# CP Violation In Heavy Flavour Physics

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# Short recap I (CP symmetry in $K^0$ mesons)

1. $K^0(\bar{s}d)$ and $\bar{K}^0(s\bar{d})$ are the eigenstates of the strong interactions but they are not eigenstates of the weak interactions.

$$H_{Strong}|K^0\rangle = m_{K^0}|K^0\rangle$$
$$H_{Strong}|\bar{K}^0\rangle = m_{\bar{K}^0}|\bar{K}^0\rangle$$
$$m_{K^0} = m_{\bar{K}^0} \approx 498\ MeV$$

2. The linear combinations of $K^0$ and $\bar{K}^0$:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$
$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$K_1^0$ and $K_2^0$ are eigenstates of $\mathcal{CP}$ operator

3. Kaons are produced as eigenstates of strong Hamiltonian but propagate through time as eigenstates of weak one.
4. But - since $|K_2^0\rangle$ is a mixture of $|K^0\rangle$ and $|\bar{K}^0\rangle$ states, even starting from pure $|K^0\rangle$ (or $|\bar{K}^0\rangle$) state we end up with a mixture of states of different strangeness (this so-called „**flavour oscillations**”)

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right) \qquad |\bar{K}^0\rangle = -\frac{1}{\sqrt{2}}\left(|K_1^0\rangle - |K_2^0\rangle\right)$$

# Short recap II (CP Violation in $K^0$ mesons)

1. If CP symmetry holds for weak interactions, then $K_1^0$ must decay to two pions and $K_2^0$ must decay to three pions.
2. You are not surprised to know that a decay $K_2^0 \to \pi^0 + \pi^0$ and $K_2^0 \to \pi^+ + \pi^-$ has been observed.
3. So a new combination:

$$|K_S^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_1^0\rangle + \epsilon|K_2^0\rangle\right) \qquad |K_L^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_2^0\rangle + \epsilon|K_1^0\rangle\right)$$

   - $K_1^0$ and $K_2^0$ are eigenstates of $\mathcal{CP}$ operator, whereas $K_S^0$ and $K_L^0$ are not.
   - $K_S^0$ and $K_L^0$ are states with definite lifetimes $\boldsymbol{\gamma(\Gamma)_{S,L}}$ and distinct mass $\boldsymbol{m(M)_{S,L}}$
   - $\epsilon$ is a small number describing the degree of $\boldsymbol{\mathcal{CPV}}$, $K_S^0$ is almost $K_1^0$, $K_L^0$ is $K_2^0$.
   - Only 0.2% of $K_L^0$ violates $\mathcal{CP}$! Very tiny effect!
   - This type of $\boldsymbol{\mathcal{CPV}}$ is called „indirect”

4. Solving the Schrödinger time dependent equation for a two-body system, we could calculate the probability of flavour oscillation phenomena.

5. **Now**: are there any other systems that could show us:
   - a flavour oscillation,
   - $\boldsymbol{\mathcal{CPV}}$ in much higher degree?

# Time evolution of neutral mesons

1. In the absence of mixing, meson $K^0$ can decay into all, allowed by energy-momentum conservation, states.
2. The exponential decay law leads to the time dependence of the wave function:

$$|K^0(t)\rangle = |K^0\rangle\, e^{-\frac{\Gamma t}{2}}\, e^{-imt}$$

$e^{-imt}$: time evolution of a stable state with mass $m$, $m = E$

$e^{-\frac{\Gamma t}{2}}$: total width such that probability of finding an undecayed meson at time $t$ is:

$$|\langle K^0(t)|K^0\rangle|^2 = e^{-\Gamma t}$$

which satisfy the equation:

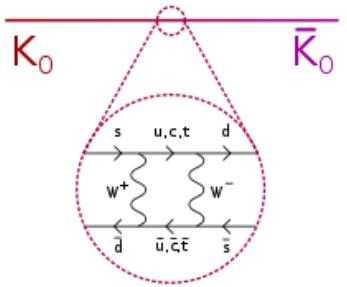
$$i\frac{\partial}{\partial t}|K^0(t)\rangle = \underbrace{\left(m - \frac{i}{2}\Gamma\right)}_{H}|K^0(t)\rangle$$

3. If $K^0$ can convert into $\overline{K^0}$ through second order mixing diagram, the time evolution of a neutral meson must include both $K^0$ and $\overline{K^0}$:

$$|K^0(t)\rangle = e^{-iHt}|K^0(t=0)\rangle = e^{-iHt}\frac{1}{\sqrt{2}}\left(|K_S^0\rangle + |K_L^0\rangle\right) =$$

$$= \frac{1}{\sqrt{2}}\left[e^{-i\left(m_S - \frac{i\Gamma_S}{2}\right)t}|K_S^0\rangle + e^{-i\left(m_L - \frac{i\Gamma_L}{2}\right)t}|K_L^0\rangle\right] = \ldots = \ldots = \ldots$$

$$\boxed{|\psi(t)\rangle = a(t)|K^0\rangle + b(t)|\overline{K^0}\rangle}$$




so let's be more general:

# Time evolution of neutral mesons

1. Assume that we have two neutral meson states: $\boldsymbol{P^0}$ and $\boldsymbol{\overline{P^0}}$ (they can be $K^0, D^0, B^0$) with an internal quantum number $F$ such that $\Delta F = 0$ for strong and ELM interaction, but $\Delta F \neq 0$ for weak interactions.

$$|\psi(t)\rangle = a(t)|P^o\rangle + b(t)|\overline{P}^o\rangle$$

only the coefficients are time-dependent, states- are not

2. The state obeys Schrödinger equation with total (effective) Hamiltonian (compare with the previous lecture):

$$i\hbar \frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = \mathbf{H}\begin{pmatrix} a \\ b \end{pmatrix} \equiv (\mathbf{M} - \frac{i}{2}\mathbf{\Gamma})\begin{pmatrix} a \\ b \end{pmatrix}$$

time evolution of coefficients only,
$H$ is not hermitian,
$H = H_0 + H_W$

3. $M$ and $\Gamma$ are $2 \times 2$ Hermitian matrices (mass and decay)

$$\mathbf{H} \equiv \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

we need to solve the eigenvalue problem for this system:
- diagonalise $H$ matrix,
- find eigenvectors and eigenvalues (try this!)

