Symmetries (a bit) revisited and current current interaction picture

Tomasz Szumlak, Agnieszka Obłąkowska-Mucha
Last time on CPV

- Symmetries are incredibly useful in building theories – strong constraints on possible scenarios
- Central place here belongs to the unitary transformations

$$\langle \psi_1 | \sigma | \psi_2 \rangle \rightarrow \langle \psi'_1 | \sigma | \psi'_2 \rangle_S = \langle \psi_1 | u^\dagger \sigma u | \psi_2 \rangle$$

$$[\sigma, u] = 0$$

$$\langle \psi | u^\dagger u \sigma | \psi \rangle$$
Last time on CPV...

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\[ |\psi\rangle \rightarrow |\psi'\rangle = U|\psi\rangle \]
\[ U^\dagger U = UU^\dagger = 1 \]
\[ \langle \psi_1 | \sigma | \psi_2 \rangle \rightarrow \langle \psi_1 | \sigma | \psi_2 \rangle_s = \langle \psi_1 | U^\dagger \sigma U | \psi_2 \rangle \]
\[ [\sigma, U] = 0 \]
\[ \langle \psi | U^\dagger U \sigma | \psi \rangle \]

Symmetry!
Last time on CPV...

- Symmetries are incredibly useful in building theories – strong constraints on possible scenarios.
- Central place here belongs to the unitary transformations

\[ |\psi\rangle \rightarrow |\psi'\rangle = U|\psi\rangle \]
\[ U^\dagger U = UU^\dagger = 1 \]

\[ \langle \psi_1 | \mathcal{O} | \psi_2 \rangle \rightarrow \langle \psi_1' | \mathcal{O} | \psi_2' \rangle_s = \langle \psi_1 | U^\dagger \mathcal{O} U | \psi_2 \rangle \]
\[ [\mathcal{O}, U] = 0 \]

\[ \langle \psi | U^\dagger U \mathcal{O} | \psi \rangle \]

- One conclusion of paramount meaning – if \( U \) is going to be a symmetry of a system the following must be always true

\[ [U, \mathcal{H}] = 0 \]
\[ U^\dagger \mathcal{H} U = \mathcal{H} \]
Last time on CPV…

- What is also awesome, is that we can connect unitary symmetry transformations with group theory
- For the continuous transformations we can write the related operators in a form
  \[ U(\alpha_1, \alpha_2, ..., \alpha_n) = \exp \left\{ \sum_l i \alpha_l G_l \right\} \]

- If \( U \) is \textbf{unitary}, then \( G_l \) must be \textbf{Hermitian} and we call it the transformation’s \textbf{generator}
- So, \( U \) being the symmetry implies \textbf{conservation of group generators}

\[ [G, H] = 0 \]
Last time on CPV...

Now the parity is one tough cookie...

\[ |\psi\rangle \rightarrow |\psi'\rangle = \mathcal{P}|\psi\rangle, \mathcal{P}^\dagger \mathcal{P} = 1 \]

\[ \langle \psi'|\mathcal{X}|\psi'\rangle = \langle (\mathcal{P}\psi)|\mathcal{X}\mathcal{P}|\psi\rangle = \langle \psi|\mathcal{P}^\dagger \mathcal{X}\mathcal{P}|\psi\rangle = -\langle \psi|\mathcal{X}|\psi\rangle \]

\[ \mathcal{P}^\dagger \mathcal{X}\mathcal{P} = -\mathcal{X} \]
Last time on CPV...

- Now the parity is one tough cookie...

\[ |\psi\rangle \rightarrow |\psi'\rangle = \mathcal{P}|\psi\rangle, \mathcal{P}^\dagger\mathcal{P} = 1 \]

\[ \langle\psi'|X|\psi'\rangle = \langle(\mathcal{P}\psi)|X\mathcal{P}|\psi\rangle = \langle\psi|\mathcal{P}^\dagger X\mathcal{P}|\psi\rangle = -\langle\psi|X|\psi\rangle \]

\[ \mathcal{P}^\dagger X\mathcal{P} = -X \]

- Now the same trick with momentum is not that easy...
- Need some doing to show that:

\[ \mathcal{P}\delta D_x = \delta D_{-x}\mathcal{P} \rightarrow \delta D_x = 1 - \frac{i}{\hbar} d x p_x \]

\[ \{\mathcal{P}, p\} = 0 \rightarrow \mathcal{P}^\dagger p\mathcal{P} = -p \]
Last time on CPV...

