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THE DYNAMIC RANGE OF AN ECG IN THE TIME-FREQUENCY DOMAIN USED FOR THE LOSSLESS SIGNAL COMPRESSION

Abstract

Time-frequency domain compression of an ECG is the new and still underestimated issue. This paper is devoted to the statistical analysis of the ECG' dynamic ranges on various aspects of time-frequency plane. The traditional approach to the signal values distribution, originally introduced by Huffman, is now studied for integer time-frequency coefficients obtained with use of reversible integer-to-integer wavelet transform. Obtained results determine limits of perfectly lossless compression ratios for an electrocardiogram. They are interesting for comparative purposes and further development of nearly lossless algorithms. The research had initially only experimental aims, but practical hardware implementation is also feasible.

1. INTRODUCTION

Time-frequency representation of an electrocardiogram is recently widely investigated, since with preserving the number of samples identical as in the original time-domain signal, it throws new light on the signal content. Certain features of the signal can be extracted much easier in the time-frequency domain making possible the mathematical derivation of corresponding diagnostic parameters. Similarly, dynamical parameters are represented in time-frequency plane differently than in time domain. These parameters are investigated during the research described in this paper. Main goal of the research was comparing the achievable compression effectiveness of the statistic-based perfectly lossless algorithms with those of nearly lossless based on pre-processed signal and assumed local electrocardiograms' properties [1], [2], [3]. However, the real time hardware implementation with use of a powerfull floating point signal processor is feasible.

As far as the perfectly lossless time-frequency domain compression algorithm is concerned, the choice of time-frequency transforms is limited to reversible ones. This condition follows from the requirement of data identity in both domains. No data loss or redundancy is allowable during the signal transform.

The second restriction on the time-frequency transform results from the traditional Huffman approach [5], assuming the not equal distribution of values in a finite-length

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values set. Although any finite-length data sets are allowed, this requirement is usually fulfilled by the use of integer values. This is the most straightforward and natural way in case of data originating from an analog-to-digital converter, but the time-frequency representation of a signal is mostly expressed in real format, and thus uses an infinite set of values. In order to preserve the perfect lossless property of the whole compression process, the time-frequency coefficient values cannot be simplified or in any way rounded to the integers. In consequence, the only way to preserve the identical values of original and de-compressed samples is the use of a wavelet transform that maps integers to integers [4].

2. MATERIALS AND METHODS

2.1. DESIGN OF THE EXPERIMENT

The numerical experiment was designed and carried out with use of Matlab environment and the raw time-frequency data was then transferred to Statistica. There were three aspects of testing the statistical properties of time-frequency signal representation:

a) dynamic ranges of raw time-frequency coefficients with reference to time-domain signal dynamic ranges,
b) dynamic ranges of differential signals (i. e. first derivative) in each frequency band – each frequency band is characterized by different sample number due to the variable time resolution involved by signal decimation by a factor of 2,
c) dynamic ranges of differential signals in temporal sections of signal – the differences are organized in a tree structure due to the variable time resolution involved by signal decimation by a factor of 2.

All three aspect of time-frequency plane insight are displayed in figure 1.

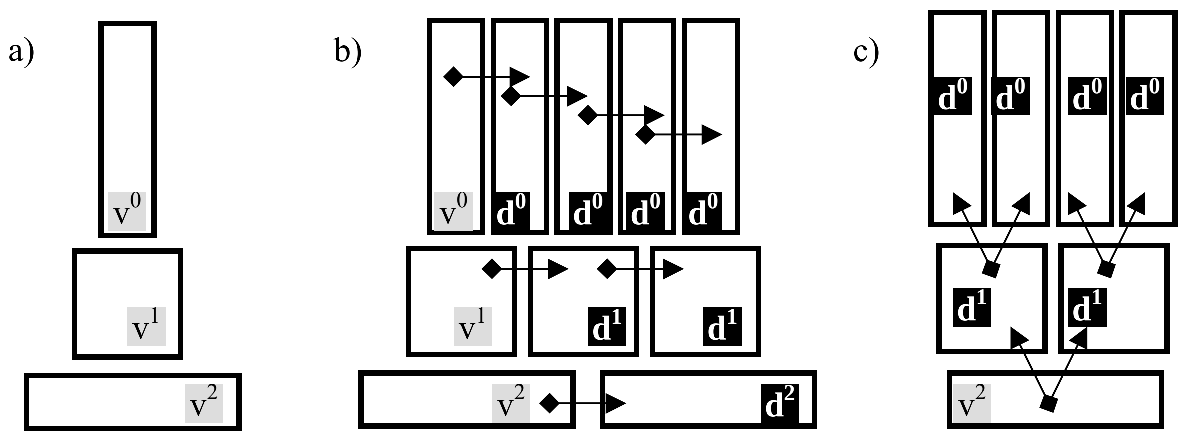

Figure 1. Three considered aspects of time frequency plane: a) raw values, b) differentiation by time – each frequency band is processed independently, c) differentiation by scale – based on the lowest frequency coefficient

2.2. THE SOURCE OF TEST SIGNALS

All test signals were really recorded ECG signals form the CSE Database [6]. The technical parameters of digitizing were:

– sampling frequency 500 Hz
– amplitude resolution 12 bits (2.44μV for Least Significant Bit).

