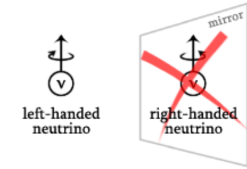

Symmetries (a bit) revisited and current current interaction picture

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WFiIS AGH

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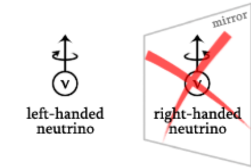


Last time on CPV

- Symmetries are incredibly useful in building theories – strong constraints on possible scenarios
- Central place here belongs to the unitary transformations

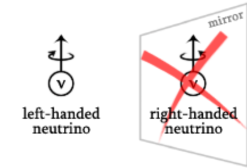
$$|\psi\rangle \to |\psi'\rangle = \mathcal{U}|\psi\rangle$$

$$\mathcal{U}^\dagger \mathcal{U} = \mathcal{U}\mathcal{U}^\dagger = 1$$

$$\boldsymbol{\langle \psi_1 | \mathcal{O} | \psi_2 \rangle} \to \langle \psi_1' | \mathcal{O} | \psi_2' \rangle_S = \boldsymbol{\langle \psi_1 | \mathcal{U}^\dagger \mathcal{O} \mathcal{U} | \psi_2 \rangle}$$

$$[\boldsymbol{\mathcal{O}}, \boldsymbol{\mathcal{U}}] = \mathbf{0}$$

$$\langle \psi | \mathcal{U}^\dagger \mathcal{U} \mathcal{O} | \psi \rangle$$



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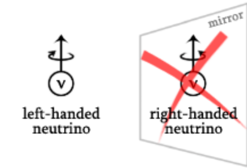
$$\langle\psi|\mathcal{U}^\dagger\mathcal{U}\mathcal{O}|\psi\rangle$$

Symmetry!



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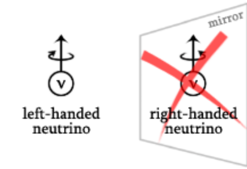
$$[\boldsymbol{\mathcal{O}},\boldsymbol{\mathcal{U}}] = \mathbf{0}$$

$$\langle\psi|\mathcal{U}^\dagger\mathcal{U}\mathcal{O}|\psi\rangle$$

- One conclusion of paramount meaning – if $\mathcal{U}$ is going to be a symmetry of a system the following must be always true

$$[\boldsymbol{\mathcal{U}},\boldsymbol{\mathcal{H}}] = \mathbf{0}$$

$$\boldsymbol{\mathcal{U}^\dagger\mathcal{H}\mathcal{U}} = \boldsymbol{\mathcal{H}}$$



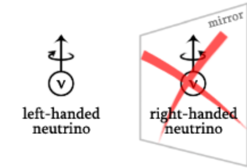
Last time on CPV…

- What is also awesome, is that we can connect unitary symmetry transformations with group theory
- For the continuous transformations we can write the related operators in a form

$$\mathcal{U}(\alpha_1, \alpha_2, \dots, \alpha_n) = exp\left\{\sum_l i\alpha_l \mathcal{G}_l\right\}$$

- If $\mathcal{U}$ is **unitary**, then $\mathcal{G}_l$ must be **Hermitian** and we call it the transformation's **generator**
- So, $\mathcal{U}$ being the symmetry implies **conservation of group generators**

$$[\mathcal{G}, \mathcal{H}] = 0$$



Last time on CPV…

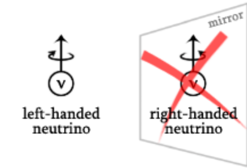
- Now the parity is one tough cookie…

$$|\psi\rangle \to |\psi'\rangle = \mathcal{P}|\psi\rangle, \mathcal{P}^\dagger\mathcal{P} = 1$$

$$\langle\psi'|\mathcal{X}|\psi'\rangle = \langle(\mathcal{P}\psi)|\mathcal{X}\mathcal{P}|\psi\rangle = \left\langle\psi\middle|\mathcal{P}^\dagger\mathcal{X}\mathcal{P}\middle|\psi\right\rangle = -\langle\psi|\mathcal{X}|\psi\rangle$$

$$\mathcal{P}^\dagger\mathcal{X}\mathcal{P} = -\mathcal{X}$$

Last time on CPV…



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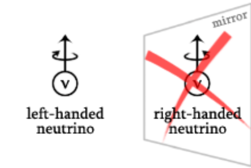
$$\langle\psi'|\mathcal{X}|\psi'\rangle = \langle(\mathcal{P}\psi)|\mathcal{X}\mathcal{P}|\psi\rangle = \langle\psi\left|\mathcal{P}^{\dagger}\mathcal{X}\mathcal{P}\right|\psi\rangle = -\langle\psi|\mathcal{X}|\psi\rangle$$

$$\mathcal{P}^{\dagger}\mathcal{X}\mathcal{P} = -\mathcal{X}$$

- Now the same trick with momentum is not that easy…
- Need some doing to show that:

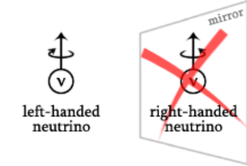
$$\mathcal{P}\delta\mathcal{D}_x = \delta\mathcal{D}_{-x}\mathcal{P} \rightarrow \delta\mathcal{D}_x = 1 - \frac{i}{\hbar}dx\,p_x$$

$$\{\mathcal{P}, p\} = 0 \rightarrow \mathcal{P}^{\dagger}p\mathcal{P} = -p$$



Last time on CPV…

- ❑ Even worse… for a particle to be an e-state of the parity operator it must be at rest!
- ❑ Thus, we define the **intrinsic parity**

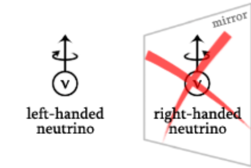


Last time on CPV…

- Even worse… for a particle to be an e-state of the parity operator it must be at rest!
- Thus, we define the intrinsic parity
- All in all, we have the following rules and naming convention

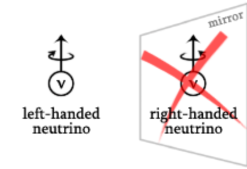
$\mathcal{P}^\dagger X \mathcal{P} = -X$	Vector
$\mathcal{P}^\dagger p \mathcal{P} = -p$	Vector
$\mathcal{P}^\dagger \vec{J} \mathcal{P} = \vec{J}$	Pseudo-Vector
$\mathcal{P}^\dagger \vec{x} \cdot \vec{p} \mathcal{P} = (-\vec{x}) \cdot (-\vec{p}) = \vec{x} \cdot \vec{p}$	Scalar
$\mathcal{P}^\dagger \vec{x} \cdot \vec{S} \mathcal{P} = (-\vec{x}) \cdot (\vec{S}) = -\vec{x} \cdot \vec{S}$	Pseudo-Scalar

And now the conclusions…



- Today:
  - Charge parity as unitary operator
  - Bilinear forms (with spinors) and why they matter so much (Lagrangian scalars)
  - C and P transformations of the bi-linears
  - Current current representation of the weak interactions

Charge conjugation



- As we can guess, $\mathcal{C}$, is a unitary operator that changes particle into its antiparticle (and vice-versa). Since $\mathcal{C}$ reverses not only the charge but also a lot of other quantum numbers it is somewhat more appropriate to call it **charge parity** operation
- It does not affect momentum, spin or helicity

$$\mathcal{C}|\psi(p,\lambda)\rangle = c_\psi|\bar{\psi}(p,\lambda)\rangle, p = (E,\vec{p}), \lambda = \vec{s}\cdot\vec{p}^0$$

$$\mathcal{P}|\psi(p,\lambda)\rangle = p_\psi|\psi(\tilde{p},-\lambda)\rangle, \tilde{p} = (E,-\vec{p})$$

- Similar to the parity, when applying $\mathcal{C}$ twice we need to arrive at the same state:

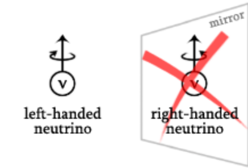
$$\mathcal{C}^2 = 1 : \mathcal{C} = \mathcal{C}^{-1} = \mathcal{C}^\dagger$$

$$|\psi\rangle = \mathcal{C}^2|\psi\rangle = c_\psi\mathcal{C}|\bar{\psi}\rangle = c_\psi c_{\bar{\psi}}|\psi\rangle$$