4. $\mathcal{CPT}$ Theorem imposes that mass and width of particle $P^0$ and antiparticle $\overline{P^0}$ are the same, so: $\boldsymbol{H_{11} = H_{22}}$

# Time evolution of neutral mesons

5. First some commonly used definitions:

$\boldsymbol{E_{1,2}}$ are eigenvalue that we have after solving the characteristic equation (H matrix should be diagonal):

$$|H - E\,I| = 0$$

$$\left(M - \frac{i}{2}\Gamma - E\right)^2 = \left(M_{12} - \frac{i}{2}\Gamma_{12} - E\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^* - E\right)$$

$$E_1 = m_1 - \frac{i\Gamma_1}{2} = \left(M - \frac{\Delta m}{2}\right) - i\left(\Gamma - \frac{\Delta\Gamma}{2}\right)$$

$$E_2 = m_2 - \frac{i\Gamma_2}{2} = \left(M + \frac{\Delta m}{2}\right) - i\left(\Gamma + \frac{\Delta\Gamma}{2}\right)$$

$$m_{1,2} = M \pm \frac{\Delta m}{2}$$

$$\Delta m = m_2 - m_1$$

$$\Gamma_{1,2} = \Gamma \pm \frac{\Delta\Gamma}{2}$$

$$\Delta\Gamma = \Gamma_2 - \Gamma_1$$

and eigenvector equation of the form:

$$(H - EI)\begin{pmatrix} p \\ \pm q \end{pmatrix} = 0$$

gives the relations:

$$\frac{p}{q} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M^*_{12} - \frac{i}{2}\Gamma^*_{12}}}$$

# Time evolution of neutral mesons

7. The eigenstates of effective Hamiltonian written in the form:

$$|P_1\rangle = p|P^0\rangle + q|\overline{P^0}\rangle$$
$$|P_2\rangle = p|P^0\rangle - q|\overline{P^0}\rangle$$

(compare with $K_S^0$ and $K_L^0$ as a mixture of $K^0$ and $\overline{K^0}$)

$p$ and $q$ are complex numbers satisfying: $|p|^2 + |q|^2 = 1$ (for $K_1^0$ and $K_2^0$ : $p = q = \frac{1}{\sqrt{2}}$ )

8. Solving Schrödinger equation we see time evolution of the eigenstates (tutorial):

$$|P_1(t)\rangle = |P_1\rangle\, e^{-i\left(m_1 - \frac{i\Gamma_1}{2}\right)t}$$

$$|P_2(t)\rangle = |P_2\rangle\, e^{-i\left(m_2 - \frac{i\Gamma_2}{2}\right)t}$$

These relations show that the original $P^0$ meson after some time can either convert to $\overline{P^0}$ or decay.

# Time evolution of neutral mesons

9. Finally the time evolution of weak eigenstates as a combination of flavour eigenstates:

$$|P^0(t)\rangle = f_+(t)|P^0\rangle + \frac{q}{p} f_-(t)|\overline{P^0}\rangle$$

$$|\overline{P^0}(t)\rangle = f_+(t)|\overline{P^0}\rangle + \frac{p}{q} f_-(t)|P^0\rangle$$

$$f_\pm(t) = \frac{1}{2}\left[e^{-i(m_1 - \frac{i}{2}\Gamma_1)t} \pm e^{-i(m_2 - \frac{i}{2}\Gamma_2)t}\right]$$

solve this!

$$|f_\pm(t)|^2 = \frac{1}{4}\left[e^{-i\Gamma_1 t} + e^{-i\Gamma_2 t} \pm 2e^{-\bar{\Gamma}t}\cos(\Delta m t)\right]$$

$$\bar{\Gamma} = \frac{\Gamma_1 + \Gamma_2}{2}$$

interference term

10. The time evolution of mixing probabilities, i.e. the probability that having started the observation with a $P^0$ meson, after some time $t$ we still have $P^0$ (or it has oscillated to $\overline{P^0}$):

$$P(P^0 \to P^0; t) = |\langle P^0|P^0(t)\rangle|^2 = |f_+(t)|^2$$

$$P(P^0 \to \overline{P^0}; t) = |\langle \overline{P^0}|P^0(t)\rangle|^2 = \left|\frac{q}{p} f_-(t)\right|^2$$

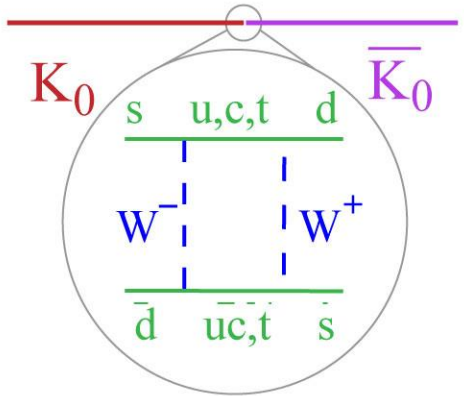
Let's look closer at the parameters of flavour oscillations:

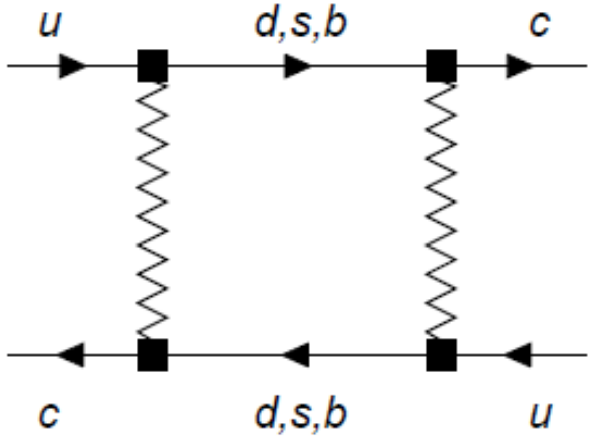
# Time evolution of neutral mesons

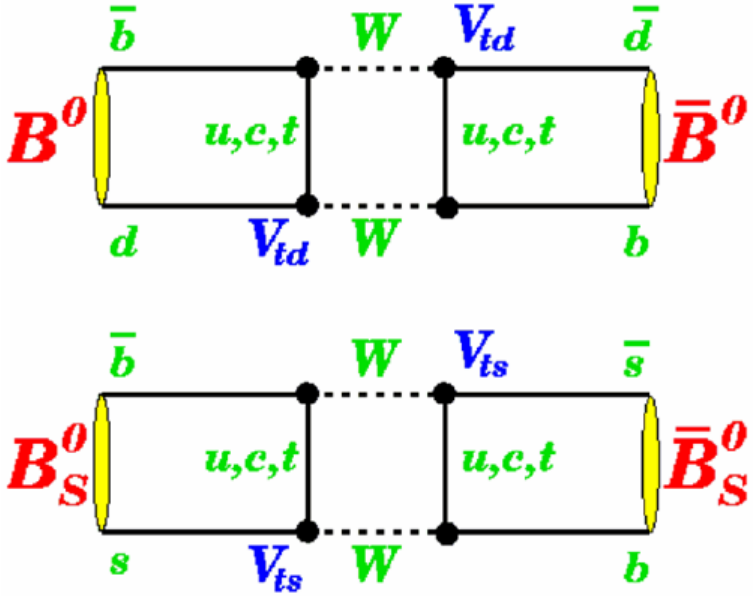
11. We can find four $P^0$ -type mesons and investigate their flavour oscillations:

| | $d$ | $s$ | $b$ |
|---|---|---|---|
| $\overline{d}$ | $\times$ | $K^0$ | $B^0$ |
| $\overline{s}$ | $\overline{K^0}$ | $\times$ | $B_s$ |
| $\overline{b}$ | $\overline{B^0}$ | $\overline{B_s}$ | $\times$ |

| | $u$ | $c$ | $t$ |
|---|---|---|---|
| $\overline{u}$ | $\times$ | $D^0$ | $\diamond$ |
| $\overline{c}$ | $\overline{D^0}$ | $\times$ | $\diamond$ |
| $\overline{t}$ | $\diamond$ | $\diamond$ | $\times$ |