- Even worse... for a particle to be an e-state of the parity operator it must be at rest!
- Thus, we define the intrinsic parity
Last time on CPV…

- Even worse… for a particle to be an e-state of the parity operator it must be at rest!
- Thus, we define the intrinsic parity

- All in all, we have the following rules and naming convention

\[
\begin{align*}
\mathcal{P}^\dagger \mathcal{X} \mathcal{P} &= -\mathcal{X} & \text{Vector} \\
\mathcal{P}^\dagger \mathcal{P} &= -\mathcal{P} & \text{Vector} \\
\mathcal{P}^\dagger \mathcal{J} \mathcal{P} &= \mathcal{J} & \text{Pseudo-Vector} \\
\mathcal{P}^\dagger \mathcal{X} \cdot \mathcal{P} &= (-\mathcal{X}) \cdot (-\mathcal{P}) = \mathcal{X} \cdot \mathcal{P} & \text{Scalar} \\
\mathcal{P}^\dagger \mathcal{X} \cdot \mathcal{S} \mathcal{P} &= (-\mathcal{X}) \cdot \mathcal{S} = -\mathcal{X} \cdot \mathcal{S} & \text{Pseudo-Scalar}
\end{align*}
\]
And now the conclusions...

- **Today:**
  - Charge parity as unitary operator
  - Bilinear forms (with spinors) and why they matter so much (Lagrangian scalars)
  - C and P transformations of the bi-linears
  - Current current representation of the weak interactions
Charge conjugation

- As we can guess, $\mathcal{C}$, is a unitary operator that changes particle into its antiparticle (and vice-versa). Since $\mathcal{C}$ reverses not only the charge but also a lot of other quantum numbers it is somewhat more appropriate to call it charge parity operation.

- It does not affect momentum, spin or helicity:

$$\mathcal{C}|\psi(p, \lambda)\rangle = c_\psi |\bar{\psi}(p, \lambda)\rangle, p = (E, \bar{p}), \lambda = \vec{s} \cdot \bar{p}^0$$

$$\mathcal{P}|\psi(p, \lambda)\rangle = p_\psi |\psi(\bar{p}, -\lambda)\rangle, \bar{p} = (E, -\bar{p})$$

- Similar to the parity, when applying $\mathcal{C}$ twice we need to arrive at the same state:

$$\mathcal{C}^2 = 1: \mathcal{C} = \mathcal{C}^{-1} = \mathcal{C}^\dagger$$

$$|\psi\rangle = \mathcal{C}^2 |\psi\rangle = c_\psi \mathcal{C} |\bar{\psi}\rangle = c_\psi c_{\bar{\psi}} |\psi\rangle$$

$$c_\psi c_{\bar{\psi}} = 1$$

- So, as a consequence of unitarity these guys must be phase factors:

$$c_\psi = e^{i\phi_c}, c_{\bar{\psi}} = e^{-i\phi_c} = c_\psi^*$$
Charge conjugation

- Ok, let’s discuss a bit… What actually can be an e-state of such operator and what about the similarity form? $C^\dagger HC = H$?
- If a state $\psi$ is e-state of C-parity operator, we have:

$$c_\psi = c_{\overline{\psi}} = \pm 1$$

- Ok, so far C-parity is very similar to P-parity. It is even a multiplicative quantum number. However, look at a proton $|p\rangle$

$$Q|p\rangle = q|p\rangle, Q|\overline{p}\rangle = -q|\overline{p}\rangle$$

$$C|p\rangle = |\overline{p}\rangle$$

- **Proton is not** an e-state of C-parity operator! Acting with the $C$ operator on a state introduces quite a change! (all additive quantum numbers reverse)

- So, only particles that have all their additive quantum numbers equal to 0 can possibly be e-states of $C$

$$C|\pi^0\rangle = c_{\pi^0}|\pi^0\rangle, C^2|\pi^0\rangle = c_{\pi^0}C|\pi^0\rangle = c_{\pi^0}^2|\pi^0\rangle \rightarrow c_{\pi^0} = \pm 1$$
Charge conjugation

