CP Violation in Heavy Flavour Physics	**Dirac Equation, Symmetries & Currents**
Tutorial 1	28th March 2018

1. What conditions should the γ matrices fulfil for the Dirac Equation $(i\gamma^\mu\partial_\mu - m)\psi = 0$ to be consistent with the Klein-Gordon equation $\left(-\frac{\partial^2}{\partial t^2}\Psi + \nabla^2\right) = m^2\psi$?
2. Postulate that the solutions of the Dirac Equation have form of the plane wave and a spinor: $\psi(x^\mu) = u(p^\mu)e^{-i(Et-\vec{p}\cdot\vec{x})}$ and write out the DE for spinors.
3. Use the previous result and find four solutions of Dirac Equation for a particle with momentum $\vec{p}$ and mass m.
4. Define the properties of Dirac spinors using their transformation properties.
5. Show that for a particle/antiparticle with momentum $\vec{p} = (0,0,p)$ the u_1 and v_1 spinors represent spin up states and u_2 and v_2 spin down states.
6. Using the properties of the γ matrices and the definition of γ^5, show that: $(\gamma^5)^2 = 1$, $\gamma^{5\dagger} = \gamma^5$, $\{\gamma^5, \gamma^\mu\} = 0$.
7. Show that the chiral projection operators $P_L = \frac{1}{2}(1-\gamma_5)$ and $P_R = \frac{1}{2}(1+\gamma_5)$ satisfy: $P_L\, P_R = P_R\, P_L = 0$, $\quad P_L + P_R = 1$, $P_L\, P_L = P_L$, $P_R\, P_R = P_R$, $P_L\, P_R = 0$. These conditions prove that these operators are projectors (applying one of them twice gives the same result as applying it once and that applying both of them results in the null state.
8. Discuss in detail Dirac equations and their solutions for particles and anti-particles considering the interaction term with the electromagnetic field, i.e., by substituting the 4-derivative with covariant 4-derivative: $i\partial_\mu \to i\partial_\mu - qA_\mu$, where A_μ is the 4-potential. Next, show, using the explicit calculation, how spinors behave w.r.t. the P- and C-parity operators.
9. Check the respective properties of (S), (P), (V) and (A) bi-linear forms under P- and C-parity operators.
10. Show that only by using a mixed (V)-(A) bi-linear form we are able to obtain matrix elements that breaks maximally P- and C-parities.