AGH

CP Violation In Heavy Flavour Physics

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The CKM Matrix

1. In two generation system (1964) – one angle – no CP violation
2. Third generation proposed by Kobayashi & Maskawa (1973) opened the Pandora's box of new ideas how to measure CPV.
3. Many possible parametrizations:
   a) original K-M: $s_i = \sin\vartheta_i$; $c_i = \cos\vartheta_i$
   b) Standard representation (PDG proposal):

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$\begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{23} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \end{pmatrix}$$

The CKM matrix is described by three rotation angles and a complex phase.

CKM matrix parametrization

4. Wolfenstein parametrization (1983):
   a) matrix elements are expanded in terms of $\sin\vartheta_i \equiv \lambda$;
   b) from kaons sector we have $V_{us} = \lambda = 0.22$, and $V_{ud} = (1-\lambda^2/2)$
   c) from B-lifetime: $V_{cb} = 0.04 - 0.006 = A\lambda^2$
   d) let's keep V_{ud}, V_{us}, V_{tb} real – expressed in term of four real parameters : λ, A, ρ, η
   e) the only complex components are in V_{ub} and V_{td}, third row/column are of order smaller (CPV effects) $A\lambda^3(\varrho - i\eta)$
   f) For the CP violation – one phase should be measurable, so η cannot be zero

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{\lambda^2}{2}) \\ -\lambda & 1-\frac{\lambda^2}{2} - i\eta A\lambda^4 & A\lambda^2(1+i\eta\lambda^2) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$|V_{td}|e^{-i\beta}$

$|V_{ub}|e^{-i\gamma}$

Higher order in CKM

5. Higher order in Wolfenstein parametrization:

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

$$\begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda-A^2\lambda^5(\rho+i\eta-\frac{1}{2}) & 1-\frac{\lambda^2}{2}-(\frac{1}{8}+\frac{A}{2})\lambda^4 & A\lambda^2 \\ A\lambda^3[1-(\rho+i\eta)(1-\frac{\lambda^2}{2})] & -A\lambda^2-A\lambda^4(\rho+i\eta-\frac{1}{2}) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

$|V_{ts}|e^{-i\beta_s}$

$\boldsymbol{\beta}$ and $\boldsymbol{\beta_s}$ are weak phases in $\boldsymbol{B^0}$ and $\boldsymbol{B_S^0}$ mixing,

$\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are the CKM angles (see next slides).

They are ones of the most important observables in experimental heavy flavour physics

Unitarity of CKM matrix

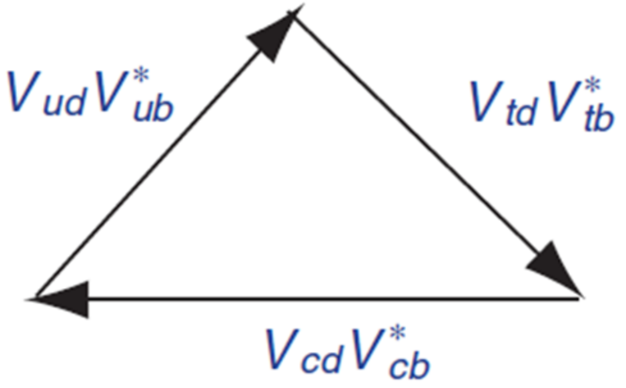
The CKM matrix is unitary $V_{CKM}^{-1} = V_{CKM}^{\dagger}$ – this give us 12 orthogonality conditions:

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$
$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$$
$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$$
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1$$
$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$$

$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$$
$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$
$$V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} = 0$$
$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$$
$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$
$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0$$

The orthogonality conditions can be regarded as a triangle condition – CKM matrix elements are complex numbers, so their sum is simply a sum of three vectors:



Unitarity of CKM matrix

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

But most of them have magnitudes of very different size and are currently useless from experimental point of view :

$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0 \qquad \lambda,\ \lambda,\ \lambda^5$$

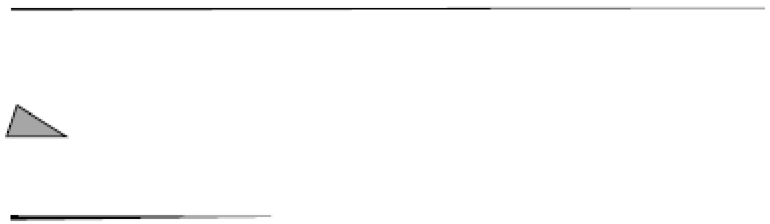
$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0 \qquad \lambda^3,\ \lambda^3,\ \lambda^3$$

$$V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} = 0 \qquad \lambda^4,\ \lambda^2,\ \lambda^2$$

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0 \qquad \lambda,\ \lambda,\ \lambda^5$$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \qquad \lambda^3,\ \lambda^3,\ \lambda^3$$

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0 \qquad \lambda^4,\ \lambda^2,\ \lambda^2$$



The most attractive are two triangles:

$$\boxed{V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0}$$

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

Unitarity of CKM matrix

But still two promising left:

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$V^*_{ud}V_{cd}+V^*_{us}V_{cs}+V^*_{ub}V_{cb}=0$	$\lambda, \lambda, \lambda^5$
$V^*_{ud}V_{td}+V^*_{us}V_{ts}+V^*_{ub}V_{tb}=0$	$\lambda^3, \lambda^3, \lambda^3$
$V^*_{cd}V_{td}+V^*_{cs}V_{ts}+V^*_{cb}V_{tb}=0$	$\lambda^4, \lambda^2, \lambda^2$
$V_{ud}V^*_{us}+V_{cd}V^*_{cs}+V_{td}V^*_{ts}=0$	$\lambda, \lambda, \lambda^5$
$V_{ud}V^*_{ub}+V_{cd}V^*_{cb}+V_{td}V^*_{tb}=0$	$\lambda^3, \lambda^3, \lambda^3$
$V_{us}V^*_{ub}+V_{cs}V^*_{cb}+V_{ts}V^*_{tb}=0$	$\lambda^4, \lambda^2, \lambda^2$

„The” unitary triangle!

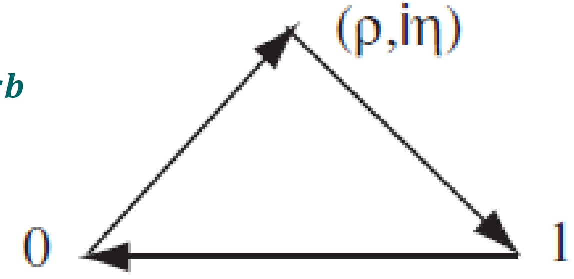
Using Wolfenstein parametrization, we can draw them on complex plane :

$$V_{ud}V^*_{ub} = A\lambda^3\left(1-\lambda^2/2\right)(\varrho+i\eta)$$

$$V_{cd}V^*_{cb} = -A\lambda^3$$

$$V_{td}V^*_{tb} = A\lambda^3(1-\varrho-i\eta)$$

if sides are divided by $V_{cd}V^*_{cb}$ the UT looks like that:



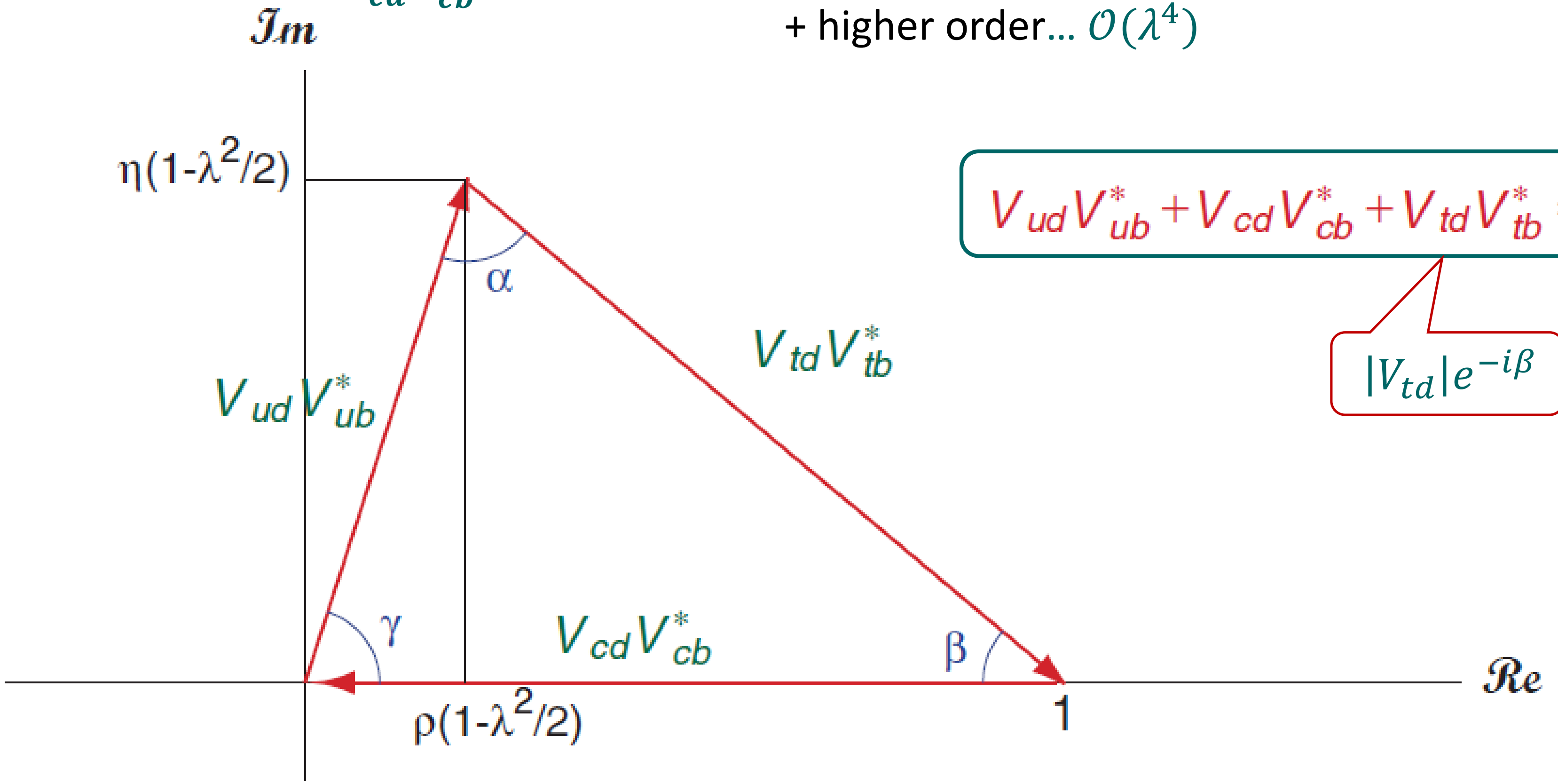
The unitary triangle

Try your vector algebra…

$$V_{ud}V_{ub}^{*} = A\lambda^3(1-\lambda^2/2)(\varrho + i\eta)$$
$$V_{td}V_{tb}^{*} = A\lambda^3(1-\varrho - i\eta)$$
$$V_{cd}V_{cb}^{*} = -A\lambda^3$$