- Another obvious candidate to be the $C$ e-state is the photon. Here the deal is a bit more tricky – let’s start from the 4-current (density):
  \[ J^\mu = (J^0, J^1, J^2, J^3) = (q, j) \]

  \[ C^\dagger J^\mu C = -J^\mu \]

- We need the photon to interact with the 4-current, QED Lagrangian that corresponds to interactions:

  \[ J_\mu A^\mu \rightarrow C^\dagger J_\mu A^\mu C =? \]

  \[ C^\dagger J_\mu A^\mu C = C^\dagger J_\mu C C^\dagger A^\mu C = -J_\mu C^\dagger A^\mu C \]

  \[ C^\dagger A^\mu C = -A^\mu \]

- This must be invariant

- This is the reason why we assume $C|n\gamma\rangle = (-1)^n |\gamma\rangle$

  \[ \pi^0 \rightarrow \gamma + \gamma, C|\pi^0\rangle = +1 |\pi^0\rangle \]

- $C$-parity is conserved in the QED and QCD it is maximally broken in the WI
Some comments...

- This is the time to wonder a little bit: what is the same and what is different about C- and P-parity...
- They both are discrete unitary transformations, but we intuitively feel that the C-parity introduces more „radical changes”
- The big difference is of course that the parity can be defined for all particles and that there can not be states of mixed parity (or in other words we either have bosons or fermions)

\[ [P, \mathcal{H}] = 0 \rightarrow P^\dagger \mathcal{H} P = \mathcal{H} \]

Parity is conserved, odd state remains odd and even remains even

\[ P \mathcal{H} |\psi\rangle = P \varepsilon |\psi\rangle = \varepsilon P |\psi\rangle = \pm \varepsilon |\psi\rangle \]

- Parity is conserved in the QED and QCD interactions and maximally violated in the WI
And now..., spinors!

- That was a lot of cool stuff, but what about Dirac equation and spinors...?
- There are two main points here – how to construct the $P$ operator that is able to act on Dirac bi-spinors and what it actually does to them

\[ |\psi\rangle \rightarrow |\psi'\rangle = P|\psi\rangle, P|\psi'\rangle = |\psi\rangle \]

\[ i\gamma^\mu \partial_\mu |\psi\rangle - m|\psi\rangle = 0 \]

\[ |\psi'(x', y', z', t')\rangle = P|\psi(x, y, z, t)\rangle \]

\[ i\gamma^1 \frac{\partial |\psi\rangle}{\partial x} + i\gamma^2 \frac{\partial |\psi\rangle}{\partial y} + i\gamma^3 \frac{\partial |\psi\rangle}{\partial z} - m|\psi\rangle = -i\gamma^0 \frac{\partial |\psi\rangle}{\partial t} \]

\[ i\gamma^1 \frac{\partial |\psi'\rangle}{\partial x'} + i\gamma^2 \frac{\partial |\psi'\rangle}{\partial y'} + i\gamma^3 \frac{\partial |\psi'\rangle}{\partial z'} - m|\psi'\rangle = -i\gamma^0 \frac{\partial |\psi'\rangle}{\partial t'} \]

\[ i\gamma^1 P \frac{\partial |\psi'\rangle}{\partial x} + i\gamma^2 P \frac{\partial |\psi'\rangle}{\partial y} + i\gamma^3 P \frac{\partial |\psi'\rangle}{\partial z} - mP|\psi'\rangle = -i\gamma^0 P \frac{\partial |\psi'\rangle}{\partial t} \]

\[ i\gamma^0 \gamma^1 P \frac{\partial |\psi'\rangle}{\partial x'} + i\gamma^0 \gamma^2 P \frac{\partial |\psi'\rangle}{\partial y'} + i\gamma^0 \gamma^3 P \frac{\partial |\psi'\rangle}{\partial z'} - m\gamma^0 P|\psi'\rangle = -i\gamma^0 \gamma^0 P \frac{\partial |\psi'\rangle}{\partial t'} \]
And now..., spinors!

