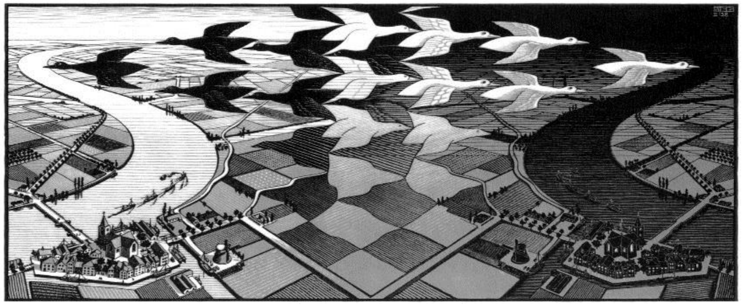

CP-Violation in Heavy Flavour Physics
Lecture 2 (part II)

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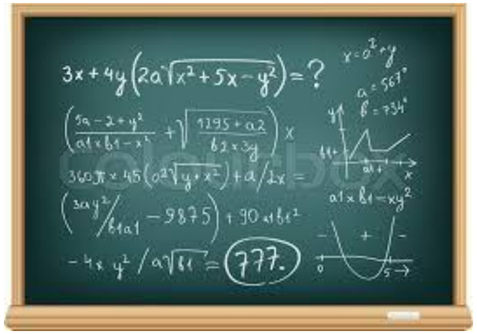
Violation of CP symmetry

- Last time we learned that **two** of the fundamental symmetries (space and charge parities) are **maximally broken** by the **weak** interactions
- Also, there is a fascinating phenomena, occurring for neutral mesons that we called **flavour oscillations**
- By analysing various processes we came to conclusion that although both $\mathcal{C}$ and $\mathcal{P}$ are broken the **combined symmetry** ,i.e., CP is **exact**
- Ah, yes… there was also this bizarre effect about kaons – we decided that the particles that are **produced** in strong interactions are not the same that **decay** later on via weak force
- A lot of new stuff! And today we are shifting to higher gear!



Violation of CP symmetry

- From the last lecture – K_1^0 and K_2^0 are eigenstates of CP operator

$$\mathcal{CP}|K_1^0\rangle = \frac{1}{\sqrt{2}}(\mathcal{CP}|K^0\rangle - \mathcal{CP}|\overline{K}^0\rangle) = \frac{1}{\sqrt{2}}(-|\overline{K}^0\rangle + |K^0\rangle) = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) = |K_1^0\rangle$$

- In other words, if combined parity is conserved, processes such these below should never happen!

$$K_2^0 \not\to \pi^0 + \pi^0$$

$$K_2^0 \not\to \pi^+ + \pi^-$$

- We should not be surprised by the fact that they **indeed happen**! An experiment has been performed to study the behaviour of the long-lived component of K^0, which **found them**!
- So, we are for another redefinition of what the kaons really are…
- Because we see clearly that CP **is broken**, thus, we must accept that neutral kaons **are not composed** out of K_1^0 and K_2^0
- The new states are called $\boldsymbol{K_S^0}$ and $\boldsymbol{K_L^0}$ instead…

Violation of CP symmetry

- This may come as yet another surprise, but the effect is **very weak**, the fractional branching ratios measured are of order of 0.1%

$$\frac{K_L^0 \to \pi^+ + \pi^-}{K_L^0 \to anything} \approx 2.0 \times 10^{-3} \qquad \frac{K_L^0 \to \pi^0 + \pi^0}{K_L^0 \to anything} \approx 9.0 \times 10^{-4}$$

- Taking into account life-times of both K_S^0 and K_L^0 one can show that

$$\frac{\Gamma(K_L^0 \to 2\pi)}{\Gamma(K_S^0 \to 2\pi)} \approx 10^{-6}$$

- Ok – a small resume…
  - $\boldsymbol{CP}$ is indeed **violated**
  - The effect is **tiny** (not so tiny for beauty decays though…)
  - Matter and anti-matter are not **symmetrical**
  - CP , apart from small number of weak processes involving neutral mesons, is conserved