$$c_\psi c_{\bar{\psi}} = 1$$

- So, as a consequence of unitarity these guys must be phase factors

$$c_\psi = e^{i\varphi_c}, c_{\bar{\psi}} = e^{-i\varphi_c} = c_\psi^*$$



Charge conjugation

- Ok, let's discuss a bit… What actually can be an e-state of such operator and what about the similarity form? $\mathcal{C}^{\dagger}\mathcal{H}\mathcal{C} = \mathcal{H}$?
- If a state ψ is e-state of C-parity operator, we have:

$$c_{\psi} = c_{\bar{\psi}} = \pm 1$$

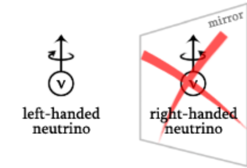
- Ok, so far C-parity is very similar to P-parity. It is even a multipicative quantum number. However, look at a proton $|p\rangle$

$$\mathcal{Q}|p\rangle = q|p\rangle, \mathcal{Q}|\bar{p}\rangle = -q|\bar{p}\rangle$$

$$\mathcal{C}|p\rangle = |\bar{p}\rangle$$

- **Proton is not** an e-state of C-parity operator! Acting with the $\mathcal{C}$ operator on a state introduces quite a change! (all additive quantum numbers reverse)
- So, only particles that have all their additive quantum numbers **equal to 0** can possibly be e-states of $\mathcal{C}$

$$\mathcal{C}|\pi^0\rangle = c_{\pi^0}|\pi^0\rangle, \mathcal{C}^2|\pi^0\rangle = c_{\pi^0}\mathcal{C}|\pi^0\rangle = c_{\pi^0}^2|\pi^0\rangle \rightarrow c_{\pi^0} = \pm 1$$



Charge conjugation

- Another obvious candidate to be the $\mathcal{C}$ e-state is the photon. Here the deal is a bit more tricky – let's start from the 4-current (density):

$$J^\mu = (J^0, J^1, J^2, J^3) = (\varrho, \vec{j})$$

$$\mathcal{C}^\dagger J^\mu \mathcal{C} = -J^\mu$$

- We need the photon to interact with the 4-current, QED Lagrangian that corresponds to interactions:

$$J_\mu A^\mu \to \mathcal{C}^\dagger J_\mu A^\mu \mathcal{C} = ?$$

This must be invariant

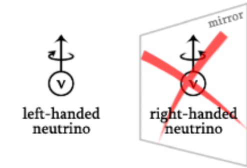
$$\mathcal{C}^\dagger J_\mu A^\mu \mathcal{C} = \mathcal{C}^\dagger J_\mu \mathcal{C}\mathcal{C}^\dagger A^\mu \mathcal{C} = -J_\mu \mathcal{C}^\dagger A^\mu \mathcal{C}$$

$$\mathcal{C}^\dagger A^\mu \mathcal{C} = -A^\mu$$

- This is the reason why we assume $\mathcal{C}|n\gamma\rangle = (-1)^n|\gamma\rangle$

$$\pi^0 \to \gamma + \gamma, \mathcal{C}|\pi^0\rangle = +1|\pi^0\rangle$$

- C-parity is conserved in the QED and QCD it is maximally broken in the WI



Some comments…

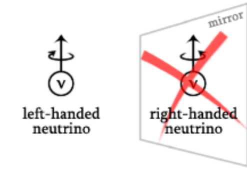
- This is the time to wonder a little bit: what is the same and what is different about C- and P-parity…
- They both are discrete unitary transformations, but we intuitively feel that the C-parity introduces more „radical changes"
- The big difference is of course that the parity can be defined for **all particles** and that there can not be states of mixed parity (or in other words we either have bosons or fermions)

$$[\mathcal{P}, \mathcal{H}] = 0 \rightarrow \mathcal{P}^{\dagger}\mathcal{H}\mathcal{P} = \mathcal{H}$$

Parity is conserved, odd state remains odd and even remains even

$$\mathcal{P}\mathcal{H}|\psi\rangle = \mathcal{P}\,\epsilon|\psi\rangle = \epsilon\mathcal{P}|\psi\rangle = \pm\epsilon|\psi\rangle$$

- Parity is conserved in the QED and QCD interactions and maximally violated in the WI



And now…, spinors!