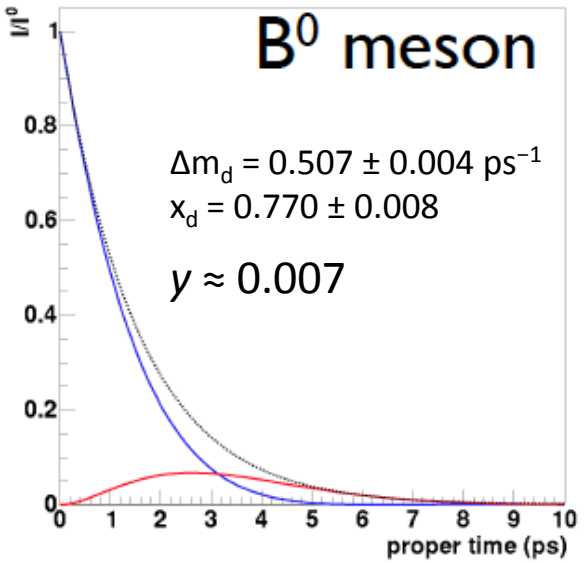


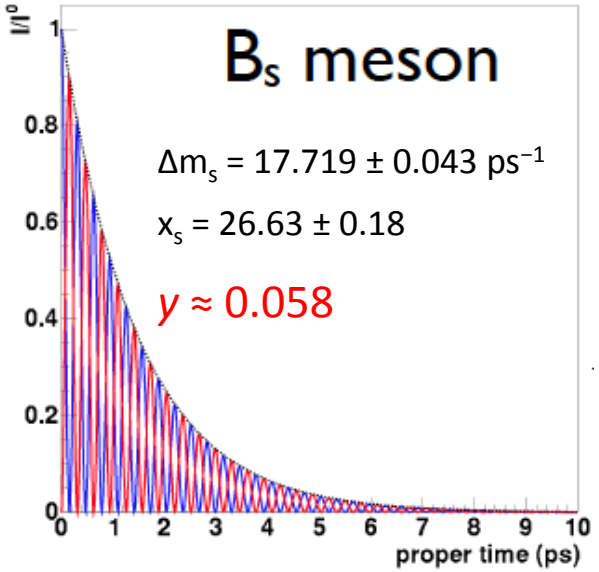



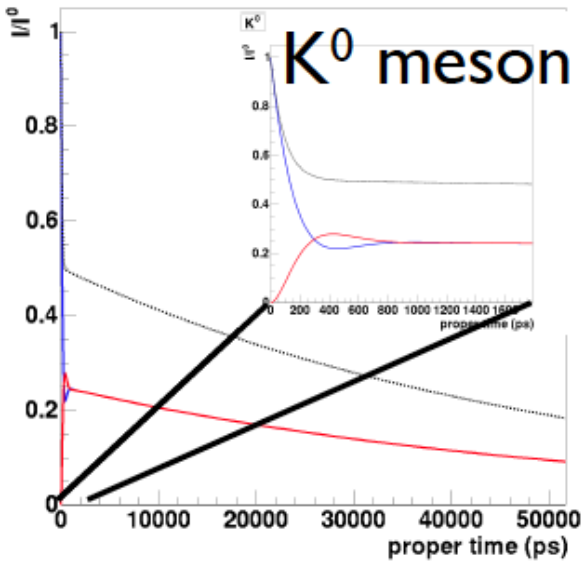



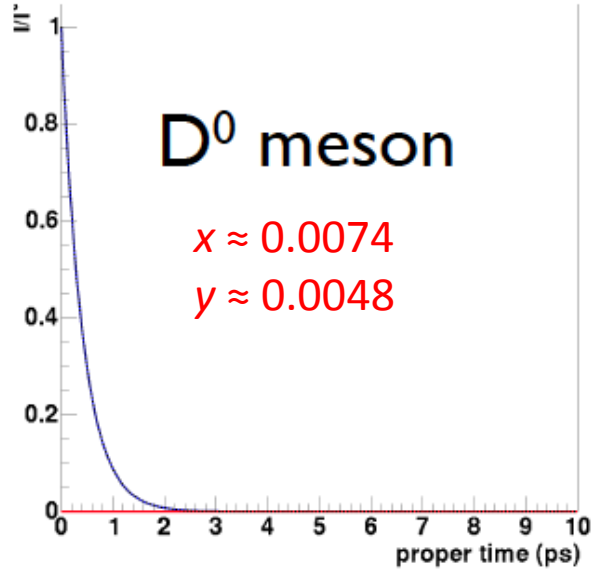

Two parameters determine period of oscillations compared to lifetime: $x = \frac{|\Delta m|}{\overline{\Gamma}}$ $y = \frac{|\Delta \Gamma|}{2\Gamma}$

# Time evolution of neutral mesons




- $D^0$ mesons: *very, very* slowly
- $K^0$ mesons: very slowly
- $B_d$ mesons: slowly
- $B_s$ mesons: fast!

The frequency of $B^0_s$ – anti-$B^0_s$ oscillations is the highest.
On average, a $B^0_s$ meson changes its flavour 9 times between production and decay

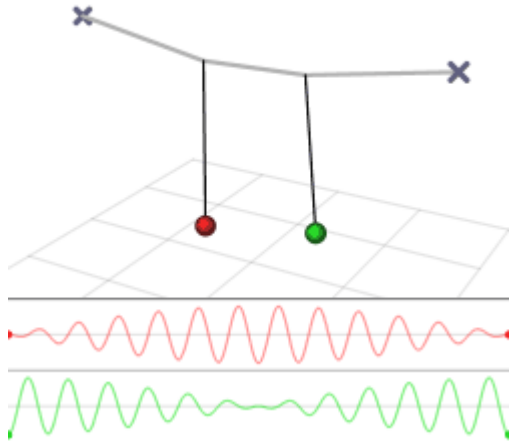
Blue:
given a $P^0$, at t=0, the probability of finding a $P^0$ at t.

Red:
given a $P^0$, at t=0, the probability of finding a $P^0$bar at t.

Project!

# Summary of oscillations

1. Experimental puzzle with strange time life of a new particle has been solved by introducing weak decays of strange mesons.
2. Since they are produced in strong interactions and decay through weak, a second order box process occurs and is a way for flavour oscillations.
3. So neutral mesons behave like coupled damped  pendulum.
4. This also holds for other neutral mesons.
5. In addition a CP violating decays were discovered in kaons decays.

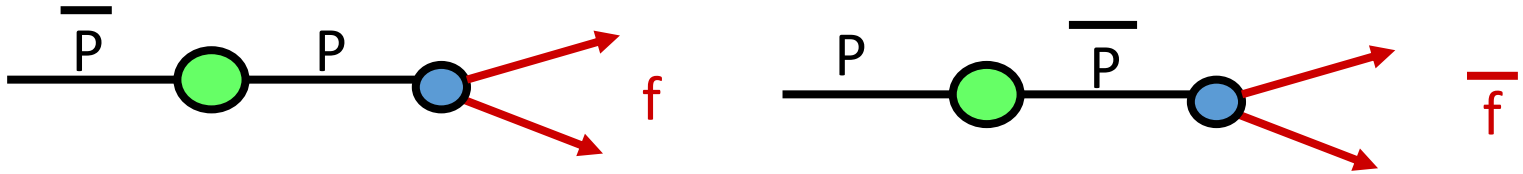


So now, the question is: how to connect mixing (flavour oscillations) with CP violation?

