar{u}\left[-i\frac{g_w}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right]\boldsymbol{V_{ud}}\,d$$

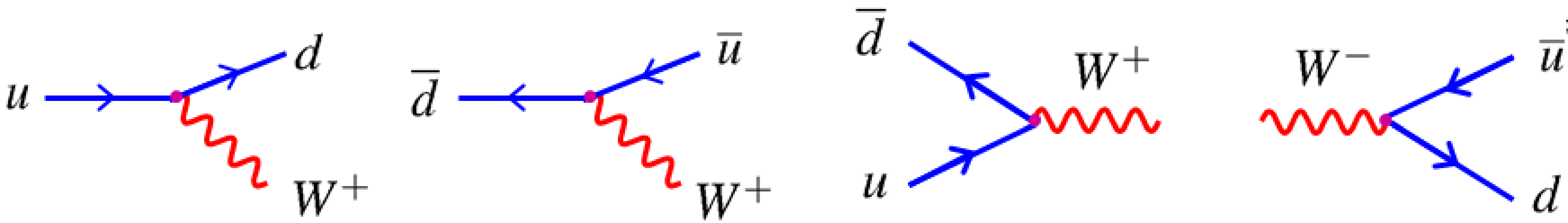
$$j^{w}_{ud'} = \bar{d'}\left[-i\frac{g_w}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right]u \rightarrow j^{w}_{ud'} = \bar{d}\boldsymbol{V^{*}_{ud}}\left[-i\frac{g_w}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right]d$$

$$\bar{d'} = (d')^{\dagger}\gamma^0 = (V_{ud}d)^{\dagger}\gamma^0 = V^{*}_{ud}d^{\dagger}\gamma^0 = V^{*}_{ud}\bar{d}$$

CKM matrix



$$\Lambda = \left[-i \frac{g_w}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \right] \boldsymbol{V_{ud}}$$



$$\Lambda' = \boldsymbol{V_{ud}^*} \left[-i \frac{g_w}{\sqrt{2}} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \right]$$

CKM matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

- Elements of the CKM mixing matrix are parameters of the quark flavour sector of the SM
- Need to be measured
- The last row filled with the question marks – hard to measure
- With unitarity assumption one can get

Cabibbo matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

- The only way to **change flavour** via charged currents in the SM
- Can introduce **change** of quark **generation** and **CP violation**!

CKM matrix

- The „standard" representation – rotation in a complex space

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

- NOTE! $U_{ij} = |V_{ij}|^2$ is independent of quark **re-phasing**
- Next simplest: Quartets: $Q_{aibj} = V_{ai}V_{bj}V^*_{aj}V^*_{bi}$ with $a \neq b$ and $i \neq j$
  - "Each quark phase appears with and without *"
- $V^\dagger V = 1$: Unitarity triangle: $V_{ud}\,V^*_{cd} + V_{us}V^*_{cs} + V_{ub}\,V^*_{cb} = 0$
- Jarlskog invariant (measure of CP violation):

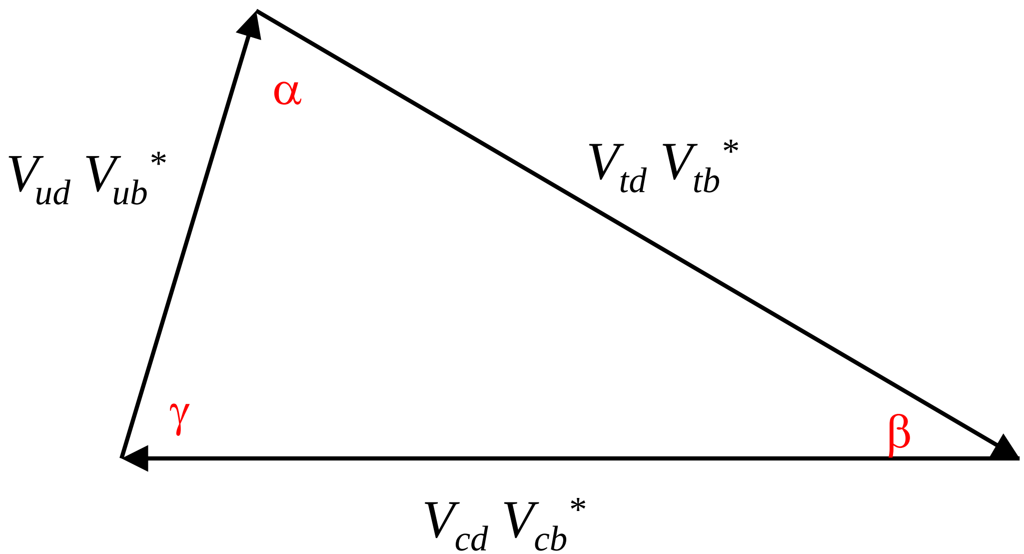
$$J = Im\,(Q_{udcs}) = -Im\,(Q_{ubcs})$$

- The imaginary part of each Quartet combination is the same (up to a sign)
  - In fact it is equal to $2x$ the **surface** of the **unitarity triangle**

Unitarity triangle

- Using unitarity of the CKM matrix one can write (for instance)

$$\boxed{V_{ud}\, V_{ub}^{*} + V_{cd} V_{cb}^{*} + V_{td}\, V_{tb}^{*} = 0}$$



$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right) = \arg\left(-Q_{ubtd}\right)$$

$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right) = \arg\left(-Q_{tbcd}\right)$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right) = \arg\left(-Q_{cbud}\right)$$

Unitarity angles are invariant w.r.t. quark fields re-phasing!

Unitarity triangle

- The most popular representation of the CKM matrix came from Wolfenstein – off-diagonal elements are small w.r.t. the diagonal ones

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- Using this representation we can also re-define unitary triangles, of course the angles are the same!