- That was a lot of cool stuff, but what about Dirac equation and spinors…?
- There are two main points here – **how** to construct the P operator that is able to act on Dirac bi-spinors and **what it actually does to them**

$$|\psi\rangle \to |\psi'\rangle = \mathcal{P}|\psi\rangle, \mathcal{P}|\psi'\rangle = |\psi\rangle$$

$$i\gamma^\mu \partial_\mu |\psi\rangle - m|\psi\rangle = 0 \qquad\qquad |\psi'(x',y',z',t')\rangle = \mathcal{P}|\psi(x,y,z,t)\rangle$$

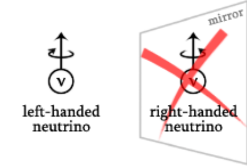
$$i\gamma^1 \frac{\partial|\psi\rangle}{\partial x} + i\gamma^2 \frac{\partial|\psi\rangle}{\partial y} + i\gamma^3 \frac{\partial|\psi\rangle}{\partial z} - m|\psi\rangle = -i\gamma^0 \frac{\partial|\psi\rangle}{\partial t}$$

$\mathcal{P}$

$$i\gamma^1 \frac{\partial|\psi'\rangle}{\partial x'} + i\gamma^2 \frac{\partial|\psi'\rangle}{\partial y'} + i\gamma^3 \frac{\partial|\psi'\rangle}{\partial z'} - m|\psi'\rangle = -i\gamma^0 \frac{\partial|\psi'\rangle}{\partial t'}$$

$$i\gamma^1 \mathcal{P} \frac{\partial|\psi'\rangle}{\partial x} + i\gamma^2 \mathcal{P} \frac{\partial|\psi'\rangle}{\partial y} + i\gamma^3 \mathcal{P} \frac{\partial|\psi'\rangle}{\partial z} - m\mathcal{P}|\psi'\rangle = -i\gamma^0 \mathcal{P} \frac{\partial|\psi'\rangle}{\partial t}$$

$$i\gamma^0\gamma^1 \mathcal{P} \frac{\partial|\psi'\rangle}{\partial x'} + i\gamma^0\gamma^2 \mathcal{P} \frac{\partial|\psi'\rangle}{\partial y'} + i\gamma^0\gamma^3 \mathcal{P} \frac{\partial|\psi'\rangle}{\partial z'} - m\gamma^0\mathcal{P}|\psi'\rangle = -i\gamma^0\gamma^0 \mathcal{P} \frac{\partial|\psi'\rangle}{\partial t'}$$



And now…, spinors!

- Using gamma-matrices algebra we have: $\gamma^0\gamma^j = -\gamma^j\gamma^0$

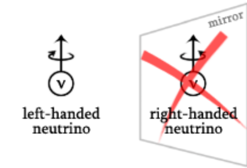
$$i\gamma^1\gamma^0\mathcal{P}\frac{\partial|\psi'\rangle}{\partial x'} + i\gamma^2\gamma^0\mathcal{P}\frac{\partial|\psi'\rangle}{\partial y'} + i\gamma^3\gamma^0\mathcal{P}\frac{\partial|\psi'\rangle}{\partial z'} - m\gamma^0\mathcal{P}|\psi'\rangle = -i\gamma^0\gamma^0\mathcal{P}\frac{\partial|\psi'\rangle}{\partial t'}$$

$$\gamma^0\mathcal{P} \propto \mathcal{I}, \mathcal{P}^2 = \mathcal{I} \rightarrow \mathcal{P} = \pm\gamma^0$$

- By convention we set: $\mathcal{P} = +\gamma^0$
- Remember, we showed that a solution of the Dirac equation was a four component bi-spinor, say u_1, u_2 correspond to a particle and v_1, v_2 to its antiparticle, then we have:

$$\mathcal{P}u_1 = +u_1, \mathcal{P}u_2 = +u_2, \mathcal{P}v_1 = -v_1, \mathcal{P}v_2 = -v_2$$

- So, picking the positive gamma-0 matrix for the parity operator makes the picture complete – intrinsic relative parities for fermions are positive and for anti-fermions are negative



Charge conjugation for spinors

- An extra step here is to understand that when applying C-parity to spinors we cannot neglect the interactions – we are changing charge!
- Interaction term must be added to the Dirac equation via modified (so called covariant) derivative:

$$i\partial_\mu \to i\partial_\mu - qA_\mu$$

$$\gamma^\mu(\partial_\mu - ieA_\mu)\psi + im\psi = 0$$

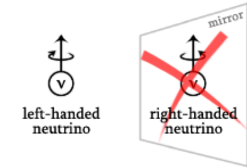
$$\gamma^\mu(\partial_\mu + ieA_\mu)\psi' + im\psi' = 0$$