Comparing to laboratory recorded electrocardiograms the use of a Standard Database has several advantages:

a) a wide variety of hearts' beats morphologies are represented in a database thus the processing skills of the algorithm under test is similar to the real scores in clinic – in laboratory the availability of patients with different diseases is limited,
b) the raw ECG data are accompanied by results issued by different processing software, so the average diagnostic results are not dependent on the quality of particular equipment,
c) the ECG recordings in a database were assessed by several cardiologist from over the world representing different approaches, but always best skilled,
d) the recordings of a Standard Database, such as CSE or MIT-BIH are identifiable by their number anywhere in the world and the experiment on data can be easily reconstructed by any laboratory,
e) the economical aspect is also not negligible, the database is not very cheap, but much more expensive would be organizing the clinical experiment on a comparable scale.

The CSE Database contains two sets of 125 signals: original, being directly recorded in 15 simultaneous derivatives (12-leads plus VCG) and artificial. The artificial set contains signals consisting of the same beat – being the most representative for corresponding original signal – repeated until a 10 second signal length is achieved.

For our experiment the most suitable signals were isolated hearts' beats synchronized in their maximum of 3-dimensional R wave fiducial point to the middle of 512 samples length decomposition section. The ECG segments were taken from the artificial set, in that way the baseline level and variability do not influence the decomposition and all border effects are far enough from the most interesting P-QRS-T section. All 15-lead signals (12-lead ECG and VCG) were considered separately for the experiment.

2.3. THE INTEGERS-TO-INTEGERS WAVELET TRANSFORM

Since the wavelet transform mapping integer to integer values is not supported by any known Matlab toolbox (even the third-party wavelet procedure set available for free in the Internet), we had to face up the problem of writing an appropriate procedure by ourselves. The need for such transform seems obvious and the great demand from the area of image processing depicts vide applicability. Nevertheless, only three reports were found during the bibliographical study. Finally, we decided to base our algorithm on the lifting scheme described in mathematical details in [4], so only main ideas are presented hereby. The advantage of this method is its relative simplicity from a programmer point of view. Detail investigations, and probably applications of other algorithms are worthwhile and considered for the research in the future.

Computing the wavelet transform using lifting steps consists of several stages (fig. 2). The key is to compute a trivial wavelet transform, also called Lazy Wavelet, and than to improve its properties using lifting and dual lifting alternately. The lifting operation means here increasing the number of vanishing moments of a wavelet without any changes of its properties.

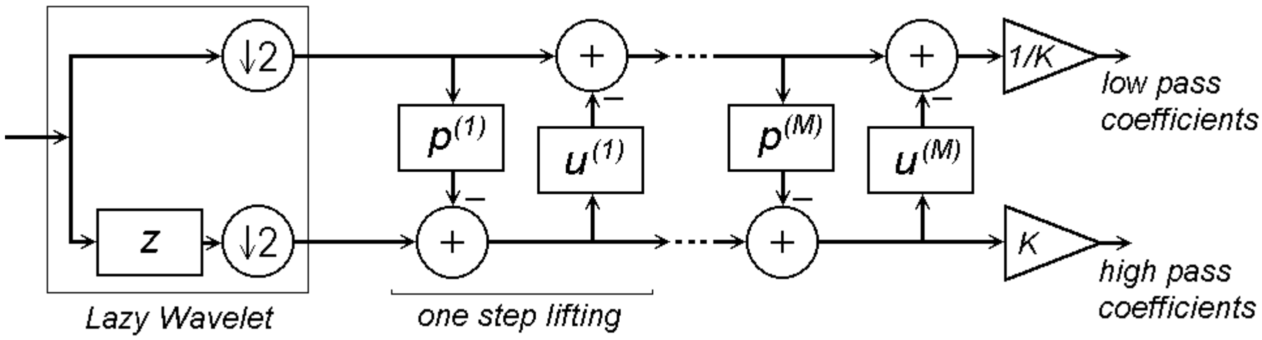

Figure 2. Computing the wavelet transform using lifting steps

The Lazy Wavelet only splits the signal into two strings:

$$
\begin{aligned}
s_{1,l}^{(0)} &= s_{1,2l} && \text{first, containing only even samples} \\
d_{1,l}^{(0)} &= s_{1,2l+1} && \text{second, containing only odd samples}
\end{aligned}
\tag{1}
$$