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

+ higher order… $\mathcal{O}(\lambda^4)$



$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

$|V_{td}|e^{-i\beta}$

$$\alpha = \arg\left(-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right)$$

$$\beta = \arg\left(-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right)$$

And another unitarity triangle

and more complex example…

$$V_{ub}V_{tb}^* = A\lambda^3(\varrho + i\eta)$$
$$V_{ud}V_{td}^* = A\lambda^3(1-\lambda^2/2)(1-\varrho-i\eta)$$
$$V_{us}V_{ts}^* = -A\lambda^3$$

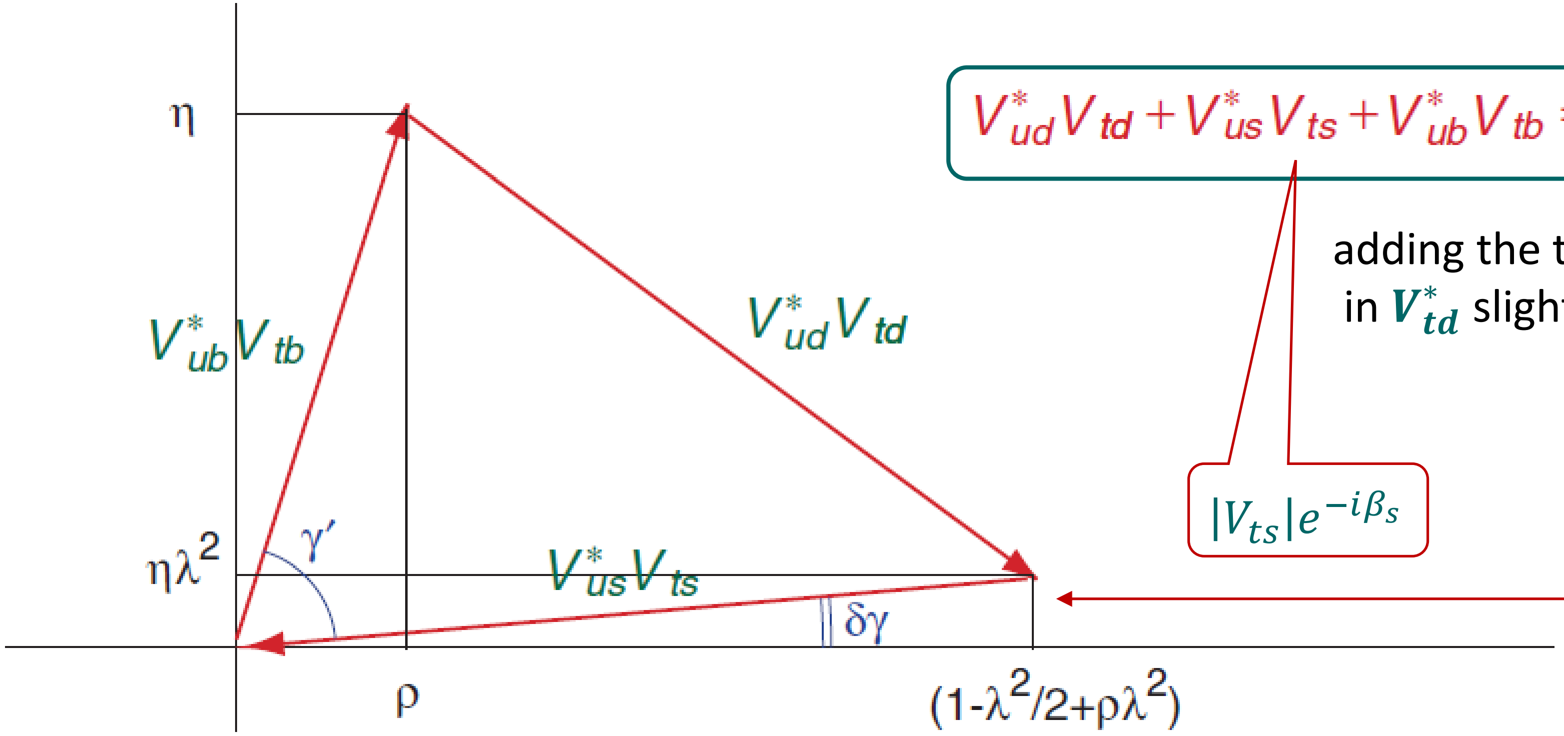
$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

+ higher order… $\mathcal{O}(\lambda^4)$

$-iA\lambda^4\eta$

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

$|V_{ts}|e^{-i\beta_s}$



adding the term $\mathcal{O}(\lambda^4)$ in V_{td}^* slightly tilts the Δ

precise measurements can prove this!

Physics with the Unitary Triangles

1. HEP has always two main aims:

   to confirm the SM

   or

   to find evidences for Physics Beyond the Standard Model.

2. Precise measurement of the UTs are able to fulfill both….
   a) If the triangle remains triangular – we have three generation of quarks with small CP violation effects.
   b) If one angle is „open"- fourth generation?
   c) If an angle is greater then predictions – new particles were exchanged?

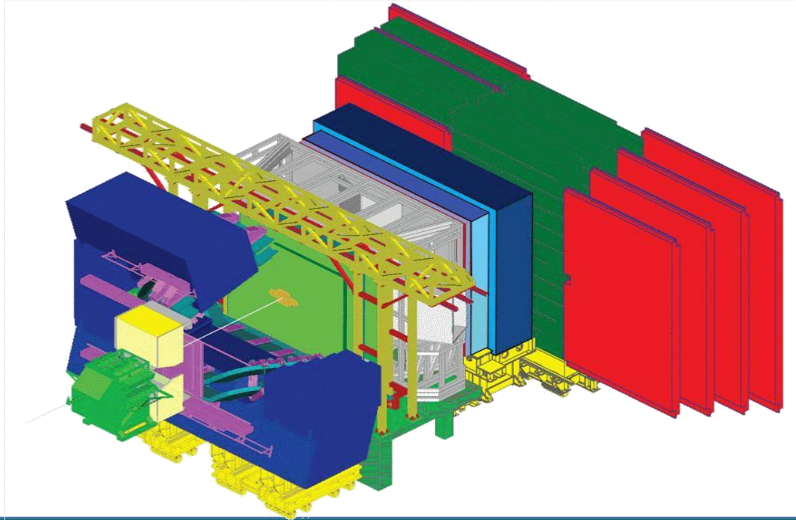
3. So the main purpose in WI is now to over constrain the UTs – measure all sides and angles with great precision and to compare them with SM predictions.

HOW?

With charm and beauty mesons decays.

WHERE?

At the LHCb spectrometer.



Sides of the Unitary Triangles

Sides of the UT can be measured with:

$\boldsymbol{V_{ud}}$	β-decay	Nuclear physics	$\cos\vartheta_i$
$\boldsymbol{V_{us}}$	K decay	$K^{+0} \to \pi^{0+} l^+ \nu_l$	$\sin\vartheta_i$
$\boldsymbol{V_{cd}}$	Neutrino scatering	$\nu_\mu d \to \mu^+ c$	$\cos\vartheta_i$
$\boldsymbol{V_{cs}}$	Charm decay	$D_S^+ \to \mu^+ \nu_\mu$	BR
$\boldsymbol{V_{ub}}$	B decay	$B^0 \to \pi^- e^+ \nu_e$	BR
$\boldsymbol{V_{cb}}$	B decay to charm		
$\boldsymbol{V_{td}}$	B mixing		

$$\boldsymbol{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0}$$

$b \to u$ transitions

$b \to c$ transitions

B^0 mixing

Angles of the Unitary Triangles

Angles of the UT can be measured with:

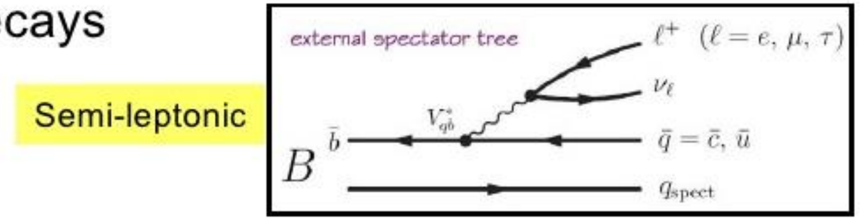
$B^0 \to J/\psi K_S$	$\sin 2\beta$
$B^0 \to \pi^+\pi^-$	$\sin 2\alpha$
$B_S^0 \to D_S^+ K^-$	$\sin 2\gamma$
Weak phase	$\boldsymbol{\beta_s}$

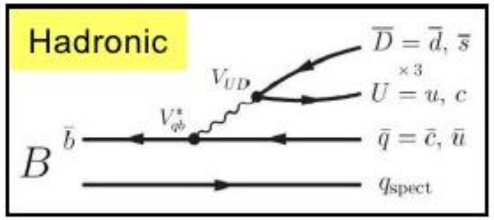
Short history of flavour physics:

1. First B physics experiments were build on symmetric electron-positron collider:
   - Petra (DESY) in 80'ties
   - LEP at CERN in 1994-2000
2. Then two asymmetric B-factories (currently not taking data):
   - Belle (Japan)
   - BaBar (SLAC,USA)
3. LHC
   - **LHCb – dedicated B physics experiment**
   - CMS, ATLAS also interested in heavy flavours