- Here, ψ' is vector (in Dirac representation) describing a particle which has the same mass as electron but with the opposite charge

$$\psi' = \mathcal{C}\psi = i\gamma^2\psi^*$$

$$\psi = u_1 e^{i(\vec{p}\cdot\vec{x}-Et)} \to \mathcal{C}\psi = i\gamma^2\psi^* = i\gamma^2 v_1^* e^{-(\vec{p}\cdot\vec{x}-Et)} = v_1 e^{-(\vec{p}\cdot\vec{x}-Et)}$$

$$\psi' = \mathcal{C}(u_2 e^{i(\vec{p}\cdot\vec{x}-Et)}) = v_2 e^{-(\vec{p}\cdot\vec{x}-Et)}$$



Covariant currents

- A general continuity equation for a conserved quantity (let it be electric charge) goes like that:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

- In quantum theory we use particle density function and probability current

$$\frac{\partial \psi^* \psi}{\partial t} + \nabla \cdot \vec{j} = 0, \vec{j} \propto \psi^* \nabla \psi - \psi \nabla \psi^*$$

- Now, using our Dirac language (a.k.a. picture), and covariant notation

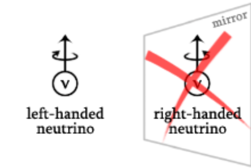
$$\psi^* \to \psi^\dagger = (\psi^*)^T, \psi^\dagger = (\psi_1^*, \psi_2^*, \psi_3^*, \psi_4^*)$$

$$\frac{\partial(\psi^\dagger \psi)}{\partial t} + \nabla \cdot (\psi^\dagger \vec{\alpha} \psi) \to \rho = \psi^\dagger \psi, \vec{j} = \psi^\dagger \vec{\alpha} \psi$$

$$\partial_\mu j^\mu = 0, j^\mu = (\rho, \vec{j}) = \psi^\dagger \gamma^0 \gamma^\mu \psi;\ \gamma^0 \gamma^0 = \mathcal{I}, \gamma^0 \gamma^\mu = \alpha_k$$

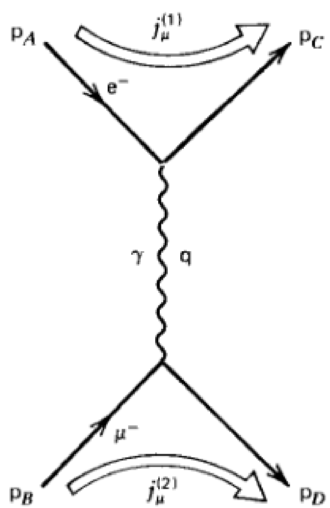
$$j^\mu = \bar{\psi} \gamma^\mu \psi, \bar{\psi} = \psi^\dagger \gamma^0$$

Adjoint spinor



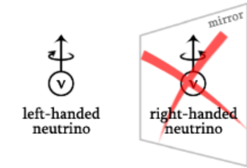
Bi-linear forms

- Last time we stated that Lagrangian invariance is essential for physics for it exposes **conservation laws** and allows to deduct equations of motions (such as Dirac equation)
  - Since the system dynamic is governed by Lagrangian it also must show the same invariance w.r.t. given transformation group
  - E.g., since QED is invariant w.r.t. parity transformation so must be the QED Lagrangian
- In quantum theory each measurement is related to a, so called, **matrix element** that also must be invariant – since it represents an observable



$$\mathcal{M}_{if}(e^-\mu^- \to e^-\mu^-) \propto (\bar{\psi}_e \gamma^\mu \psi_e) \frac{\alpha_{qed}}{q^2} (\bar{\psi}_\mu \gamma^\mu \psi_\mu)$$

$$\mathcal{M}_{if}(e^-\mu^- \to e^-\mu^-) \propto \frac{\alpha_{qed}}{q^2} j_\mu^{(e)} j^\mu_{(\mu)}$$



Bi-linear forms

- We use **charged current interaction** picture to express matrix elements
- „Physics” sits in the **propagator** (4-momentum exchange) and in the **coupling constant**
- The **covariant currents** are used to represent initial and final state
- **Relation** between the i- and f-state (spinors) is given by the **gamma-matrices**
- **The electromagnetic interactions have vector nature**