Kaons revisited

- What we did was an attempt to describe time evolution of kaons which are produced as **strong** Hamiltonian **e-states** that, in turn, decay as weak e-states: $(K^0, \overline{K}^0) \rightarrow (K_1^0, K_2^0)$
- This fails because K_1^0 and K_2^0 **are e-states of $\boldsymbol{CP}$**, so we need new particles, namely K_S^0 and K_L^0 that have the necessary behavior of K_1^0 and K_2^0 (i.e., long and short life-time) but are not CP e-states
- One remark – since the violation effect is small – this would be a hint that these new states are almost identical to K_1^0 and K_2^0

$$|K_S^0\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left((1+\epsilon)|K^0\rangle - (1-\epsilon)|\overline{K}^0\rangle\right) =$$

$$= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left((|K^0\rangle - |\overline{K}^0\rangle) + \epsilon(|K^0\rangle + |\overline{K}^0\rangle)\right) = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_1^0\rangle + \epsilon|K_2^0\rangle\right)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left((1+\epsilon)|K^0\rangle + (1-\epsilon)|\overline{K}^0\rangle\right) =$$

$$= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left((|K^0\rangle + |\overline{K}^0\rangle) + \epsilon(|K^0\rangle - |\overline{K}^0\rangle)\right) = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_2^0\rangle + \epsilon|K_1^0\rangle\right)$$

Kaons Revisited

- How should we thing about what is going on…, so,…
- The transitions $\langle 2\pi|H_w|K_L^0\rangle$ violate CP invariance. This can happen:
  - because the e-states of the weak Hamiltonian, K_S^0 and K_L^0, are not e-states of the CP operator
  - we say, that the physical states are **mixtures** of **CP-even** and **CP-odd** components
  - In other words – we observe small violation of CP in $K_L^0 \to 2\pi$ decays, because of small admixture of K_1^0
  - this type of violation is called **indirect**, and implies, that the Hamiltonian itself is even under CP symmetry
  - again… to add confusion, it turns out that the **direct** violation is also possible for kaons (i.e., violation induced via the weak Hamiltonian) $\langle 2\pi|H_w|K_2^0\rangle \neq 0$
  - But that is another story…

Measure of CP-violation

- How can we express the degree of CP-violation?

$$|K_S^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_1^0\rangle + \epsilon|K_2^0\rangle\right) \qquad |K_L^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_2^0\rangle + \epsilon|K_1^0\rangle\right)$$

ϵ represents deviation of K_S^0 and K_L^0 from true CP e-states (in general this is complex number!)

$$\mathcal{CP}|K_S^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(\mathcal{CP}|K_1^0\rangle + \epsilon\mathcal{CP}|K_2^0\rangle\right) = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(|K_1^0\rangle - \epsilon|K_2^0\rangle\right) \neq |K_S^0\rangle$$

$$\mathcal{CP}|K_L^0\rangle = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(\mathcal{CP}|K_2^0\rangle + \epsilon\mathcal{CP}|K_1^0\rangle\right) = \frac{1}{\sqrt{(1+|\epsilon|^2)}}\left(-|K_1^0\rangle + \epsilon|K_2^0\rangle\right) \neq -|K_L^0\rangle$$

$$\langle K_L^0|K_S^0\rangle = \frac{1}{1+|\epsilon|^2}\left(\langle K_2^0| + \epsilon^*\langle K_1^0|\right)\left(|K_1^0\rangle + \epsilon|K_2^0\rangle\right) =$$

$$\frac{1}{1+|\epsilon|^2}\left(\epsilon\langle K_2^0|K_2^0\rangle + \epsilon^*\langle K_1^0|K_1^0\rangle\right) = \frac{\epsilon+\epsilon^*}{1+|\epsilon|^2} = \frac{2Re(\epsilon)}{1+|\epsilon|^2} = \langle K_S^0|K_L^0\rangle$$

K_S^0 and K_L^0 are not orthogonal states!