A dual lifting step consists of applying a low-pass integer filter p to the even samples and subtracting the results from the corresponding odd samples:

$$d_{1,l}^{(i)} = d_{1,l}^{(i-1)} - \sum_k p_k^{(i)} \cdot s_{1,l-k}^{(i-1)} \tag{2}$$

A primal lifting step, on the opposite, consists of applying a high-pass integer filter u to the odd samples and subtracting the results from the corresponding even samples:

$$s_{1,l}^{(i)} = s_{1,l}^{(i-1)} - \sum_k u_k^{(i)} \cdot d_{1,l-k}^{(i)} \tag{3}$$

After M lifting steps the even samples become the low-pass coefficients and the odd samples become high-pass coefficients, with applying the scaling coefficient K:

$$
\begin{aligned}
s_{1,l} &= \frac{1}{K} \cdot s_{1,l}^{(M)} \\
d_{1,l} &= K \cdot d_{1,l}^{(M)}
\end{aligned}
\tag{4}
$$

In our application, we used the simplest Haar filters for p and u. The first difference acts as high-pass filter, and the average acts as low-pass filter:

$$d_{1,l} = s_{0,2l+1} - s_{0,2l}$$
$$s_{1,l} = \frac{1}{2}(s_{0,2l} + s_{0,2l+1}) \quad (5)$$

It is worth a remark, that the lifting algorithm generates two subsampled strings: the decimated low-pass coarse signal and the detail high-pass signal, exactly like one decomposition step of a traditional wavelet transform does. The lifting is a reversible process, thus the resulting stings contain complete original information. Thanks to losslessness, the lifting corresponds to invertible wavelet decomposition. All operation can be performed in the integer format. The only doubt may concern the average, where truncation of the least significant bit is possible. Technically, this problem can be solved by rounding towards -∞ or +∞, depending on the condition whether the difference is even or odd, because the sum and difference of two integers may only both be even or odd.

3. RESULTS

3.1. RESULTS FOR RAW TIME-FREQUENCY COEFFICIENTS

For the comparative purpose, first displayed results are statistic and dynamic parameters of time-domain ECG signal. Two versions of each time-domain signal were considered: raw values and values differentiated by time. The appropriate histograms of values, bits per value, and main statistic properties are displayed in figure 3 and 4 respectively.