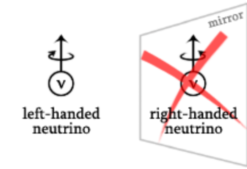
$$\mathcal{M}_{if}(e^-\mu^- \to e^-\mu^-) \propto \frac{\alpha_{qed}}{q^2} j_\mu^{(e)} j_{(\mu)}^{\mu} = \frac{\alpha_{qed}}{q^2} \eta_{\mu\nu} j_\mu^{(e)} j_{(\mu)}^{\mu}$$

$$\eta_{\mu\nu} j_\mu^{(e)} j_{(\mu)}^{\mu} = j_{(e)}^0 j_{(\mu)}^0 - \vec{j}_{(e)} \cdot \vec{j}_{(\mu)}$$

Scalar (a complex number)

- Why we use such bi-linear forms? Well, they are the simplest expression there is wich allows to formulate invariant $\mathcal{M}_{if}$

Weak bi-linears



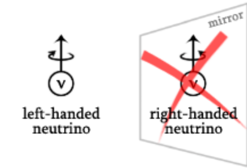
- The QED picture, as we know, is very successful in describing physics
- Use lazy approach and try to re-use and extend it to describe the WI
- The basic premise: we use a **generalised four-current** (bi-linear) to calculate weak matrix elements

$$j^{\mu} = \bar{\psi}\mathcal{O}_k\psi$$

- Where $\mathcal{O}_k$ is the operator that tells us all there is about the interaction type and is expressed via gamma-matrices
- The complication arises from the fact, that the bi-linear now is required to behave (depending on the particular interaction type) as: a scalar (S), a pseudo-scalar (P), a vector (V), an axial-vector (A) and a tensor (T)

$$j^{\mu} = \bar{\psi}\mathcal{O}_k\psi, k = \{S, P, V, A, T\}$$

- As usual, the symmetries (in this case broken symmetries) determine the form of the four-current
- It can be shown (tutorial) that the WI have **mixed vector–axial-vector** nature which allows for the parity to be broken and bosons to couple to the particles of specific handedness



C- and P-parity against weak bi-linears

- P-parity

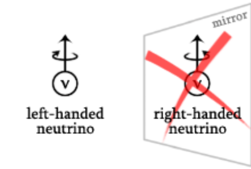
$$\mathcal{P}: x = (\vec{x}, t) \to x' = (-\vec{x}, t)$$

Scalar	$\bar{\psi}_1\psi_2$	$\to$	$\bar{\psi}_1\psi_2$
Pseudo-scalar	$\bar{\psi}_1\gamma^5\psi_2$	$\to$	$-\bar{\psi}_1\gamma^5\psi_2$
Vector	$\bar{\psi}_1\gamma^\mu\psi_2$	$\to$	$\bar{\psi}_1\gamma^\mu\psi_2$
Axial-vector	$\bar{\psi}_1\gamma^\mu\gamma^5\psi_2$	$\to$	$-\bar{\psi}_1\gamma^\mu\gamma^5\psi_2$

- C-parity

$$\mathcal{C}: x = (\vec{x}, t) \to x' = (\vec{x}, t)$$

Scalar	$\bar{\psi}_1\psi_2$	$\to$	$\bar{\psi}_2\psi_1$
Pseudo-scalar	$\bar{\psi}_1\gamma^5\psi_2$	$\to$	$\bar{\psi}_2\gamma^5\psi_1$
Vector	$\bar{\psi}_1\gamma^\mu\psi_2$	$\to$	$-\bar{\psi}_2\gamma^\mu\psi_1$
Axial-vector	$\bar{\psi}_1\gamma^\mu\gamma^5\psi_2$	$\to$	$\bar{\psi}_2\gamma^\mu\gamma^5\psi_1$