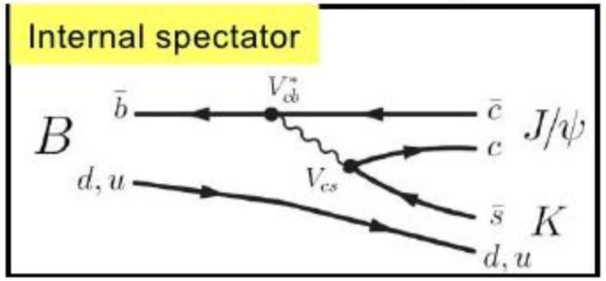
Market with diagrams

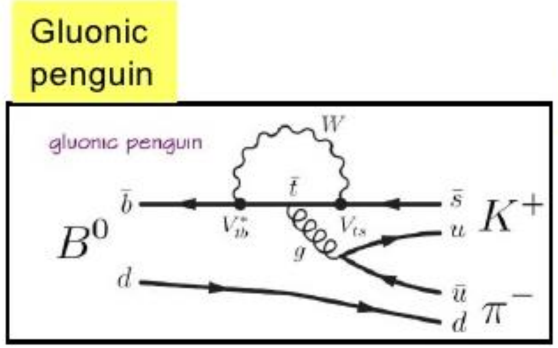
Dominant decays

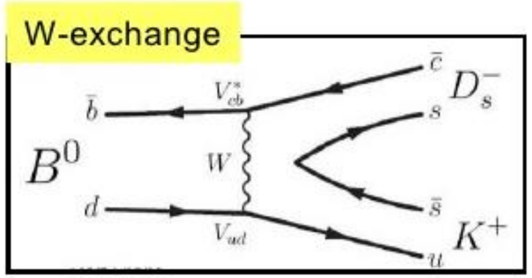




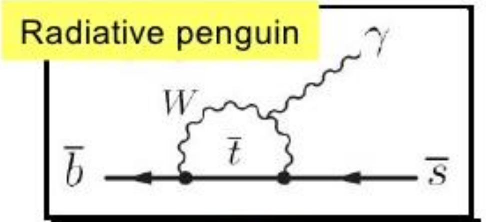
Rare hadronic decays

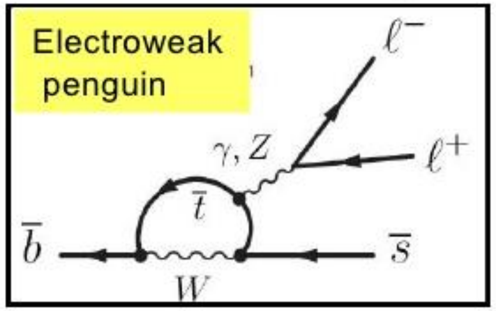


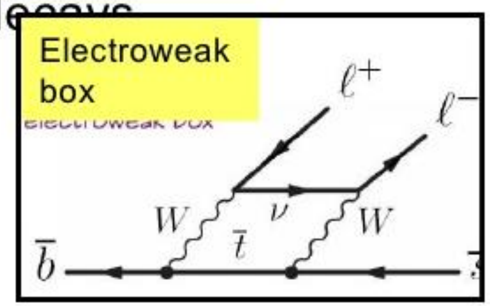


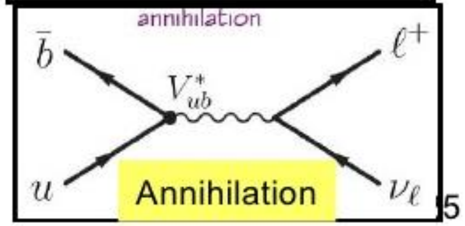


Radiative and leptonic decays









Summer School KPI 15 August 2009

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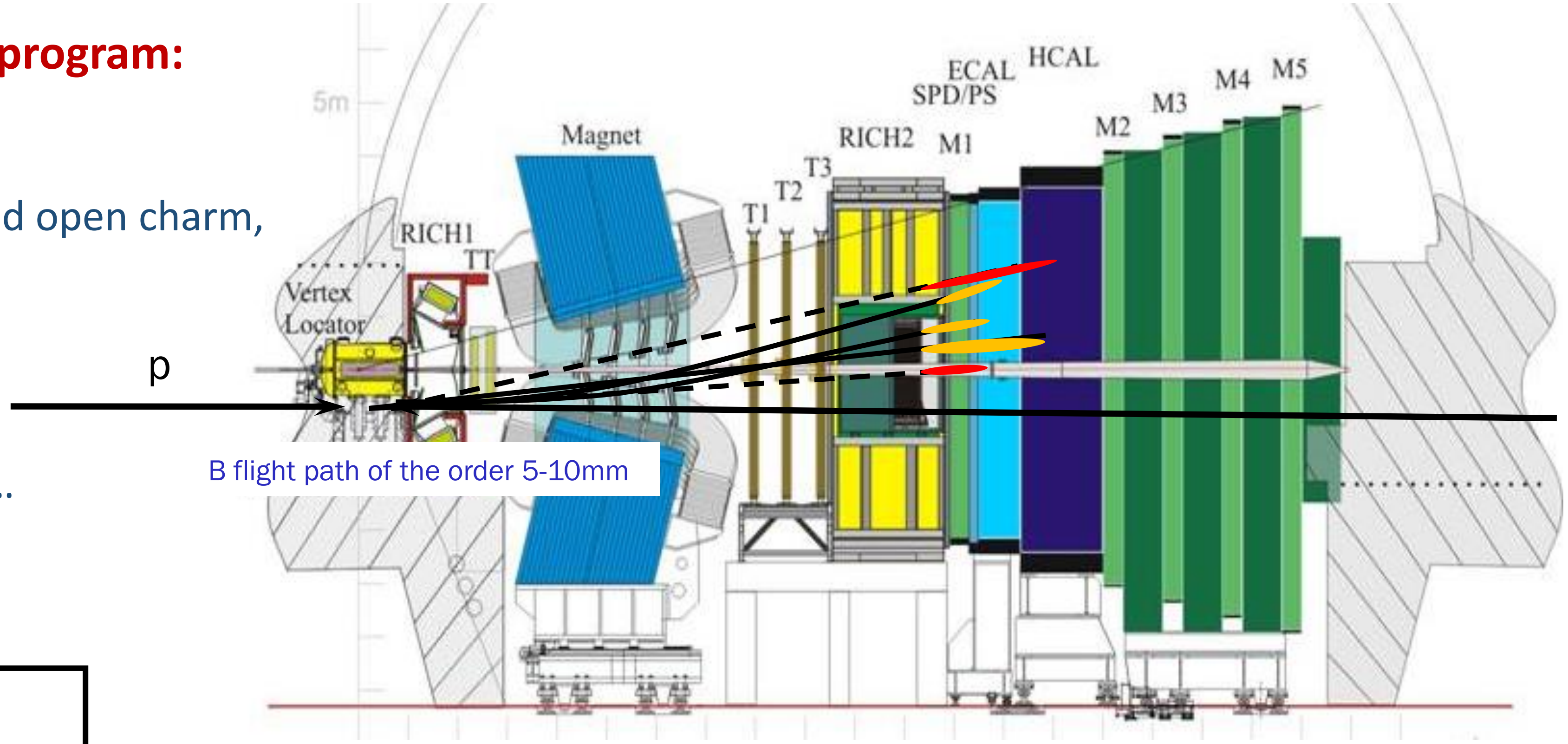
CPV – how to measure?

The experiment – LHCb spectrometer

Physics program:

- CP Violation ,
- Rare B decays,
- B decays to charmonium and open charm,
- Charmless B decays,
- Semileptonic B decays,
- Charm physics
- B hadron and quarkonia
- QCD, electroweak, exotica …



Tracking:
Silicon & Straw tubes
Magnetic field

Vertexing:
High precision silicon detectors (10μm position resolution) very close to collision point

RICH performance:
Cherenkov radiation.
Measures velocity, combine with momentum to get mass
Particle identification in p range 1-100 GeV
π, K ID efficiency > 90%, misID<~10%

Calorimeters:
Electromagnetic &
Hadronic calorimeters
- Critical (with muons) for triggering

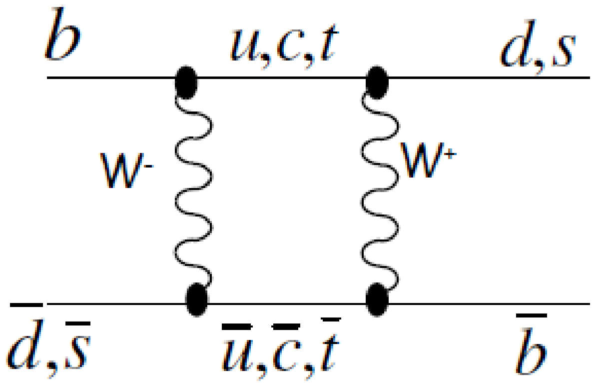
Mixing of B^0 and B_S^0 meson

1. Like neutral kaon system, neutral B mesons may also oscillate:
2. The top quark transition has the dominant amplitude:

$$A \propto \sum all\ pair\ of\ quarks\ A_{bi}A_{jb}^*$$

$$\begin{pmatrix} B^0 = d\bar{b} \\ \overline{B^0} = \bar{d}b \end{pmatrix}$$

$$\begin{pmatrix} B_S^0 = s\bar{b} \\ \overline{B_S^0} = \bar{d}s \end{pmatrix}$$



	$\boldsymbol{B^0 = d\bar{b}\ \ \overline{B^0} = \bar{d}b}$	$\boldsymbol{B_S^0 = s\bar{b}\ \ \overline{B_S^0} = \bar{d}s}$
Oscillations parameter	$x_d = \frac{\Delta m_d}{\overline{\Gamma_d}} \approx 0.72$	$x_s = \frac{\Delta m_s}{\overline{\Gamma_s}} \approx 24$
Large mass difference	$\Delta m_d \approx 3.3 \cdot 10^{-13}\ GeV$ $\approx 0.5\ ps^{-1}$	$\Delta m_s \approx 17.8\ ps^{-1}$
Small lifetime difference	$x_d = \frac{\Delta\Gamma_d}{\overline{\Gamma_d}} \approx 5 \cdot 10^{-3}$	$x_d = \frac{\Delta\Gamma_s}{\overline{\Gamma_s}} \approx 0.1$
$\frac{q}{p}$ - sensitivity to weak phase	$\frac{q}{p} = \frac{V_{td}V_{tb}^*}{V_{tb}V_{td}^*} \sim \beta$	$\frac{q}{p} = \frac{V_{ts}V_{tb}^*}{V_{tb}V_{ts}^*} \sim \beta_s$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^*}{M_{12}}}$$

Mixing of B^0 and B_S^0 meson

1. The weak B-meson states are a combination of flavour states:

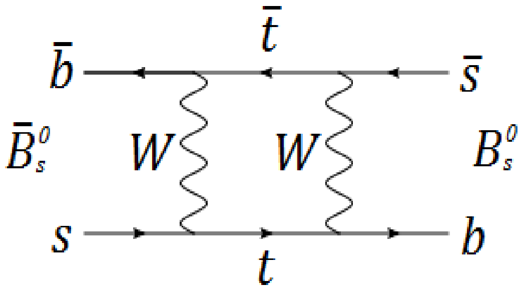
$$|B_L\rangle = p|B^0\rangle + q|\overline{B^0}\rangle \qquad |B_H\rangle = p|B^0\rangle - q|\overline{B^0}\rangle$$