- Using gamma-matrices algebra we have: $\gamma^0\gamma^j = -\gamma^j\gamma^0$

$$i\gamma^1\gamma^0\mathcal{P}\frac{\partial|\psi\rangle}{\partial x'} + i\gamma^2\gamma^0\mathcal{P}\frac{\partial|\psi\rangle}{\partial y'} + i\gamma^3\gamma^0\mathcal{P}\frac{\partial|\psi\rangle}{\partial z'} - m\gamma^0\mathcal{P}|\psi\rangle = -i\gamma^0\gamma^0\mathcal{P}\frac{\partial|\psi\rangle}{\partial t'}$$

$\gamma^0\mathcal{P} \propto I, \mathcal{P}^2 = I \rightarrow \mathcal{P} = \pm \gamma^0$

- By convention we set: $\mathcal{P} = +\gamma^0$

- Remember, we showed that a solution of the Dirac equation was a four component bi-spinor, say $u_1, u_2$ correspond to a particle and $v_1, v_2$ to its antiparticle, then we have:

$$\mathcal{P}u_1 = +u_1, \mathcal{P}u_2 = +u_2, \mathcal{P}v_1 = -v_1, \mathcal{P}v_2 = -v_2$$

- So, picking the positive gamma-0 matrix for the parity operator makes the picture complete – intrinsic relative parities for fermions are positive and for anti-fermions are negative
Charge conjugation for spinors

- An extra step here is to understand that when applying C-parity to spinors we cannot neglect the interactions – we are changing charge!
- Interaction term must be added to the Dirac equation via modified (so called covariant) derivative:

\[ i \partial_\mu \rightarrow i \partial_\mu - qA_\mu \]

\[ \gamma^\mu (\partial_\mu - ieA_\mu)\psi + im\psi = 0 \]

\[ \gamma^\mu (\partial_\mu + ieA_\mu)\psi' + im\psi' = 0 \]

- Here, \( \psi' \) is vector (in Dirac representation) describing a particle which has the same mass as electron but with the opposite charge

\[ \psi' = C\psi = i\gamma^2\psi^* \]

\[ \psi = u_1 e^{i(\vec{p}\cdot\vec{x} - Et)} \rightarrow C\psi = i\gamma^2\psi^* = i\gamma^2 v_1^* e^{-(\vec{p}\cdot\vec{x} - Et)} = v_1 e^{-(\vec{p}\cdot\vec{x} - Et)} \]

\[ \psi' = C(u_2 e^{i(\vec{p}\cdot\vec{x} - Et)}) = v_2 e^{-(\vec{p}\cdot\vec{x} - Et)} \]
Covariant currents

- A general continuity equation for a conserved quantity (let it be electric charge) goes like that:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0 \]

- In quantum theory we use particle density function and probability current

\[ \frac{\partial \psi^* \psi}{\partial t} + \nabla \cdot \vec{j} = 0, \quad \vec{j} \propto \psi^* \nabla \psi - \psi \nabla \psi^* \]

- Now, using our Dirac language (a.k.a. picture), and covariant notation

\[ \psi^* \rightarrow \psi^\dagger = (\psi^*)^T, \quad \psi^\dagger = (\psi_1^*, \psi_2^*, \psi_3^*, \psi_4^*) \]

\[ \frac{\partial (\psi^\dagger \psi)}{\partial t} + \nabla \cdot (\psi^\dagger \alpha \psi) \rightarrow \rho = \psi^\dagger \psi, \quad \vec{j} = \psi^\dagger \alpha \psi \]

\[ \partial_\mu j^\mu = 0, \quad j^\mu = (\rho, \vec{j}) = \psi^\dagger \gamma^0 \gamma^\mu \psi; \quad \gamma^0 \gamma^0 = \mathbb{I}, \quad \gamma^0 \gamma^\mu = \alpha_k \]

\[ j^\mu = \bar{\psi} \gamma^\mu \psi, \quad \bar{\psi} = \psi^\dagger \gamma^0 \]

Adjoint spinor
Bi-linear forms

- Last time we stated that Lagrangian invariance is essential for physics for it exposes **conservation laws** and allows to deduct equations of motions (such as Dirac equation)
  - Since the system dynamic is governed by Lagrangian it also must show the same invariance w.r.t. given transformation group
  - E.g., since QED is invariant w.r.t. parity transformation so must be the QED Lagrangian
- In quantum theory each measurement is related to a, so called, **matrix element** that also must be invariant – since it represents an observable

\[
\mathcal{M}_{if}(e^- \mu^- \rightarrow e^- \mu^-) \propto (\bar{\psi}_e \gamma^\mu \psi_e) \frac{\alpha_{\text{qed}}}{q^2} (\bar{\psi}_\mu \gamma^\mu \psi_\mu)
\]

\[
\mathcal{M}_{if}(e^- \mu^- \rightarrow e^- \mu^-) \propto \frac{\alpha_{\text{qed}}}{q^2} j_{(e)}^\mu j_{(\mu)}^\mu
\]
We use **charged current interaction** picture to express matrix elements