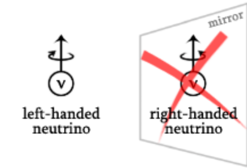


CP (or PC) transformations

- **Spoiler alert**! This will be also discussed during later lectures

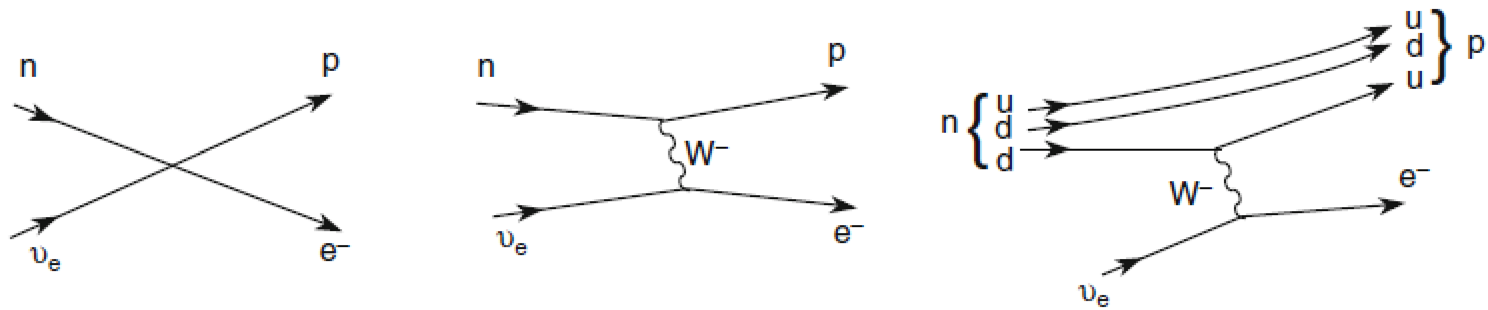
$\mathcal{CP}: x = (\vec{x}, t) \rightarrow x' = (-\vec{x}, t)$

Scalar	$\bar{\psi}_1\psi_2$	$\rightarrow$	$\bar{\psi}_2\psi_1$
Pseudo-scalar	$\bar{\psi}_1\gamma^5\psi_2$	$\rightarrow$	$-\bar{\psi}_2\gamma^5\psi_1$
Vector	$\bar{\psi}_1\gamma^\mu\psi_2$	$\rightarrow$	$-\bar{\psi}_2\gamma^\mu\psi_1$
Axial-vector	$\bar{\psi}_1\gamma^\mu\gamma^5\psi_2$	$\rightarrow$	$-\bar{\psi}_2\gamma^\mu\gamma^5\psi_1$



The weak four-current

- Let's build up now the weak four-current using the knowledge gained so far



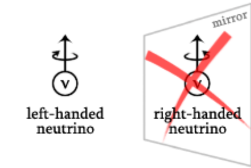
$$\mathcal{M}_{if}(n+\nu_e \to pe^-) \propto j_\mu^{(l)} j_{(b)}^\mu$$

$$j_{(l)}^\mu = \bar{\psi}_e \mathcal{O}_k \psi_{\nu_e} = \langle \psi_e | \mathcal{O}_k | \psi_{\nu_e} \rangle, j_{(b)}^\mu = \bar{\psi}_p \mathcal{O}_k \psi_n = \langle \psi_p | \mathcal{O}_k | \psi_n \rangle$$

- As mentioned already, in order to accommodate the experimental results we need to write the currents in a mixed V-A form

$$j_{(l)}^\mu = \left(c_V V_{(l)}^\mu + c_A A_{(l)}^\mu\right), c_V = -c_A = 1$$

$$j_{(l)}^\mu = \bar{\psi}_e \gamma^\mu \psi_{\nu_e} - \bar{\psi}_e \gamma^\mu \gamma^5 \psi_{\nu_e} = \bar{\psi}_e \gamma^\mu (1-\gamma^5) \psi_{\nu_e}$$



The weak four-current

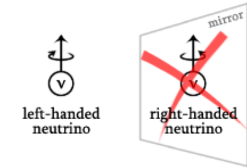
- The inner combination of gamma-matrices looks almost like the projection operator used for the chirality representation of spinors, just a touch…

$$\gamma^\mu(1-\gamma^5) = \frac{1}{2}(1+\gamma^5)\gamma^\mu(1-\gamma^5)$$

$$j^\mu_{(l)} = 2\bar{\psi}_e\left(\frac{1+\gamma^5}{2}\right)\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)\psi_{\nu_e} = 2(\bar{\psi}_e)_L\gamma^\mu(\psi_{\nu_e})_L$$

- Now, what about the barionic current…? Well here we have some additional players – quarks!
- In principle, one can consider neutrons and protons and notice some rules regarding changing angular momenta and things, but it is much easier if we use quarks, that are ½ spin fermions that looks a lot like Dirac particles…