Measure of CP-violation

- Lack of orthogonality of K_S^0 and K_L^0 is expected – both of them **share** the same decay channels
- This effect is at the same time a **measure** of CP-violation via $\boldsymbol{\epsilon}$
- In this picture the symmetry violation is a consequence of small admixture of K_1^0 state into the K_L^0, so, we observe its decays to 2π final state because the K_1^0 can decay into it – once again this is **indirect process**
- These kind of processes are referred to as $\boldsymbol{\Delta S = 2, \Delta I = \frac{1}{2}}$ transitions
- Much smaller direct contribution to CP-violation is a consequence of the **weak Hamiltonian having a $\boldsymbol{CP}$-violating term** (it does not commute with the CP operator)
- These kind of processes proceed via $\boldsymbol{\Delta S = 1, \Delta I = \frac{3}{2}}$ transitions and are called **penguin** (or loop) decays

Time Evolution of the Kaon System

- A phenomenological „**effective**" theoretical framework has been introduced to describe what is going on with kaons produced in strong interactions
- It is based on perturbation theory and describe the behavior of such system in terms of an effective Hamiltonian
- We start with describing kaons in the absence of weak interactions
  - In this case K^0 and $\overline{K}^0$ are distinct e-states of the strong Hamiltonian
  - Since the strong interactioins respect conservation of strangeness these are **stationary states**!

$$|K^0\rangle = \binom{1}{0} \qquad |\overline{K}^0\rangle = \binom{0}{1}$$

Base vectors in 2-dim Hilbert space

$$|\psi\rangle = \frac{1}{\sqrt{(a^2+b^2)}}\left(a|K^0\rangle + b|\overline{K}^0\rangle\right) = \frac{1}{\sqrt{(a^2+b^2)}}\binom{a}{b}$$

Time Evolution of the Kaon System

- Oh, well, unfortunately weak interaction cannot be switched off and the kaons **do decay**
- Theory offers two approaches to attack this problem
  - We could expand the 2-dim Hilbert space and take into account all the possible final states
  - or…, we could stay in the 2-dim space and introduce **effective Hamiltonian** that is responsible for the kaons disintegration
  - Usually the later option is picked up!
- Now, the leap to the Schrodinger equation describing **two state** system with the effective Hamiltonian is done by noticing that we no longer deal with **stationary** states – they can **decay**
- The consequence is that the Hamiltonian is no longer a **Hermitian operator** – the probability is no longer conserved for decaying states!

Effective Hamiltonian

$$i\hbar \frac{\partial|\psi(t)\rangle}{\partial t} = \mathcal{H}_{eff}|\psi(t)\rangle$$

Complex 2×2 matrix

$$\mathcal{H}_{eff} = \mathcal{M} - \frac{i}{2}\Gamma$$

$$\mathcal{M} = \frac{1}{2}\left(\mathcal{H}_{eff} + \mathcal{H}_{eff}^{\dagger}\right), \Gamma = i\left(\mathcal{H}_{eff} - \mathcal{H}_{eff}^{\dagger}\right)$$

$$\mathcal{M} = \mathcal{M}^{\dagger} \;\rightarrow\; \mathcal{M}_{ij} = \mathcal{M}_{ji}^{*}$$

Both Hermitian

$$\Gamma = \Gamma^{\dagger} \;\rightarrow \Gamma_{ij} = \Gamma_{ji}^{*}$$

- **Mass matrix** – its e-values represents masses of the states in their CM frame (real parts of the energy levels)
- **Decay matrix** – introduced to describe decay characteristics of the system

Effective Hamiltonian

- The main purpose here is to provide explicit form of the $\mathcal{H}_{eff}$, and one can start from writing down the $\mathcal{H}_{eff}$ matrix in the most generic form

$$\mathcal{H}_{eff} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\langle K^0 | \mathcal{H}_{eff} | K^0 \rangle = A \qquad \langle \bar{K}^0 | \mathcal{H}_{eff} | \bar{K}^0 \rangle = D = A$$