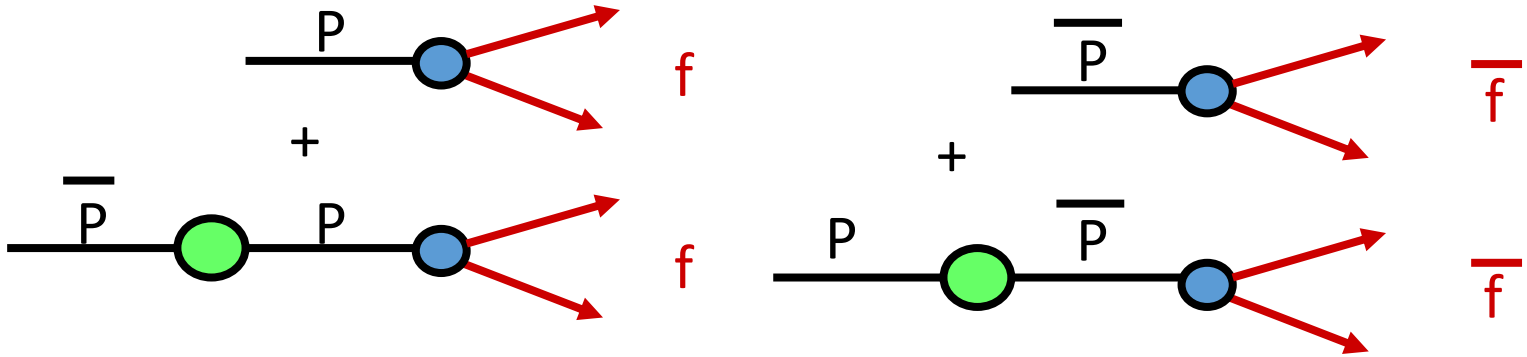
# Types of CP Violation

I. CP violation in decay (direct CP Violation)

II. CP violation in mixing (indirect CP Violation)




III. CP violation in interference between mixing and decay



# CP Violation in decay (direct)

1. One of the simplest way to discover $\mathcal{CPV}$ is to compare the decay rates $\Gamma(P \to f)$ with $\Gamma(\overline{P}) \to \overline{f}$
2. This is a method for direct $\mathcal{CPV}$ in decay amplitudes, when two amplitudes with diffrent phases interfere.
3. If we define the asymmetry between $\mathcal{CP}$ conjugated decays, for charged and neutral mesons:

$$A_{CP,dir} = \frac{\Gamma\{P \to f\} - \Gamma\{\overline{P} \to \overline{f}\}}{\Gamma\{P \to f\} + \Gamma\{\overline{P} \to \overline{f}\}}$$

where: $\Gamma(P \to f) \propto |A_f|^2$

3. Amplitude $A_f$:

- is defined as a matrix element that describes the transition between state $P$ and $f$, such that $P \to f$ depends on:

  $A_f = \langle f|H|P\rangle$ and $\overline{P} \to f$ on: $\overline{A_f} = \langle f|H|\overline{P}\rangle$

- is a complex number that can be written as a value $A$ and phase: $A_f = A\, e^{i\phi}\, e^{i\delta}$
- Usually the amplitude $A_f$ has a strong phase $\delta$ that is invariant under CP transformation and weak phase $\phi$ that changes sign under CP.

# CP Violation in decay

4. Final state $f$ can be $\mathcal{CP}$ eigenstate or not $\mathcal{CP}$ eigenstate. In the former additional amplitudes are written: $\overline{A_{\bar{f}}}$ and $A_{\bar{f}}$

5. The phase of the amplitude emerges only if we could find two different amplitudes that lead to the same final state, and:
   - their amplitudes had both different strong and weak phases,
   - then we would see evidence for direct $\mathcal{CP}$ violation (in decay) and decay rates will be different :

$$\Gamma(P \to f) \neq \Gamma(\overline{P} \to \bar{f})$$

   - most general form of asymmetry:

$$A = \frac{|\overline{A_f}|^2 - |A_f|^2}{|\overline{A_f}|^2 + |A_f|^2} = \frac{2|A_1|\,|A_2| sin(\delta_1 - \delta_2)\, sin(\phi_1 - \phi_2)}{|A_1|^2 + |A_2|^2 + |A_1|\,|A_2|\, cos(\delta_1 - \delta_2)\, cos(\phi_1 - \phi_2)}$$

amplitude interference!

# CP Violation in decay

6. We can also write a couple of asymmetries in a different form, e.g.:

$$A_f \equiv A(B^- \to f) = A_1\, e^{i\phi_1}\, e^{i\delta_1} + A_2\, e^{i\phi_2}\, e^{i\delta_2}$$

$$\bar{A}_{\bar{f}} \equiv \bar{A}\left(B^+ \to \bar{f}\right) = A_1\, e^{-i\phi_1}\, e^{i\delta_1} + A_2\, e^{-i\phi_2}\, e^{i\delta_2}$$

$$\left|A_f\right|^2 - \left|\bar{A}_{\bar{f}}\right|^2 = 2|A_1|\,|A_2| sin(\delta_1 - \delta_2)\, sin(\phi_1 - \phi_2)$$

$$\boldsymbol{\Gamma(P \to f) \neq \Gamma(\bar{P} \to \bar{f})}$$

- or (if there are more amplitudes leading to the state $f$) we can express this by:

show this!