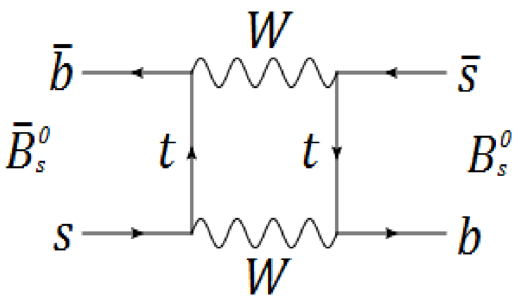
2. In terms of the CKM elements $\boldsymbol{q/p}$ is given by:

$$\frac{q}{p} = \frac{V_{td}V_{tb}^*}{V_{tb}V_{td}^*} = e^{-i2\beta}$$

here d is replaced by s in case of B_S^0

$$\frac{q}{p} = \frac{V_{ts}V_{tb}^*}{V_{tb}V_{ts}^*} = e^{-i2\beta_S}$$





so now the physical states are written as:

$$|B_L\rangle = 1/\sqrt{2}\left[|B^0\rangle + e^{-i2\beta}|\overline{B^0}\rangle\right]$$
$$|B_H\rangle = 1/\sqrt{2}\left[|B^0\rangle - e^{-i2\beta}|\overline{B^0}\rangle\right]$$

the eigenstates of the effective Hamiltonian, with definite mass and lifetime, are mixtures of the flavour eigenstates

and β is also called the $\boldsymbol{B^0}$ **mixing phase**

3. The states B_L and B_H are lighter and heavier state, with almost identical lifetimes: $\Gamma_L = \Gamma_H \equiv \Gamma$

4. The mass difference $\boldsymbol{\Delta m}$ between them is greater then in kaons.

Mixing of B^0 and B_S^0 meson

5. If we write the flavour states as a combination of weak states:

$$|B^0\rangle = 1/\sqrt{2}\,[|B_L\rangle + |B_H\rangle]$$

then the wavefunction evolves according to the time dependence of physical states:

$$|\boldsymbol{B(t)}\rangle = \mathbf{1/\sqrt{2}}\{\boldsymbol{a(t)}|\boldsymbol{B_L}\rangle + \boldsymbol{b(t)}|\boldsymbol{B_H}\rangle\}$$

where time dependence of coefficients is:

$$\boldsymbol{a(t)} = e^{-i(m_L - \frac{i}{2}\Gamma)t} \qquad \boldsymbol{b(t)} = e^{-i(m_H - \frac{i}{2}\Gamma)t}$$

Now substitute $a(t)$ and $b(t)$ and $|B_{L,H}\rangle$ into time-dependent wave function.

Do not forget to express mass states as a combination of flavour states….

$$|B_L\rangle = 1/\sqrt{2}\,[|B^0\rangle + e^{-i2\beta}|\overline{B^0}\rangle]$$
$$|B_H\rangle = 1/\sqrt{2}\,[|B^0\rangle - e^{-i2\beta}|\overline{B^0}\rangle]$$

Mixing of B^0 and B_S^0 meson

6. Now substitute $a(t)$ and $b(t)$ and $|B_{L,H}\rangle$ into time-dependent wave function:

$$|\boldsymbol{B(t)}\rangle = \boldsymbol{1/\sqrt{2}}\{\boldsymbol{a(t)}|\boldsymbol{B_L}\rangle + \boldsymbol{b(t)}|\boldsymbol{B_H}\rangle\}$$

$$|B_L\rangle = 1/\sqrt{2}\left[|B^0\rangle + e^{-i2\beta}|\overline{B^0}\rangle\right]$$

$$|B_H\rangle = 1/\sqrt{2}\left[|B^0\rangle - e^{-i2\beta}|\overline{B^0}\rangle\right]$$

$$\boldsymbol{a(t)} = e^{-i(m_L - \frac{i}{2}\Gamma)t}$$

$$\boldsymbol{b(t)} = e^{-i(m_H - \frac{i}{2}\Gamma)t}$$

…. and calculate the probabilities of the state to stay as a $|B^0\rangle$

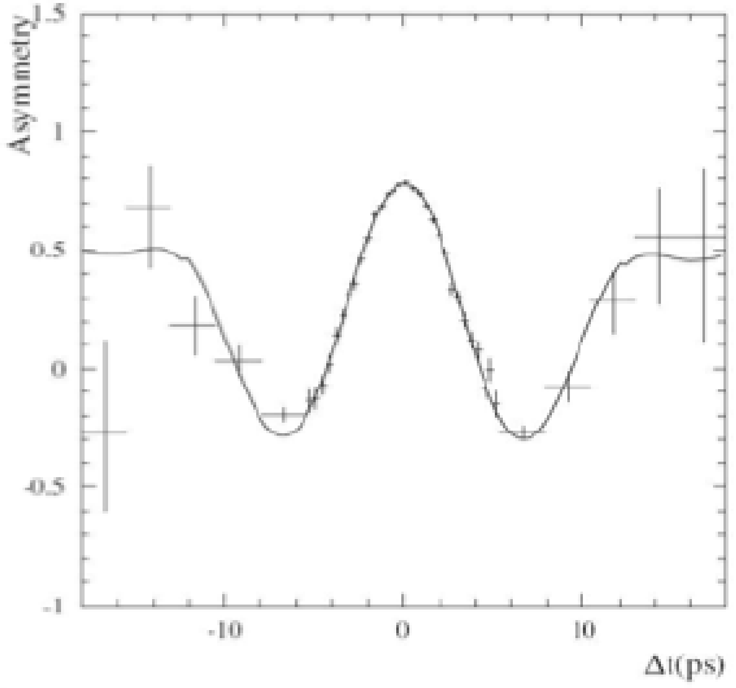
$$P(B^0(t=0) \to B^0; \mathrm{t}) = |\langle B^0(t)|B^0\rangle|^2 = .. = .. = e^{-\Gamma t} cos^2\left(\frac{\Delta m}{2}t\right)$$

7. The same calculation can be done for B_S^0

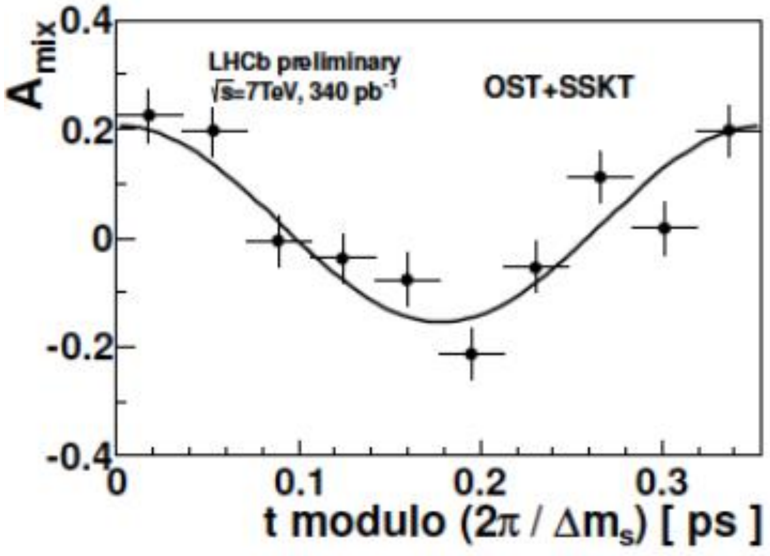
try to do it!

Mixing of B^0 and B_S^0 meson

BaBar: $\Delta m = 0.511 \pm 0.007\ ps^{-1}$



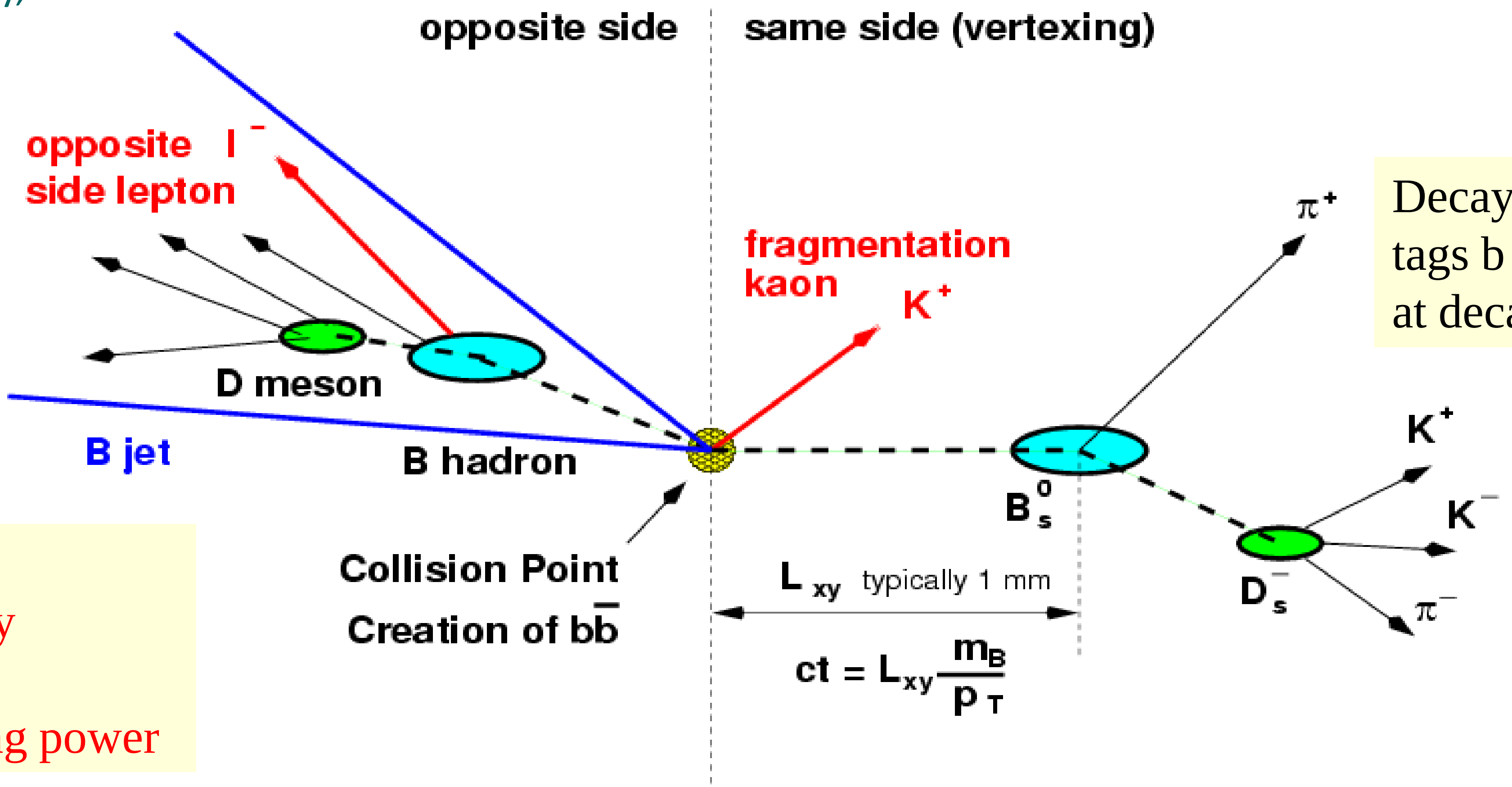
LHCb: $\Delta m_S = 17.768 \pm 0.023\ ps^{-1}$