$\mathcal{CPT}$ theorem states that the masses of K^0 and $\bar{K}^0$ must be the same

$$\mathcal{H}_{eff} = \begin{pmatrix} A & B \\ C & A \end{pmatrix}$$

The most generic form of the $\mathcal{H}_{eff}$ consistent with $\mathcal{CPT}$ theorem

- Next, let's express the e-states of the **effective** Hamiltonian in terms of our base states of the strong interactions

Effective Hamiltonian and its e-states

$$|K_S^0\rangle = \frac{1}{\sqrt{(|p|^2 + |q|^2)}}(p|K^0\rangle + q|\overline{K}^0\rangle) = \frac{1}{\sqrt{(|p|^2 + |q|^2)}}\binom{p}{q}$$

$$|K_L^0\rangle = \frac{1}{\sqrt{(|r|^2 + |s|^2)}}(r|K^0\rangle + s|\overline{K}^0\rangle) = \frac{1}{\sqrt{(|r|^2 + |s|^2)}}\binom{r}{s}$$

p, q, r, s are complex numbers defining
the decomposition of K_S^0 and K_L^0

- e-states of the effective Hamiltonian, K_S^0 and K_L^0, have e-values in their CM frame as follow:

masses of the e-states

$$m_S - \frac{i}{2}\gamma_S, \; m_L - \frac{i}{2}\gamma_L$$

widths of the e-states

$$\mathcal{H}_{eff}\left|K_S^0\right\rangle = \left(m_S - \frac{i}{2}\gamma_S\right)\left|K_S^0\right\rangle \quad (*)$$

$$\mathcal{H}_{eff}|K_L^0\rangle = \left(m_L - \frac{i}{2}\gamma_L\right)|K_L^0\rangle$$

Effective Hamiltonian and its e-states

- Now, in the basis of K_S^0 and K_L^0 e-states, the diagonal elements of the effective Hamiltonian are as above
- We can relate them to the diagonal elements of the same operator expressed in the K^0 and $\overline{K}^0$ basis using the **trace of matrix** (trace is invariant w.r.t. base transformations)

$$Tr(\mathcal{H}_{eff}) = 2A = \left(m_S - \frac{i}{2}\gamma_S\right) + \left(m_L - \frac{i}{2}\gamma_L\right)$$

$$A = \frac{1}{2}(m_S + m_L) - \frac{i}{4}(\gamma_S + \gamma_L)$$

- Now, rewrite the equation (*)

$$\begin{pmatrix} A & B \\ C & A \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \left(m_S - \frac{i}{2}\gamma_S\right)\begin{pmatrix} p \\ q \end{pmatrix}$$

$$\begin{pmatrix} A - m_S + \frac{i}{2}\gamma_S & B \\ C & A - m_S + \frac{i}{2}\gamma_S \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = 0$$

A system of coupled linear homogenous equations!

Effective Hamiltonian and its e-states

- Non trivial solution exists only if:

$$det\begin{pmatrix} A - m_S + \frac{i}{2}\gamma_S & B \\ C & A - m_S + \frac{i}{2}\gamma_S \end{pmatrix} = 0$$

$$BC = \left(A - m_S + \frac{i}{2}\gamma_S\right)^2 = \left[\frac{1}{2}(m_S + m_L) - \frac{i}{4}(\gamma_S + \gamma_L)\right]^2$$

$$\pm\sqrt{BC} = \frac{1}{2}(m_S + m_L) - \frac{i}{4}(\gamma_S + \gamma_L)$$

- Substituting to equations describing short and long states respectively one can get:

$$\boxed{\frac{p}{q} = \pm\sqrt{\frac{B}{C}}, \frac{r}{s} = \mp\sqrt{\frac{B}{C}} = -\frac{p}{q}}$$

$$\boldsymbol{r = p, s = -q}$$

Effective Hamiltonian and its e-states

- So, the e-states of the effective Hamiltonian are:

$$|K_S^0\rangle = \frac{1}{\sqrt{(|p|^2 + |q|^2)}}(p|K^0\rangle + q|\overline{K}^0\rangle)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{(|p|^2 + |q|^2)}}(p|K^0\rangle - q|\overline{K}^0\rangle)$$