„Physics” sits in the **propagator** (4-momentum exchange) and in the **coupling constant**

The **covariant currents** are used to represent initial and final state

**Relation** between the i- and f-state (spinors) is given by the **gamma-matrices**

The electromagnetic interactions have vector nature

\[
\mathcal{M}_{if}(e^- \mu^- \rightarrow e^- \mu^-) \propto \frac{\alpha_{\text{qed}}}{q^2} j_\mu^{(e)} j_{\mu}^{(\mu)} = \frac{\alpha_{\text{qed}}}{q^2} \eta_{\mu\nu} j_\mu^{(e)} j_{\mu}^{(\mu)}
\]

\[
\eta_{\mu\nu} j_\mu^{(e)} j_{\mu}^{(\mu)} = j_{\mu}^{(e)} j_{\mu}^{(\mu)} - j_{(e)} \cdot j_{(\mu)}
\]

**Scalar (a complex number)**

Why we use such bi-linear forms? Well, they are the simplest expression there is which allows to formulate invariant \( \mathcal{M}_{if} \)
Weak bi-linears

- The QED picture, as we know, is very successful in describing physics
- Use lazy approach and try to re-use and extend it to describe the WI
- The basic premise: we use a generalised four-current (bi-linear) to calculate weak matrix elements
  \[ j^\mu = \overline{\psi}O_k\psi \]
- Where \( O_k \) is the operator that tells us all there is about the interaction type and is expressed via gamma-matrices
- The complication arises from the fact, that the bi-linear now is required to behave (depending on the particular interaction type) as: a scalar (S), a pseudo-scalar (P), a vector (V), an axial-vector (A) and a tensor (T)
  \[ j^\mu = \overline{\psi}O_k\psi, \ k = \{S, P, V, A, T\} \]
- As usual, the symmetries (in this case broken symmetries) determine the form of the four-current
- It can be shown (tutorial) that the WI have mixed vector–axial-vector nature which allows for the parity to be broken and bosons to couple to the particles of specific handedness
C- and P-parity against weak bi-linears

- **P-parity**
  \[ \mathcal{P}: x = (\hat{x}, t) \rightarrow x' = (-\hat{x}, t) \]
  - Scalar: \[ \bar{\psi}_1 \psi_2 \rightarrow \bar{\psi}_1 \psi_2 \]
  - Pseudo-scalar: \[ \bar{\psi}_1 \gamma^5 \psi_2 \rightarrow -\bar{\psi}_1 \gamma^5 \psi_2 \]
  - Vector: \[ \bar{\psi}_1 \gamma^\mu \psi_2 \rightarrow \bar{\psi}_1 \gamma^\mu \psi_2 \]
  - Axial-vector: \[ \bar{\psi}_1 \gamma^\mu \gamma^5 \psi_2 \rightarrow -\bar{\psi}_1 \gamma^\mu \gamma^5 \psi_2 \]

- **C-parity**
  \[ C: x = (\hat{x}, t) \rightarrow x' = (\hat{x}, t) \]
  - Scalar: \[ \bar{\psi}_1 \psi_2 \rightarrow \bar{\psi}_2 \psi_1 \]
  - Pseudo-scalar: \[ \bar{\psi}_1 \gamma^5 \psi_2 \rightarrow \bar{\psi}_2 \gamma^5 \psi_1 \]
  - Vector: \[ \bar{\psi}_1 \gamma^\mu \psi_2 \rightarrow -\bar{\psi}_2 \gamma^\mu \psi_1 \]
  - Axial-vector: \[ \bar{\psi}_1 \gamma^\mu \gamma^5 \psi_2 \rightarrow \bar{\psi}_2 \gamma^\mu \gamma^5 \psi_1 \]
CP (or PC) transformations

- **Spoiler alert!** This will be also discussed during later lectures

\[ C\mathcal{P}: x = (\bar{x}, t) \rightarrow x' = (-\bar{x}, t) \]

- **Scalar**
  \[ \bar{\psi}_1 \psi_2 \rightarrow \bar{\psi}_2 \psi_1 \]