$B_S^o \rightarrow D_S^- \pi^+$

Experimental challenges for mixing

1. Need to determine:
   a) Flavour at production ⇔ **tagging**
   b) Flavour at decay, from final state
   c) B decay length

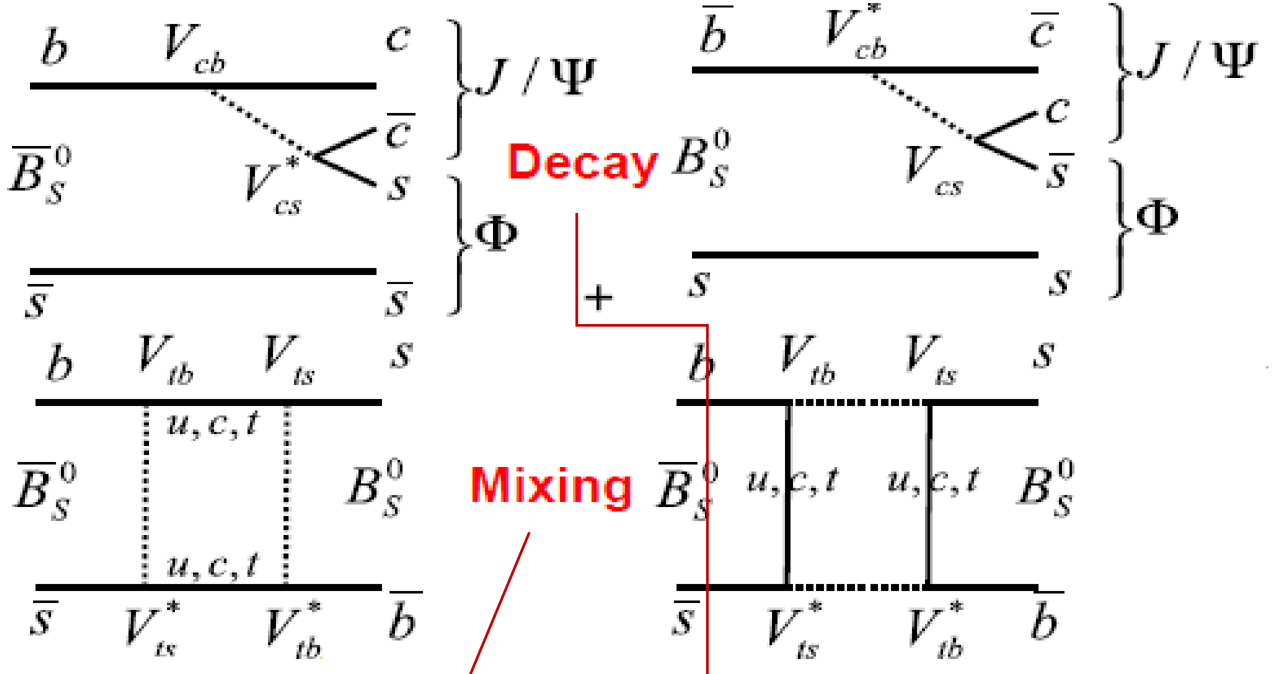


Decay mode tags b flavor at decay

Dilution $D = 1 - 2w$
w = mistag probability
ε = efficiency
εD^2 = effective tagging power

Weak phase ϕ_S

The weak phase ϕ_S can be extracted from tagged Bs decays to CP eigenstates: $B_S \to J/\psi\phi$



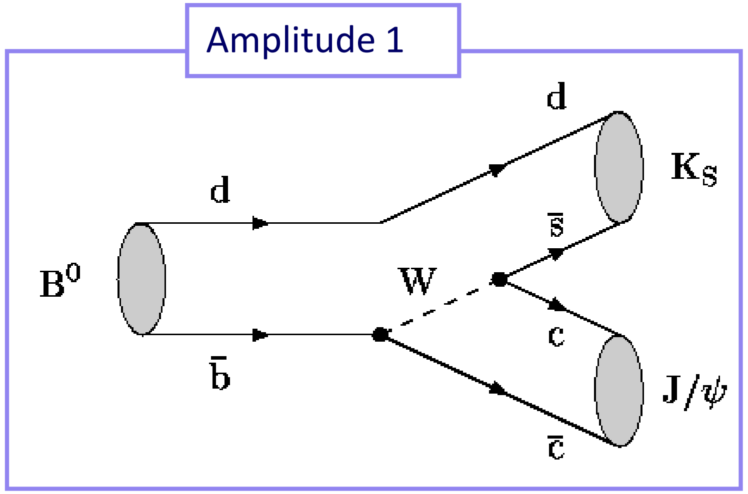
$$\lambda = \frac{q}{p}\frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}} = \left(\frac{V_{ts}V_{tb}^*}{V_{tb}V_{ts}^*}\right)\left(\frac{V_{cb}V_{cs}^*}{V_{cs}V_{cb}^*}\right) = \frac{|V_{ts}|e^{i\beta_s}}{|V_{ts}|e^{-i\beta_s}} = e^{2i\beta_s} = e^{i\phi_s}$$

$$\phi_s = -0.036 \pm 0.002$$

Very small value of ϕ_s is predicted in SM. So any deviation from zero is a sign of new particle exchanged – Physics Beyond the Standard Model

Golden channel for sin 2β

1. The process $B^0 \to J/\psi\ K_S$ is called the „golden mode" for measurement of the β angle:
   a) clean theoretical description,
   b) clean experimental signature,
   c) large (for a B meson) branching fraction of order ~10^{-4}.
2. This is a process with interference of amplitudes with and without mixing:



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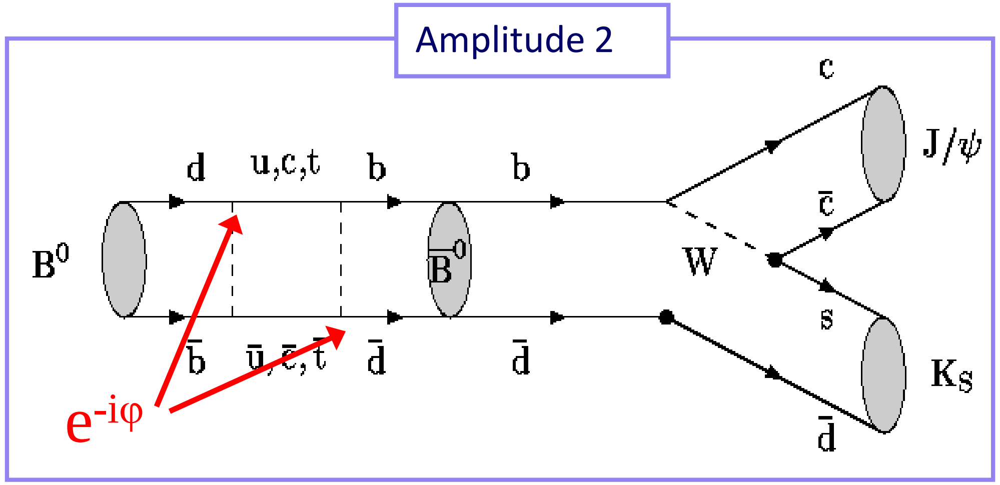


(φ=2β)

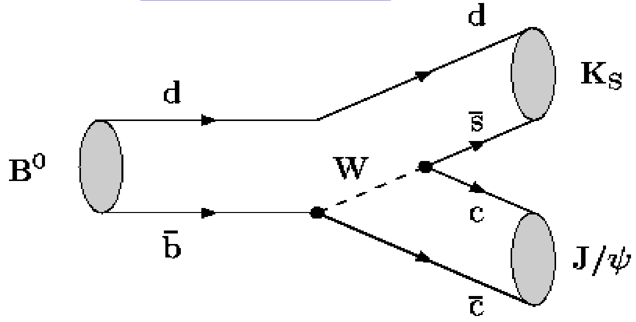
3. The β angle sensitivity comes from the $B^0 \leftrightarrow \overline{B^0}$ mixing due to the $\bar{t} \to \bar{d}$ and $t \to d$ transitions.

Golden channel for $\sin 2\beta$

4. We need to calculate the asymmetry of the type: $$A_{CP}(t) = \frac{\Gamma_f - \overline{\Gamma_f}}{\Gamma_f + \overline{\Gamma_f}}$$

and remember that decay rate depends on (see lect 4): $\Gamma(B \to f) \propto |A_f|^2 = |A_1 + A_2|^2$

Amplitude 1



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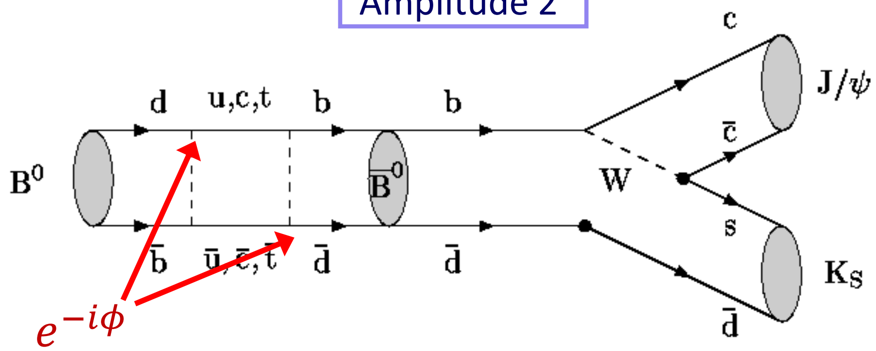
Amplitude 2



$\phi = 2\beta$

$$\Gamma(B \to J/\psi\ K_S) = \left| A e^{-imt-\Gamma t} \left(\cos\frac{\Delta m t}{2} + e^{-i\phi} \sin\frac{\Delta m t}{2} \right) \right|^2$$

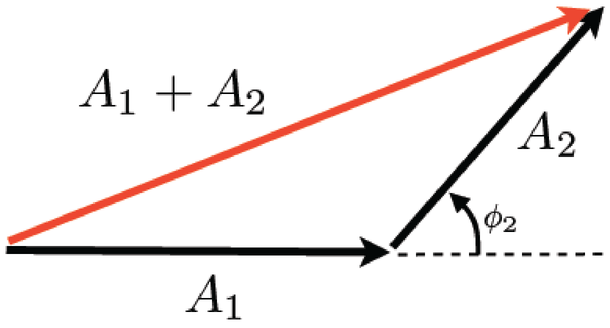
$$A_{CP}(t) = \frac{\Gamma\{B \to J/\psi\ K_S\} - \Gamma\{\bar{B} \to J/\psi\ K_S\}}{\Gamma\{B \to J/\psi\ K_S\} + \Gamma\{\bar{B} \to J/\psi\ K_S\}} = - \sin 2\boldsymbol{\beta} \sin \Delta m t$$

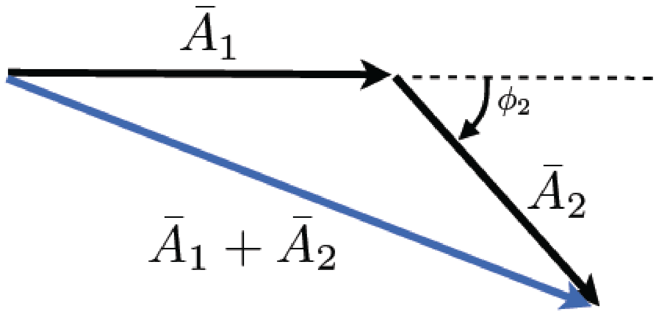
Essence of amplitude interference

$$A_j = \langle \mathrm{final}|H_j|\mathrm{initial}\rangle = |A_j| e^{+i\phi_j^{\mathrm{weak}}}$$

CP

$$\bar{A}_j = A_j^* = |A_j| e^{-i\phi_j^{\mathrm{weak}}}$$





$$P(i \to f) = |A_1 + A_2|^2$$

$$P(\bar{i} \to \bar{f}) = |\bar{A}_1 + \bar{A}_2|^2$$

$$= |A_1|^2 + 2|A_1||A_2|\cos\phi_2 + |A_2|^2$$

In case of only one decay amplitude – the decay rates are equal:

$$\Gamma(P \to f) = \Gamma(\bar{P} \to \bar{f})$$

and no CP violation occurs.