- We can now express the parameters p, q in terms of ϵ

$$\mathrm{p} = 1 + \epsilon, q = -(1 - \epsilon)$$

- And the strong e-states can be written as:

$$|K^0\rangle = \frac{\sqrt{(|p|^2 + |q|^2)}}{2p}\left(|K_S^0\rangle + |K_L^0\rangle\right)$$

$$|\overline{K}^0\rangle = \frac{\sqrt{(|p|^2 + |q|^2)}}{2q}\left(|K_S^0\rangle - |K_L^0\rangle\right)$$

Effective Hamiltonian and its e-states

- K_S^0 and K_L^0 are the e-states of the $\mathcal{H}_{eff}$, thus, the solutions of our Schrodinger equation are

$$\boxed{|K_S^0(t)\rangle = e^{-\frac{i}{\hbar}\left(m_S-\frac{i}{2}\gamma_S\right)t}|K_S^0\rangle}$$

$$\boxed{|K_L^0(t)\rangle = e^{-\frac{i}{\hbar}\left(m_L-\frac{i}{2}\gamma_L\right)t}|K_L^0\rangle}$$

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \mathcal{H}_{eff}|\psi(t)\rangle$$

- These states decay with the lifetimes

$$\tau_S = \frac{\hbar}{\gamma_S} = 0.9\times 10^{-10}\ s \qquad \tau_L = \frac{\hbar}{\gamma_L} = 5.0\times 10^{-8}\ s$$

- Note! Unlike K^0 and $\bar{K}^0$ the e-states of $\mathcal{H}_{eff}$ - K_S^0 and K_L^0 are not each other's antiparticle! Thus, $m_S \neq m_L$ and $\tau_S \neq \tau_L$
- **Awesome**!

Time Evolution Final!

- Finally we are able to write down equations that **govern the time** evolution of kaons, let's assume that we start with a pure beam of K^0

$$|K^0(t)\rangle = \frac{\sqrt{(|p|^2 + |q|^2)}}{2p}\left(|K_S^0(t)\rangle + |K_L^0(t)\rangle\right) =$$

$$\frac{\sqrt{(|p|^2 + |q|^2)}}{2p}\left(e^{-\frac{i}{\hbar}\left(m_S - \frac{i}{2}\gamma_S\right)t}|K_S^0\rangle + e^{-\frac{i}{\hbar}\left(m_L - \frac{i}{2}\gamma_L\right)t}|K_L^0\rangle\right) =$$

$$\frac{\sqrt{(|p|^2 + |q|^2)}}{2p}\left[e^{-\frac{i}{\hbar}\left(m_S - \frac{i}{2}\gamma_S\right)t}\frac{1}{\sqrt{(|p|^2 + |q|^2)}}(p|K^0\rangle + q|\bar{K}^0\rangle) + e^{-\frac{i}{\hbar}\left(m_L - \frac{i}{2}\gamma_L\right)t}\frac{1}{\sqrt{(|p|^2 + |q|^2)}}(p|K^0\rangle - q|\bar{K}^0\rangle)\right] =$$

$$\frac{1}{2p}\left[p\left(e^{-\frac{i}{\hbar}\left(m_S - \frac{i}{2}\gamma_S\right)t} + e^{-\frac{i}{\hbar}\left(m_L - \frac{i}{2}\gamma_L\right)t}\right)|K^0\rangle\right] +$$

$$+\frac{1}{2p}\left[p\left(e^{-\frac{i}{\hbar}\left(m_S - \frac{i}{2}\gamma_S\right)t} - e^{-\frac{i}{\hbar}\left(m_L - \frac{i}{2}\gamma_L\right)t}\right)|\bar{K}^0\rangle\right]$$

Time Evolution Final!