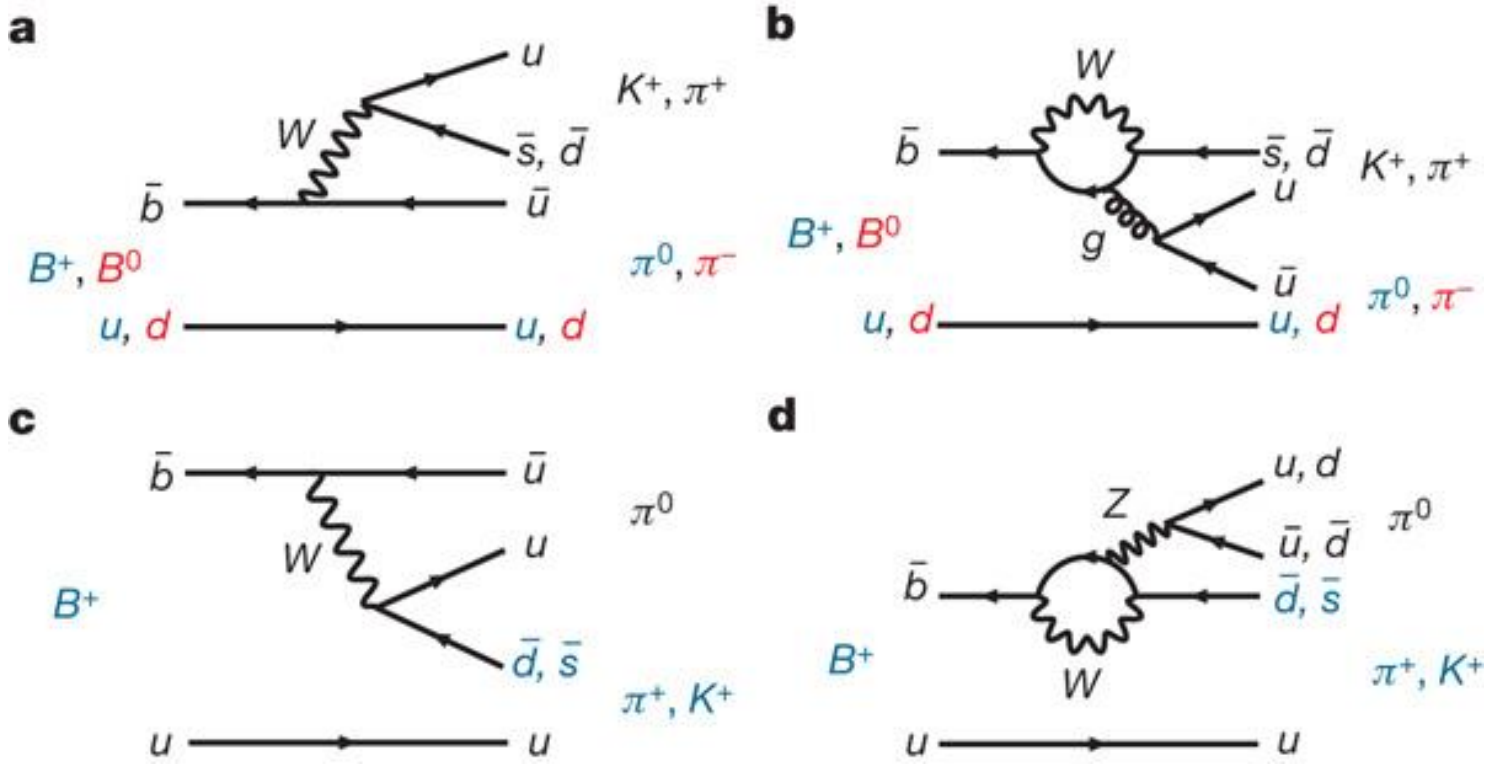
if $\mathcal{CP}$ is **NOT** conserved:

$$\left|\frac{A_f}{\bar{A}_{\bar{f}}}\right| = \left|\frac{\sum_i A_i e^{i\varphi_i} e^{i\delta_i}}{\sum_i A_i e^{-i\varphi_i} e^{i\delta_i}}\right| \neq 1$$

if $\mathcal{CP}$ is conserved:

$$\left|\frac{A_f}{\bar{A}_f}\right| = 1$$

# CPV in decay




Think about experimental challenges!

- combinatorics,
- tagging,
- probability….

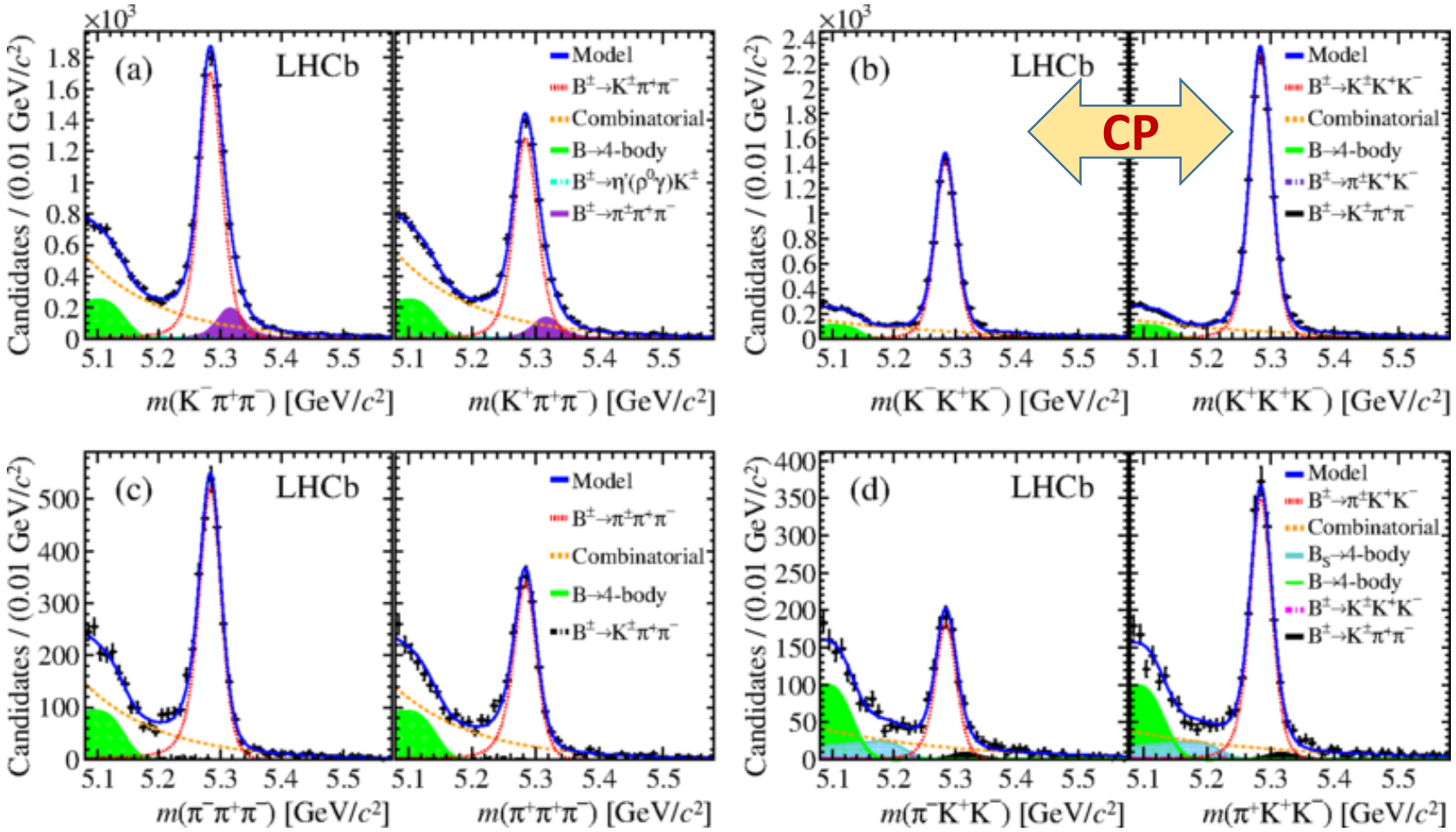
It is very common in flavour physics that simple ideas ($\mathcal{CPV}$ in differences in decay rates) are the most difficult for experiment.