- **Pseudo-scalar**
  \[ \bar{\psi}_1 \gamma^5 \psi_2 \rightarrow -\bar{\psi}_2 \gamma^5 \psi_1 \]

- **Vector**
  \[ \bar{\psi}_1 \gamma^\mu \psi_2 \rightarrow -\bar{\psi}_2 \gamma^\mu \psi_1 \]

- **Axial-vector**
  \[ \bar{\psi}_1 \gamma^\mu \gamma^5 \psi_2 \rightarrow -\bar{\psi}_2 \gamma^\mu \gamma^5 \psi_1 \]
The weak four-current

- Let’s build up now the weak four-current using the knowledge gained so far

\[ M_{if}(n + \nu_e \rightarrow p e^-) \propto j_{(l)}^{(1)} j_{(b)}^{\mu} \]

\[ j_{(l)}^{\mu} = \bar{\psi}_e O_k \psi_{\nu_e} = \langle \psi_e | O_k | \psi_{\nu_e} \rangle, \quad j_{(b)}^{\mu} = \bar{\psi}_p O_k \psi_n = \langle \psi_p | O_k | \psi_n \rangle \]

- As mentioned already, in order to accommodate the experimental results we need to write the currents in a mixed V-A form

\[ j_{(l)}^{\mu} = \left( c_V V_{(l)}^{\mu} + c_A A_{(l)}^{\mu} \right), \quad c_V = -c_A = 1 \]

\[ j_{(l)}^{\mu} = \bar{\psi}_e \gamma^{\mu} \psi_{\nu_e} - \bar{\psi}_e \gamma^{\mu} \gamma^5 \psi_{\nu_e} = \bar{\psi}_e \gamma^{\mu} (1 - \gamma^5) \psi_{\nu_e} \]
The weak four-current

- The inner combination of gamma-matrices looks almost like the projection operator used for the chirality representation of spinors, just a touch...

\[ \gamma^\mu (1 - \gamma^5) = \frac{1}{2} (1 + \gamma^5) \gamma^\mu (1 - \gamma^5) \]

\[ j_{(v)}^\mu = 2 \bar{\psi}_e \left( \frac{1 + \gamma^5}{2} \right) \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) \psi_{ve} = 2 (\bar{\psi}_e)_L \gamma^\mu (\psi_{ve})_L \]

- Now, what about the barionic current...? Well here we have some additional players – quarks!

- In principle, one can consider neutrons and protons and notice some rules regarding changing angular momenta and things, but it is much easier if we use quarks, that are \( \frac{1}{2} \) spin fermions that looks a lot like Dirac particles...

\[ j_{(b)}^\mu = g_V \bar{\psi}_u \gamma^\mu \psi_d + g_A \bar{\psi}_u \gamma^\mu \gamma^5 \psi_d \]
The weak four-current

- The complication, we need to deal with is now related to the fact that the quark states that couple to weak bosons are not "pure".
- This will be discussed in later lectures and is called quark mixing.
- That is quite different from the pure lepton states and requires to introduce a set of effective coupling constants describing probabilities of different \( qq \) transitions.
- Using spinors and covariant formalism we are able to prepare an elegant picture for all quarks.

\[
\begin{align*}
\text{"down" type quark} & \quad D \in \{d, s, b\} \\
\text{"up" type quark} & \quad U \in \{u, c, t\}
\end{align*}
\]

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

Coupling constant
The weak four-current

- Another spoiler alert! CP-violation

For CP-violation to occur we need:

- Massive quarks
- At least three generations of quarks
- Coupling constants must "somehow" be complex numbers!

\[ V_{ij} \neq V_{ij}^* \]
Summary

- So, what have we learned...?
- **C-parity** is very special, and affects not only a particle but has impact on the interaction the particle undergo.
- Special place belongs to **bi-linear forms** that can be created by combining spinors – in this way we can define 4-currents and describe interactions using current-current model.
- Returning to symmetries: first we have elegant **covariant notation** that is guaranteed to be invariant w.r.t. to Lorentz group and second we can combine currents to create various „**scalars**“ that, in turn, make the respective matrix elements invariant.
- The weak interactions are again very intriguing and have much more complicated structure than the QED.
- To accommodate all observed effects we need to assume that the WI is a mixture of vector and axial-vector currents.