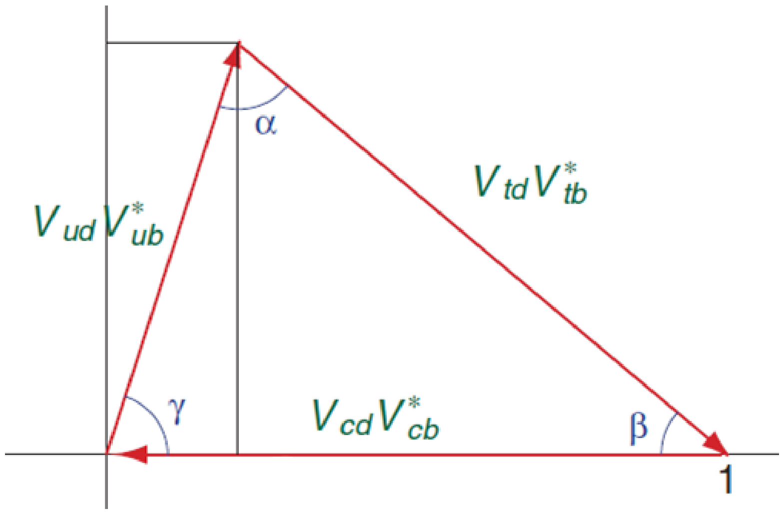
For two amplitudes the decay rates may differ and the asymmetry is sensitive to relative phase

$$A = \frac{|\bar{A}_f|^2 - |A_f|^2}{|\bar{A}_f|^2 + |A_f|^2}$$

Measurement of CKM γ angle

1. The CKM γ angle can be measured through plenty of processes:
   a) time integrated decays (GLW or ADS method)
   b) time dependent CP asymmetries in transition $b \to c\bar{u}d(s)$
2. We consider B decays of a type $B \to DK$ with different charges and B flavours:

$$B_q \to D_q h_q$$



$$B^0 \to D^+K^-$$

$$B^+ \to D^*K^+$$

$$\boldsymbol{B_s^0 \to D_s^- K}$$

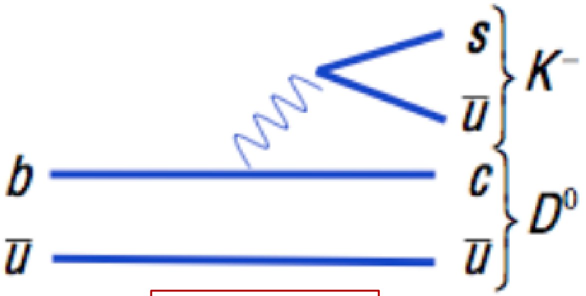
$$\boldsymbol{B_s^0 \to D_s^- K^{*+}}$$

$$\boldsymbol{B_s^0 \to D_s^{*-} K^{*+}}$$

Time integrated method

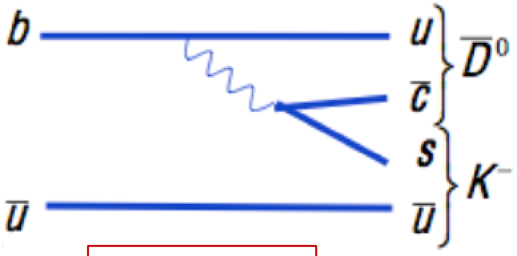
This is a measurement of angle γ with the processes $B^{\pm} \to D^0 K^{\pm}$. Plenty of methods which differ by the final states

Interference between two diagrams:



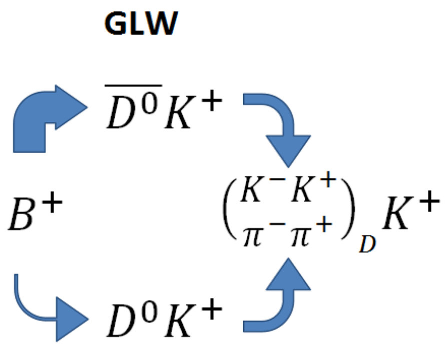
$\propto V_{cb}V_{us}^*$

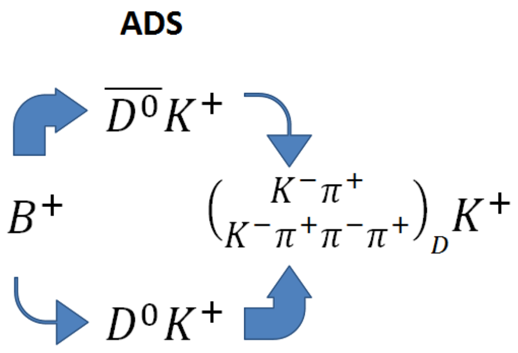
colour allowed



$\propto V_{ub}V_{cs}^*$

colour suppressed



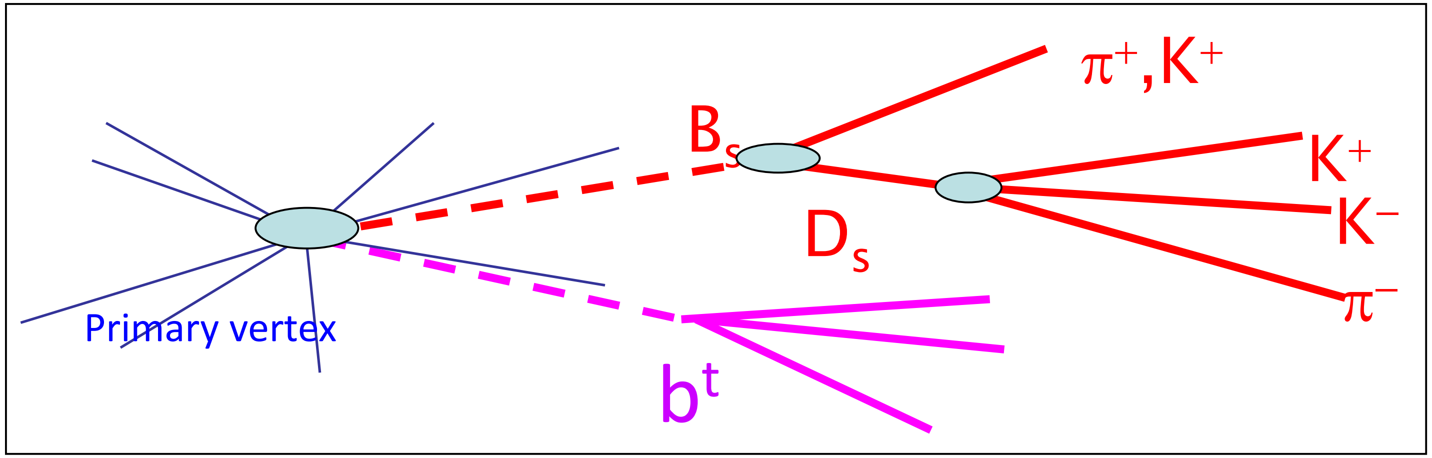


$$A_{CP} = \frac{\Gamma\{\boldsymbol{B}^- \to D^0K^-\} - \Gamma\{\boldsymbol{B}^+ \to D^0K^+\}}{\Gamma\{\boldsymbol{B}^- \to D^0K^-\} + \Gamma\{\boldsymbol{B}^+ \to D^0K^+\}} \propto \sin\gamma$$

Time dependent $B_s^0 \to D_s^- K$

$B_s^0 \to D_s^- K$

This family of processes are very experimentally challenging:

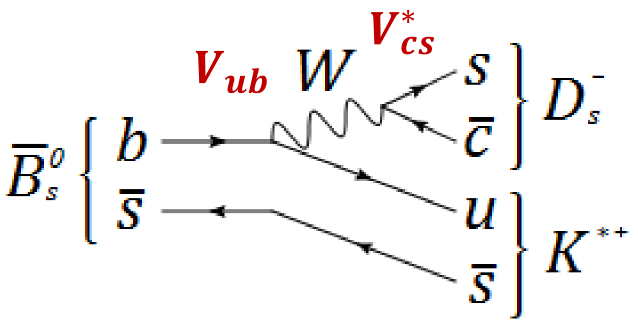


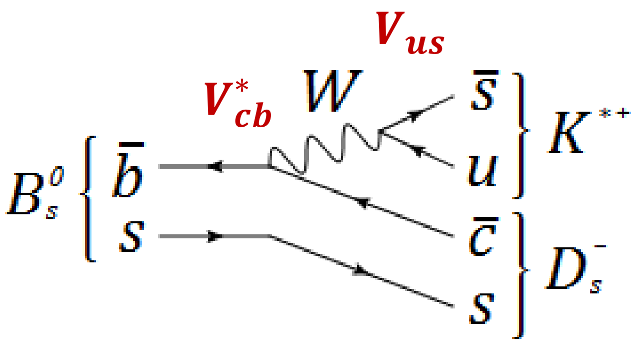
- six hadrons in the final state – very good PID and mass resolution
- high-P_T tracks and displaced vertices - *efficient trigger*
- *efficient tagging and good tagging power (small mistag rate)*
- *good decay-time resolution*

Time dependent $B_s^0 \to D_s^- K$

1. B_s^0 and $\overline{B_s^0}$ decay to the same final state.

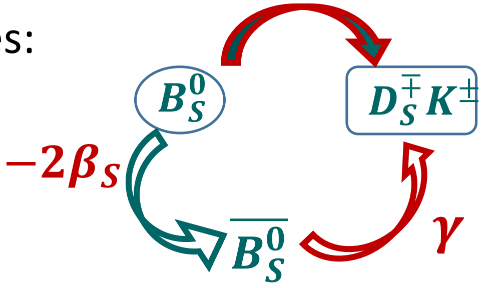
$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$





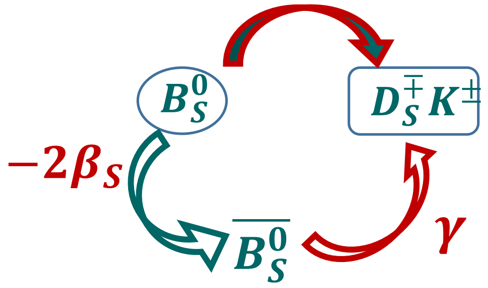
2. B_s^0 and $\overline{B_s^0}$ can oscillate into one another.