# CPV in decay

$B^+ \to K^-K^+K^+$

Huge direct **CP** violation in decay amplitudes seen in $B^-/B^+$ decays

$B^- \to K^-K^+K^-$

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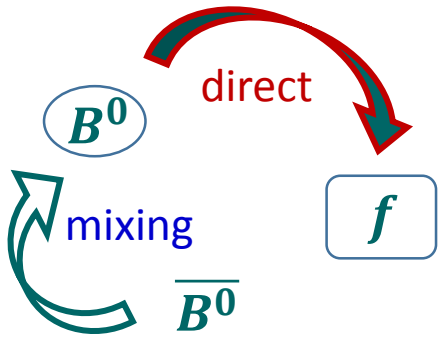
# CP Violation in mixing

1. $\mathcal{CP}$ Violation in mixing (indirect $\mathcal{CP}$ violation) based on the spontaneous oscilations of a particle into antiparticle (mixing).
2. Mass eigenstates are different from $\mathcal{CP}$ eigenstates.
3. Only for neutral mesons! See kaons section.
4. Mixing does not necessarily mean $\mathcal{CP}$ violation, but can provide additional amplitude that can interfere.
5. Mixing rate for $P^0 \to \overline{P^0}$ is different from $\overline{P^0} \to P^0$.
6. If the weak states of neutral meson is:

$$|P_1\rangle = p|P^0\rangle + q|\overline{P^0}\rangle$$

$$|P_2\rangle = p|P^0\rangle - q|\overline{P^0}\rangle$$




the $\mathcal{CP}$ symmetry is violated if:

$$\left|\frac{q}{p}\right|^2 = \left|\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right| \neq 1$$

- very small effect,
- hard to measure because of hadronic uncertainties,
- so far no evidences of $\mathcal{CPV}$ in mixing

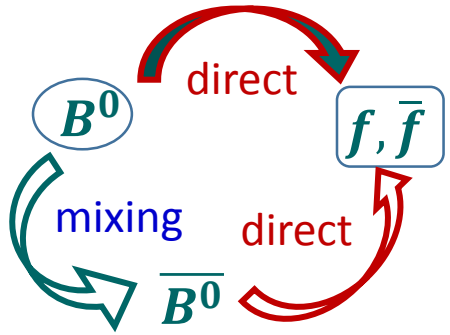
# CP Violation in interference

1. $\mathcal{CP}$ violation in interference between direct amplitude and amplitude after mixing.
2. Only for neutral mesons.
3. Some definitions:

$$A_f = \langle f|H|P^0\rangle \qquad \bar{A}_f = \langle f|H|\overline{P^0}\rangle$$

$$A_{\bar{f}} = \langle \bar{f}|H|P^0\rangle \qquad \bar{A}_{\bar{f}} = \langle \bar{f}|H|\overline{P^0}\rangle$$

$$\lambda \equiv \left(\frac{q}{p}\right)\left(\frac{\bar{A}_f}{A_f}\right) \qquad \bar{\lambda} \equiv \left(\frac{q}{p}\right)\left(\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}\right)$$




4. $\mathcal{CP}$ is conserved if: $\lambda = 1$ and $\bar{\lambda} = 1$
5. Try to calculate time dependent rates:

$$\Gamma_f = \left|\langle f|H|P^0(t)\rangle\right|^2 \qquad \bar{\Gamma}_f = \left|\langle f|H|\overline{P^0}(t)\rangle\right|^2$$

$$\Gamma_{\bar{f}} = \left|\langle \bar{f}|H|P^0(t)\rangle\right|^2 \qquad \bar{\Gamma}_{\bar{f}} = \left|\langle \bar{f}|H|\overline{P^0}(t)\rangle\right|^2$$

homework!

$$|P^0(t)\rangle = f_+(t)|P^0\rangle + \frac{q}{p} f_-(t)|\overline{P^0}\rangle$$