- So, the probability of finding K^0 in the beam at some time t is:

$$P(K^0,t) = |\langle K^0|K^0(t)\rangle|^2 = \frac{1}{4}\left|e^{-\frac{i}{\hbar}\left(m_S-\frac{i}{2}\gamma_S\right)t} + e^{-\frac{i}{\hbar}\left(m_L-\frac{i}{2}\gamma_L\right)t}\right|^2$$

$$= \frac{1}{4}\left(e^{-\frac{\gamma_S t}{\hbar}} + e^{-\frac{\gamma_L t}{\hbar}} + e^{-\frac{1}{2\hbar}(\gamma_S+\gamma_L)t} \times 2cos(m_L - m_S)\frac{t}{\hbar}\right)$$

$$= \frac{1}{4}e^{-\frac{t}{\tau_S}} + \frac{1}{4}e^{-\frac{t}{\tau_L}} + \frac{1}{2}e^{-\left(\frac{1}{\tau_S}+\frac{1}{\tau_S}\right)t} cos\frac{\Delta mt}{\hbar}$$

- And by analogy one can calculate the same for $\overline{K}^0$

$$P(\overline{K}^0,t) = |\langle \overline{K}^0|K^0(t)\rangle|^2$$

$$= \left|\frac{q}{p}\right|^2 \left[\frac{1}{4}e^{-\frac{t}{\tau_S}} + \frac{1}{4}e^{-\frac{t}{\tau_L}} - \frac{1}{2}e^{-\left(\frac{1}{\tau_S}+\frac{1}{\tau_S}\right)t} cos\frac{\Delta mt}{\hbar}\right]$$

$$\Delta m = m_L - m_S \approx 3.5 \times 10^{-12}\ MeV$$

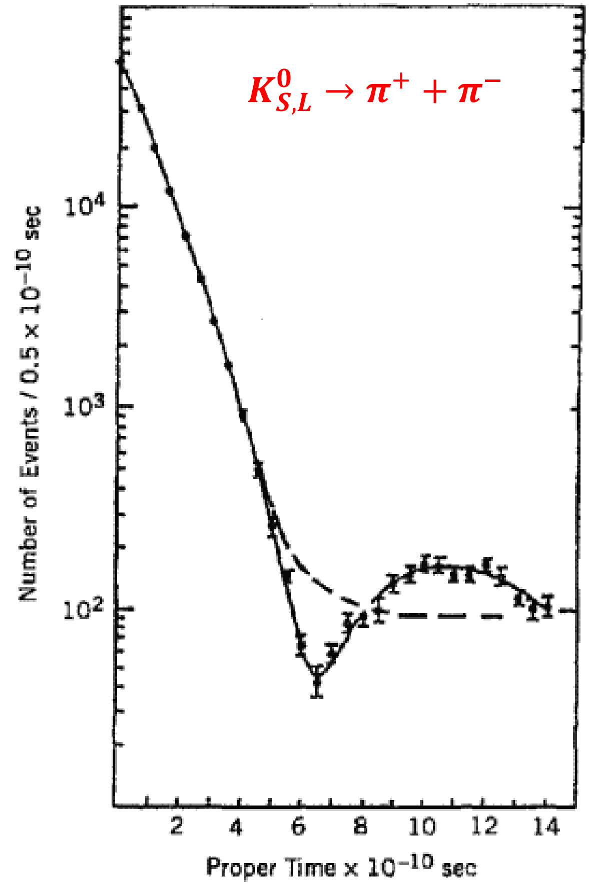
Mass difference is not zero!!

Time Evolution Final!

- The mass splitting of the weak Hamiltonian e-states can be translated into mass splitting of the strong interactions

$$m_{K^0} - m_{\overline{K}^0} < 10^{-18}\, m_{K^0}$$

- Very precise test of $\mathcal{CPT}$ symmetry



- Using our theoretical framework we could also estimate the prob. of observing weak e-states in the beam as a function of time
- By studying the number of decays as a function of the proper time one can observe QM interference in the 2π decay modes of the K_S^0 and K_L^0

Next time…

- Analogical calculations can be done for beauty mesons
- We are going to derive selected results presented today during our tutorial sessions (2 or 3 weeks time)