3. So we have interference between two processes:



Time dependent $B_s^0 \to D_s^- K$

We have some experience in decay rate equation…
The probability of B meson decay to final state f is given by the Fermi golden rule:



$$\Gamma_{B_s^0 \to f}(t) \sim |\langle f|T|B_s^0(t)\rangle|^2$$

and we can try to calculate it…

$$\Gamma_{B_s^0 \to f}(t) = |A_f|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma_s t}}{2} \cdot \left(\cosh \frac{\Delta\Gamma_s t}{2} + D_f \sinh \frac{\Delta\Gamma_s t}{2} + C_f \cos \Delta m_s t - S_f \sin \Delta m_s t\right)$$

$$\Gamma_{\bar{B}_s^0 \to f}(t) = |A_f|^2 \left|\frac{p}{q}\right|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma_s t}}{2} \cdot \left(\cosh \frac{\Delta\Gamma_s t}{2} + D_f \sinh \frac{\Delta\Gamma_s t}{2} - C_f \cos \Delta m_s t + S_f \sin \Delta m_s t\right)$$

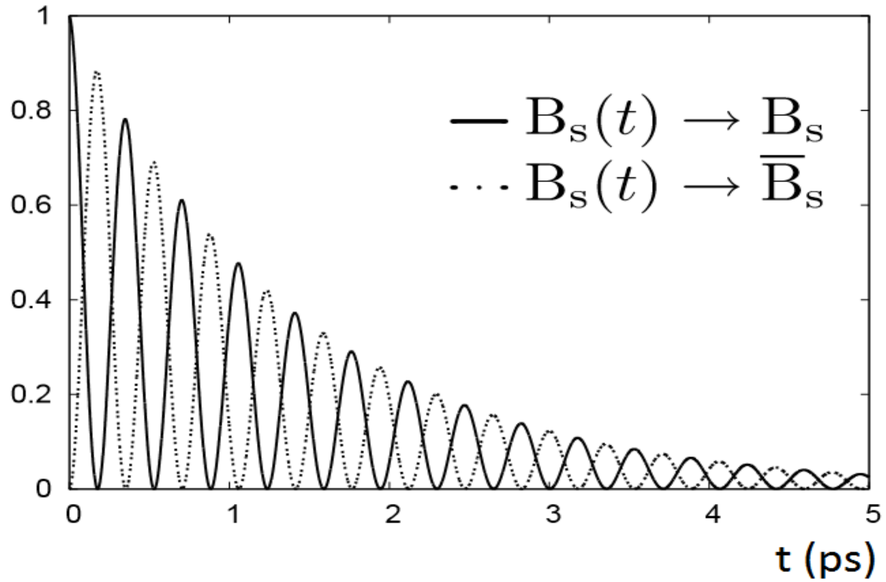
$$D_f = \frac{2Re\lambda_f}{1 + |\lambda_f|^2} \qquad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \qquad S_f = \frac{2Im\lambda_f}{1 + |\lambda_f|^2}$$

$$\lambda_f \equiv \frac{1}{\bar{\lambda}_f} = \frac{q}{p}\frac{\bar{A}_f}{A_f} \qquad A_f = \langle f|T|B_s^0\rangle \qquad \bar{A}_{\bar{f}} = \langle \bar{f}|T|\bar{B}_s^0\rangle$$

good luck!

Time dependent $B_s^0 \to D_s^- K$

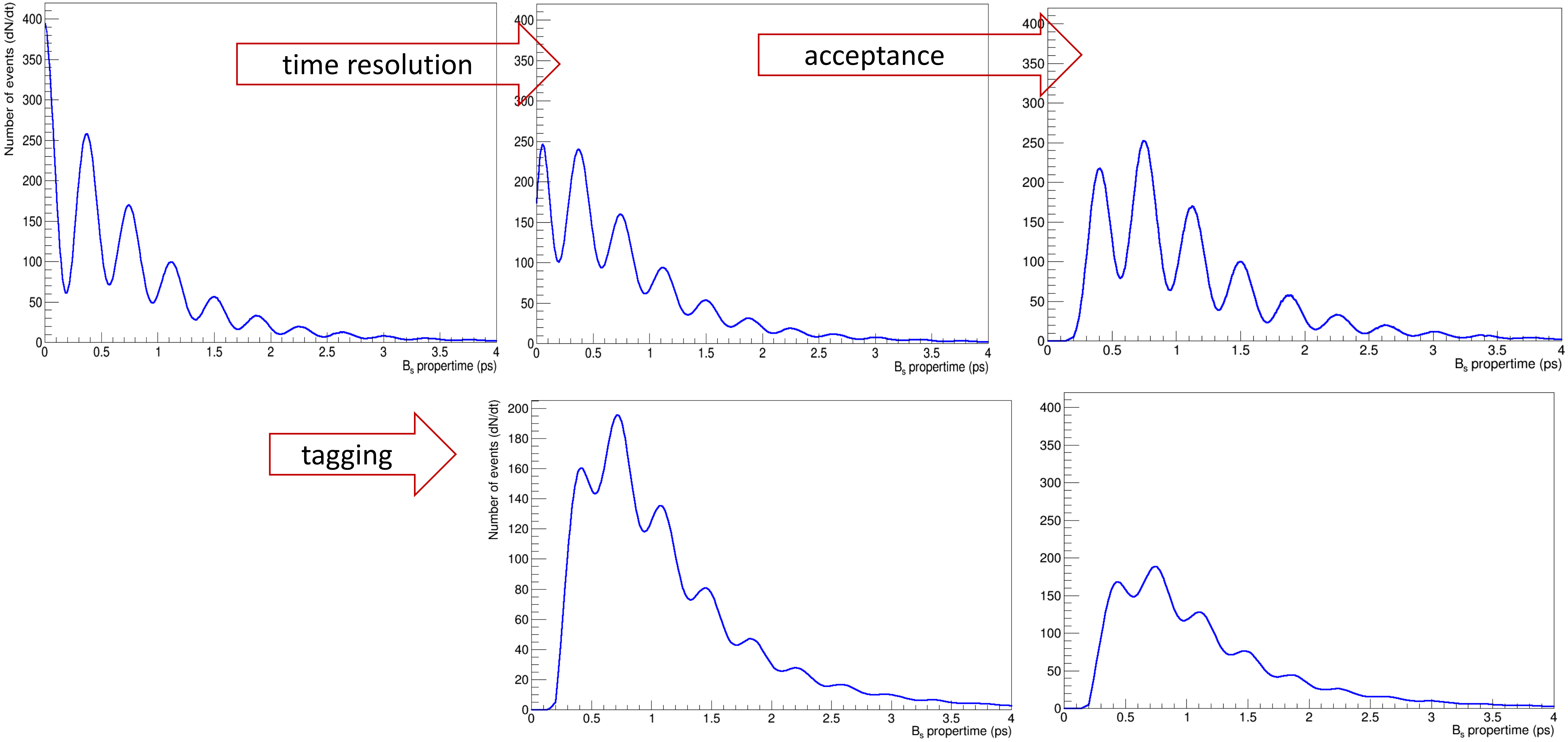
These relations should lead to the distribution like this:



$$A_{CP}(t) = \frac{\Gamma\{B(t) \to f\} - \Gamma\{\overline{B}(t) \to \bar{f}\}}{\Gamma\{B(t) \to f\} + \Gamma\{\overline{B}(t) \to \bar{f}\}}$$

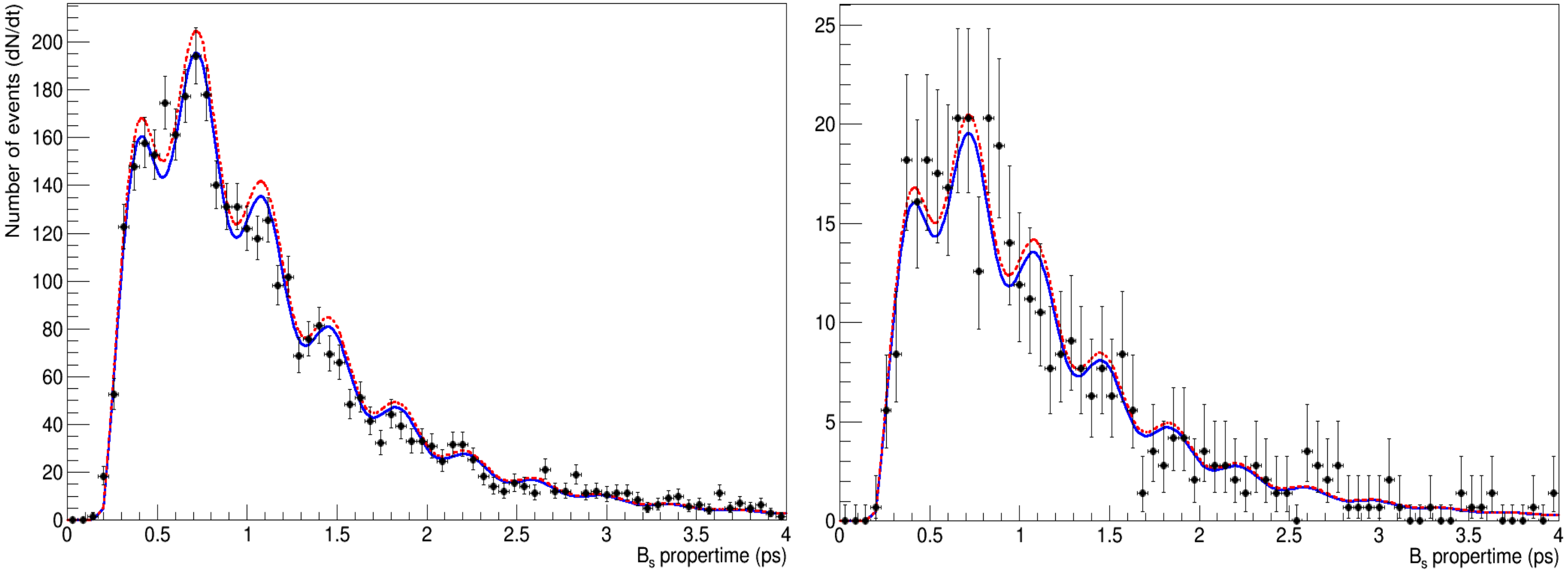
… but various detector effects have a major impact on time dependent decay rates:

Time dependent $B_s^0 \to D_s^- K$ detector effects

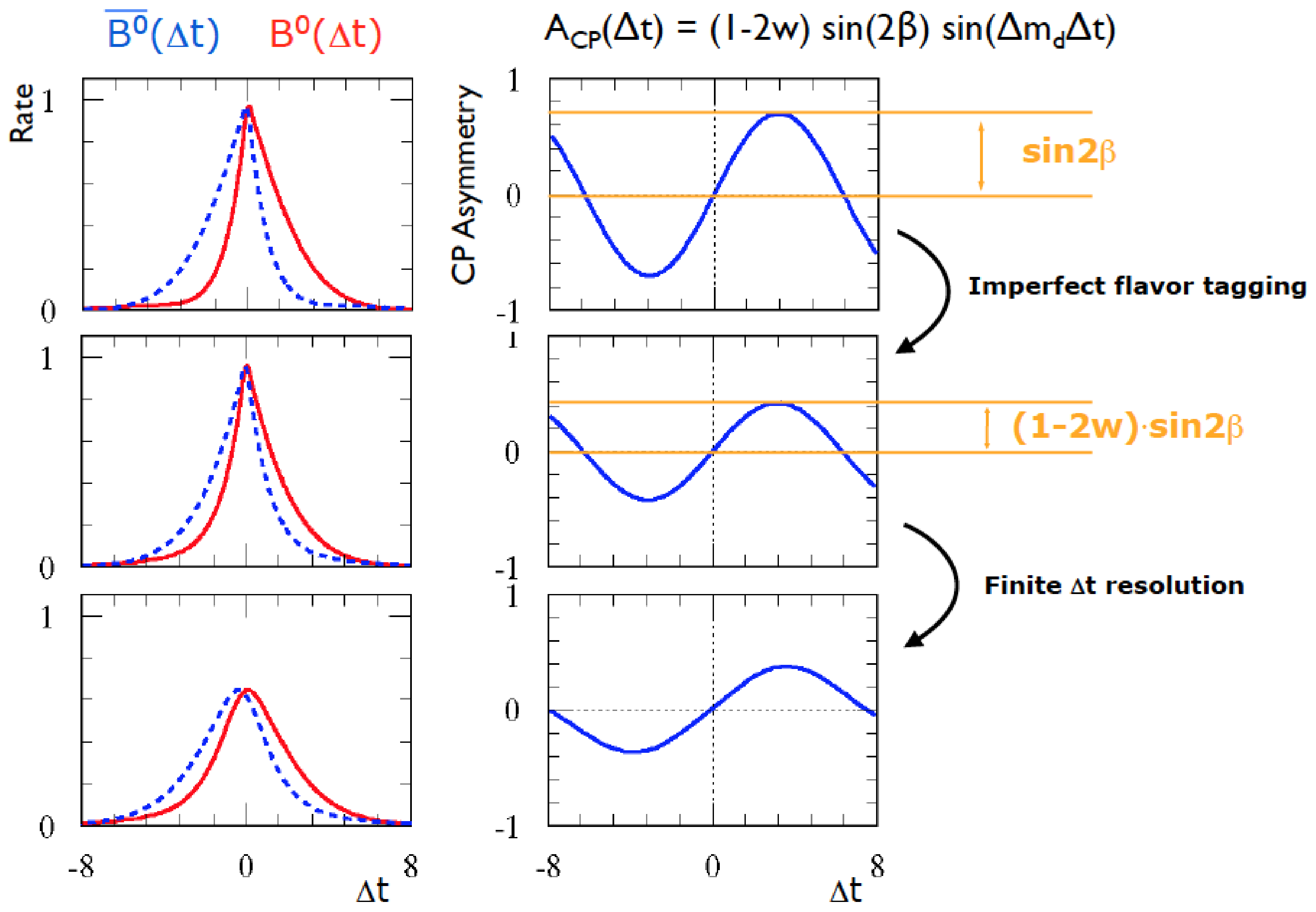


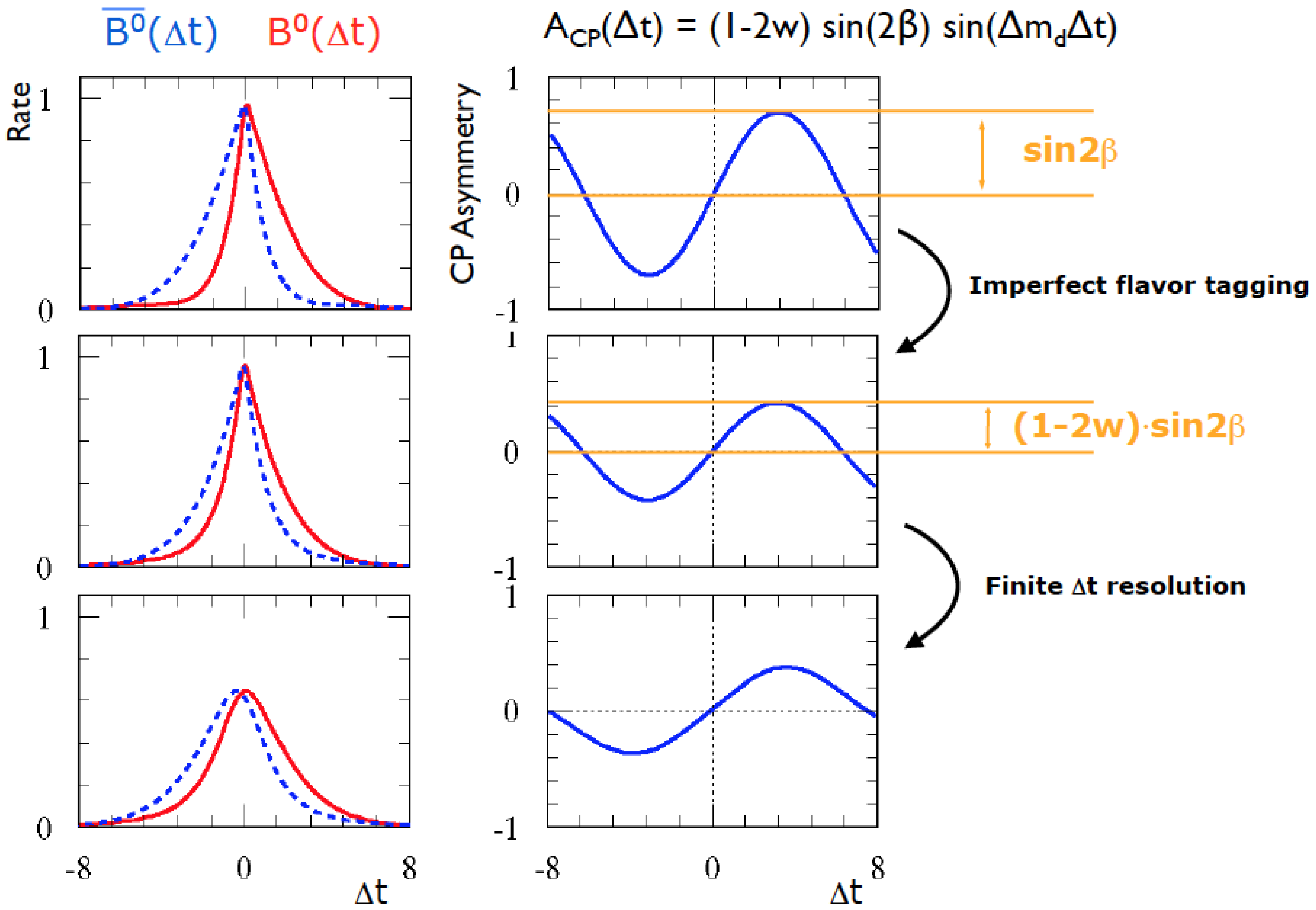
Time dependent $B_s^0 \to D_s^- K$ detector effects

Roofit simulation of 10 years of LHCb data taking for this process….



Mistag & dilution



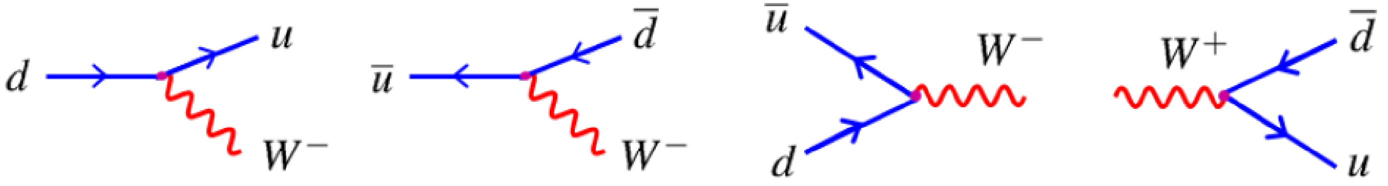


$\overline{B^0}(\Delta t)$ $B^0(\Delta t)$
$A_{CP}(\Delta t) = (1-2w)\sin(2\beta)\sin(\Delta m_d \Delta t)$
Rate
CP Asymmetry
sin2β
Imperfect flavor tagging
(1-2w)·sin2β
Finite Δt resolution
-8
0
8
Δt

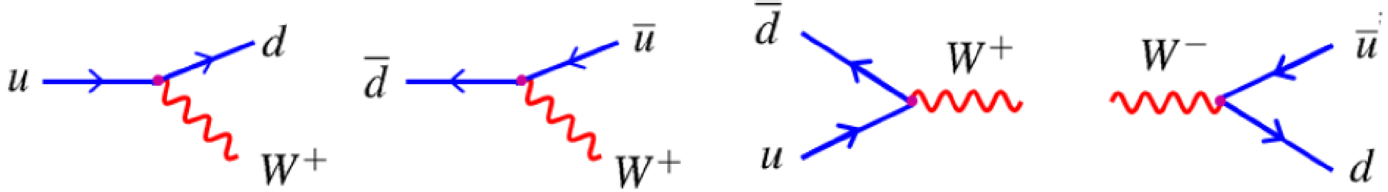
One practical remark

When we write the equation for amplitudes and matrix elements we use standard Feynman rules:

- V_{ud} when d-quark is incoming or anti-d is outgoing,
- V_{ud}^{*} if incoming u quark or outgoing anti-u quark



$$\Lambda = \left[-i\frac{g_W}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right] V_{ud}$$



$$\Lambda' = V_{ud}^{*}\left[-i\frac{g_W}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right]$$