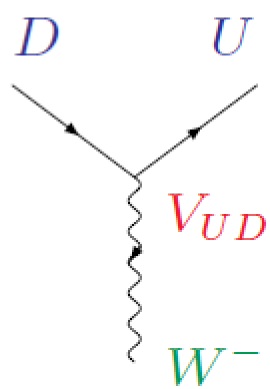
$$j^\mu_{(b)} = g_V\bar{\psi}_u\gamma^\mu\psi_d + g_A\bar{\psi}_u\gamma^\mu\gamma^5\psi_d$$



The weak four-current

- The complication, we need to deal with is now related to the fact that the quark states that couple to weak bosons are **not „pure"**
- This will be discussed in later lectures and is called quark mixing
- That is quite different from the pure lepton states and requires to introduce a set of effective coupling constants describing probabilities of different qq transitions
- Using spinors and covariant formalism we are able to prepare an elegant picture for all quarks

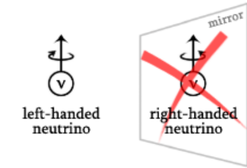
„down" type quark
$D \in \{d, s, b\}$



„up" type quark
$U \in \{u, c, t\}$

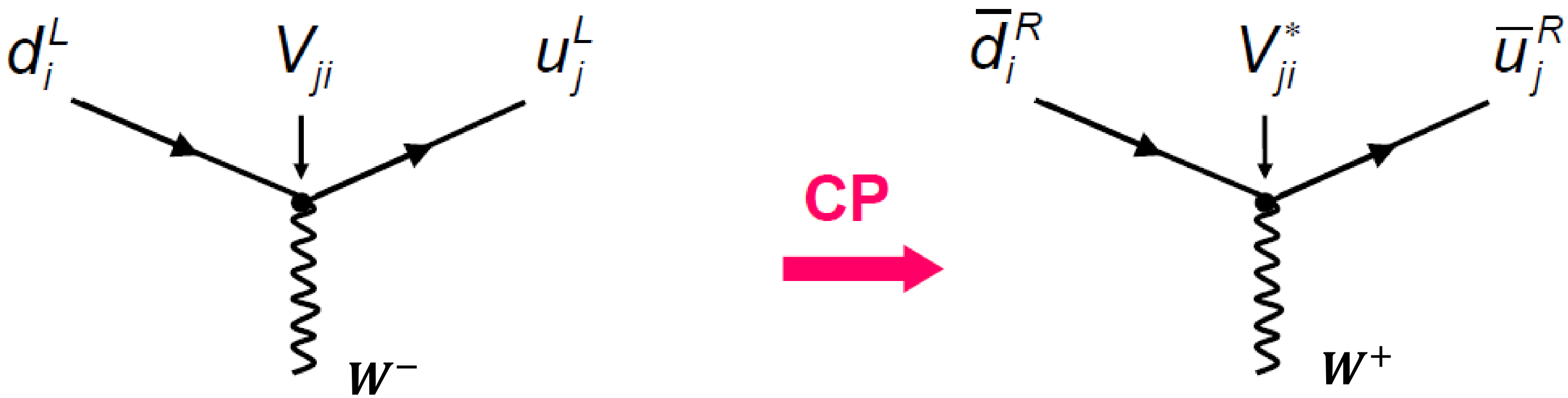
Coupling constant

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



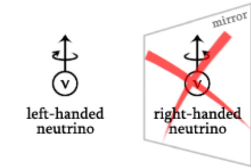
The weak four-current

- ❑ Another spoiler alert! CP-violation



- ❑ For CP-violation to occur we need:
  - ❑ Massive quarks
  - ❑ At least three generations of quarks
  - ❑ Coupling constants must „somehow” be **complex numbers**!

$$V_{ij} \neq V_{ij}^*$$



Summary

- So, what have we learned…?
- **C-parity** is very special, and affects not only a particle but has impact on the interaction the particle undergo
- Special place belongs to **bi-linear forms** that can be created by combining spinors – in this way we can define 4-currents and describe interactions using current-current model
- Returning to symmetries: first we have elegant **covariant notation** that is guaranteed to be invariant w.r.t. to Lorentz group and second we can combine currents to create various „**scalars**" that, in turn, make the respective matrix elements invariant
- The weak interactions are again very intriguing and have much more complicated structure than the QED
- To accommodate all observed effects we need to assume that the WI is a mixture of vector and axial-vector currents