6. And asymmetry (**time dependent!**)

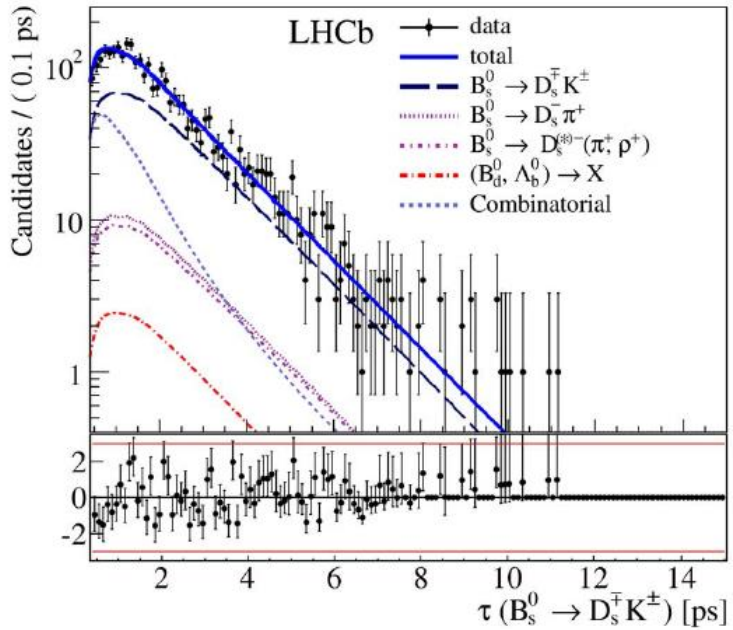
$$A_{CP}(t) = \frac{\Gamma\{P(t) \to f\} - \Gamma\{\bar{P}(t) \to \bar{f}\}}{\Gamma\{P(t) \to f\} + \Gamma\{\bar{P}(t) \to \bar{f}\}}$$

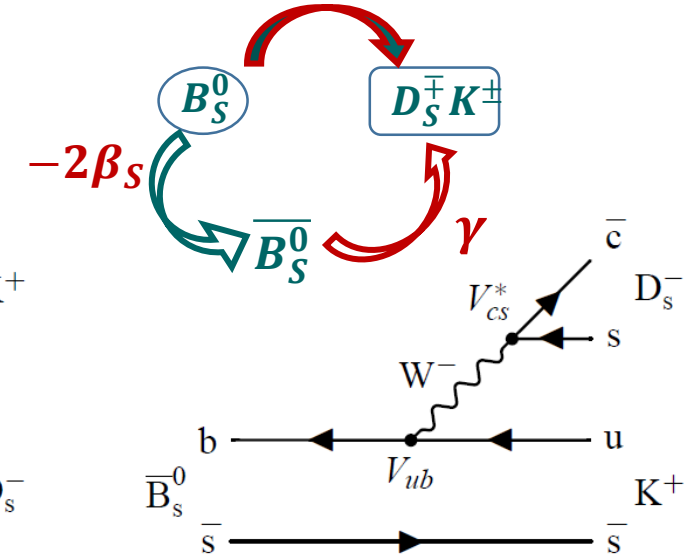
# CP Violation in interference

$B_S^0 \to D_S^{\mp(*)} K^{\pm(*)}$

**Time dependent**

1. Interference between mixing and direct decay, large effect because decays are not colour suppressed,
2. Sensitive to $(\gamma + \phi_s)$, strong phase $\delta$,
3. Need to measure 4 time dependent decay rates




$$\Gamma_{B_S^0 \to f}(t) = |A_f|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma_s t}}{2} \cdot \left( \cosh \frac{\Delta\Gamma_s t}{2} + \boldsymbol{D_f} \sinh \frac{\Delta\Gamma_s t}{2} + C_f \cos \Delta m_s t - \boldsymbol{S_f} \sin \Delta m_s t \right)$$

$\boldsymbol{D_f \propto \cos(\delta - (\gamma - 2\beta_S))}$

$\boldsymbol{S_f \propto \sin(\delta - (\gamma - 2\beta_S))}$

First measurement with this technique, 1fb$^{-1}$

# CP Violation summary

1. The discovery of flavour oscillations and kaon mixings opened new roads for $\mathcal{CP}$ violation searches.
2. The heavier the meson is – the more decays channels are available, among them should be the ones that violate $\mathcal{CP}$.
3. We have tree types of $\mathcal{CP}$ violation.
4. The connection between experimental observables achieved by this method – next lecture (in two weeks time)

Next week – tutorial.
Then – two lectures
and another tutorial.
Problems for tutorial have been prepared:

# Tutorial next week

## CP-Violation – Problem sheet 2015

P1. Using appropriate conservation rules explain why the following decays are not allowed

$$K_2^0 \to \pi^0 + \pi^0$$

$$K_2^0 \to \pi^+ + \pi^-$$

P2. Justify the form of the operator equations (with the effective Hamiltonian), especially the e-values, describing the two state kaon system:

$$\mathcal{H}_{eff}|K_S^0\rangle = \left(m_S - \frac{i}{2}\gamma_S\right)|K_S^0\rangle$$

$$\mathcal{H}_{eff}|K_L^0\rangle = \left(m_L - \frac{i}{2}\gamma_L\right)|K_L^0\rangle$$

Give the geometrical interpretation of these equations (remember that both $K_S^0$ and $K_L^0$ can be thought as vectors in the Hilbert space).

P3. Calculate the e-values of the Hamiltonian matrix describing two state system of decaying kaons. Hint: use the partially finished calculation shown during the lecture.

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# Paper Reading Club

LHCb-PUB-2009-029
16 February 2010

# Roadmap for selected key measurements of LHCb

The LHCb Collaboration[1]

Paper

2 The tree-level determination of $\gamma$

3 Charmless charged two-body B decays

4 Measurement of mixing-induced CP violation in $B_s^0 \to J/\psi\phi$

5 Analysis of the decay $B_s^0 \to \mu^+\mu^-$

6 Analysis of the decay $B^0 \to K^{*0}\mu^+\mu^-$