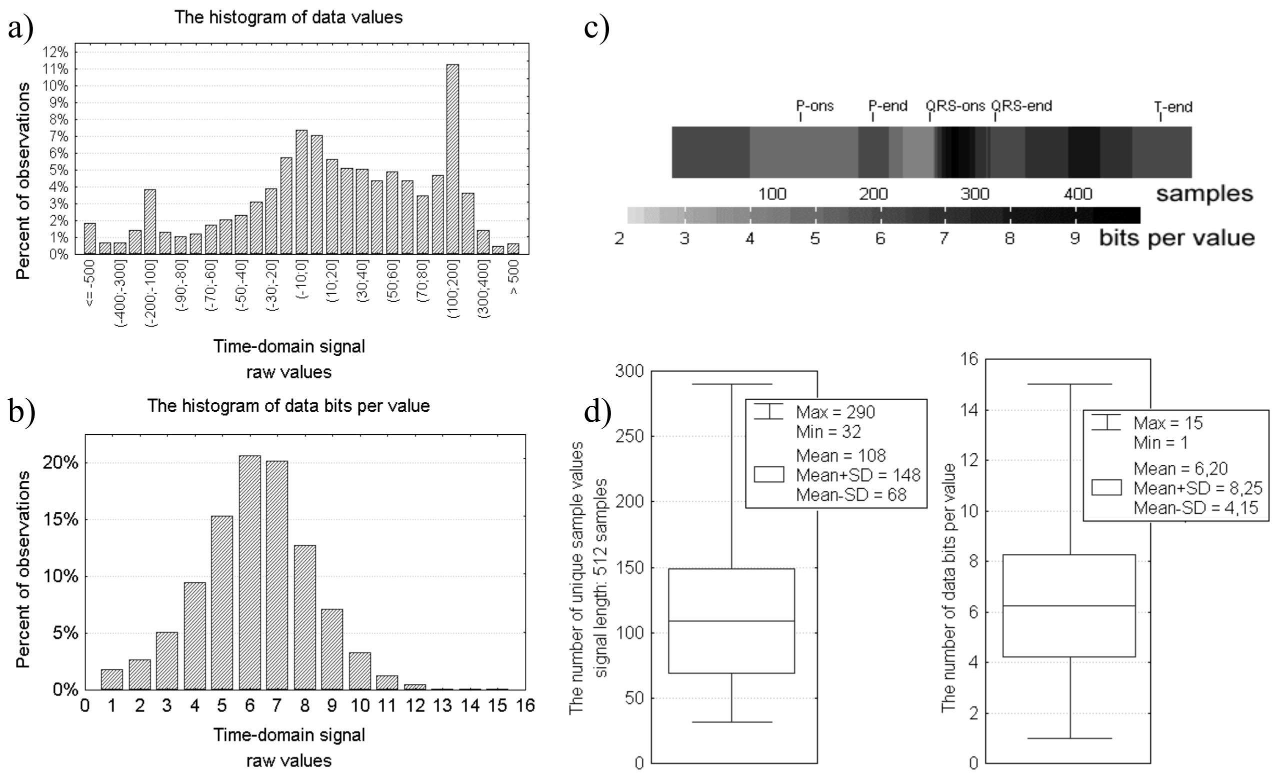

Figure 3. Results for time-domain raw data ECG signals: a) histogram of values, b) histogram of bits per value representation, c) bits per value density distribution d) unique values and bits per value statistic properties

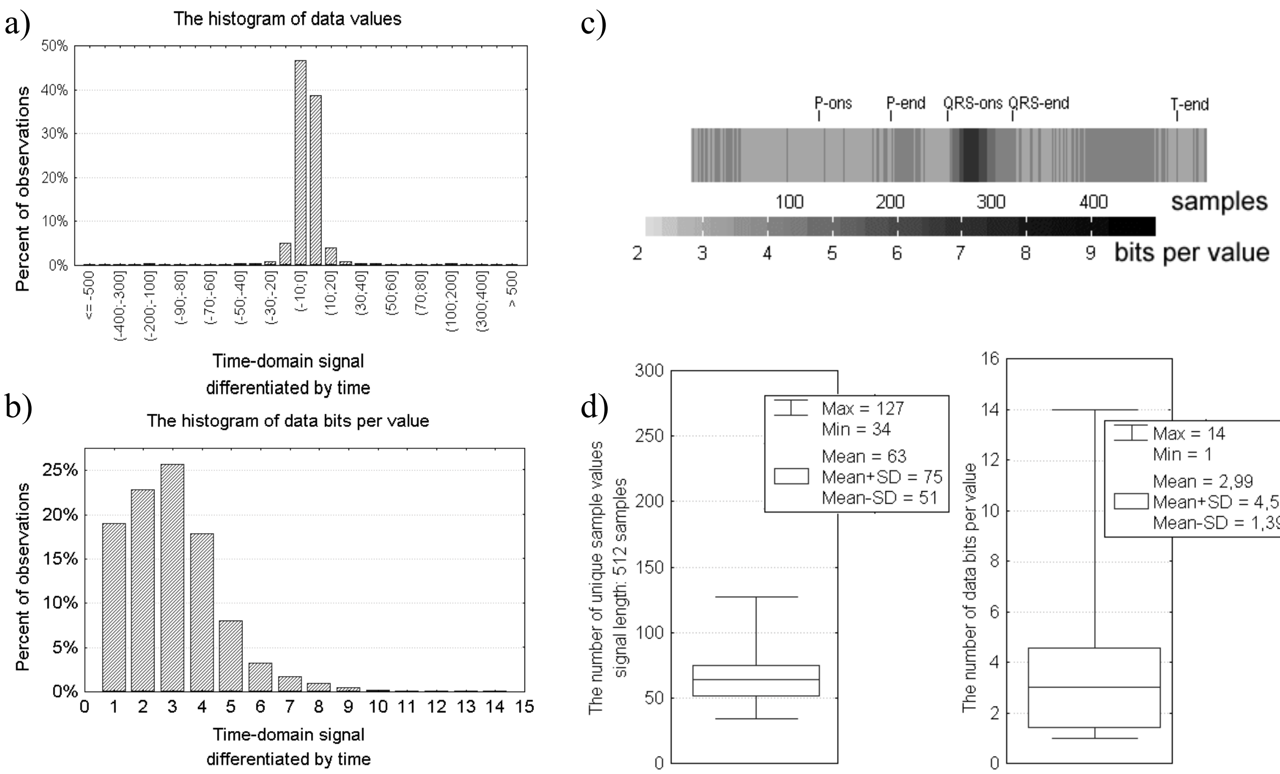

Figure 4. Results for time-domain ECG signals differentiated by time: a) histogram of values, b) histogram of bits per value representation, c) bits per value density distribution d) unique values and bits per value statistic properties

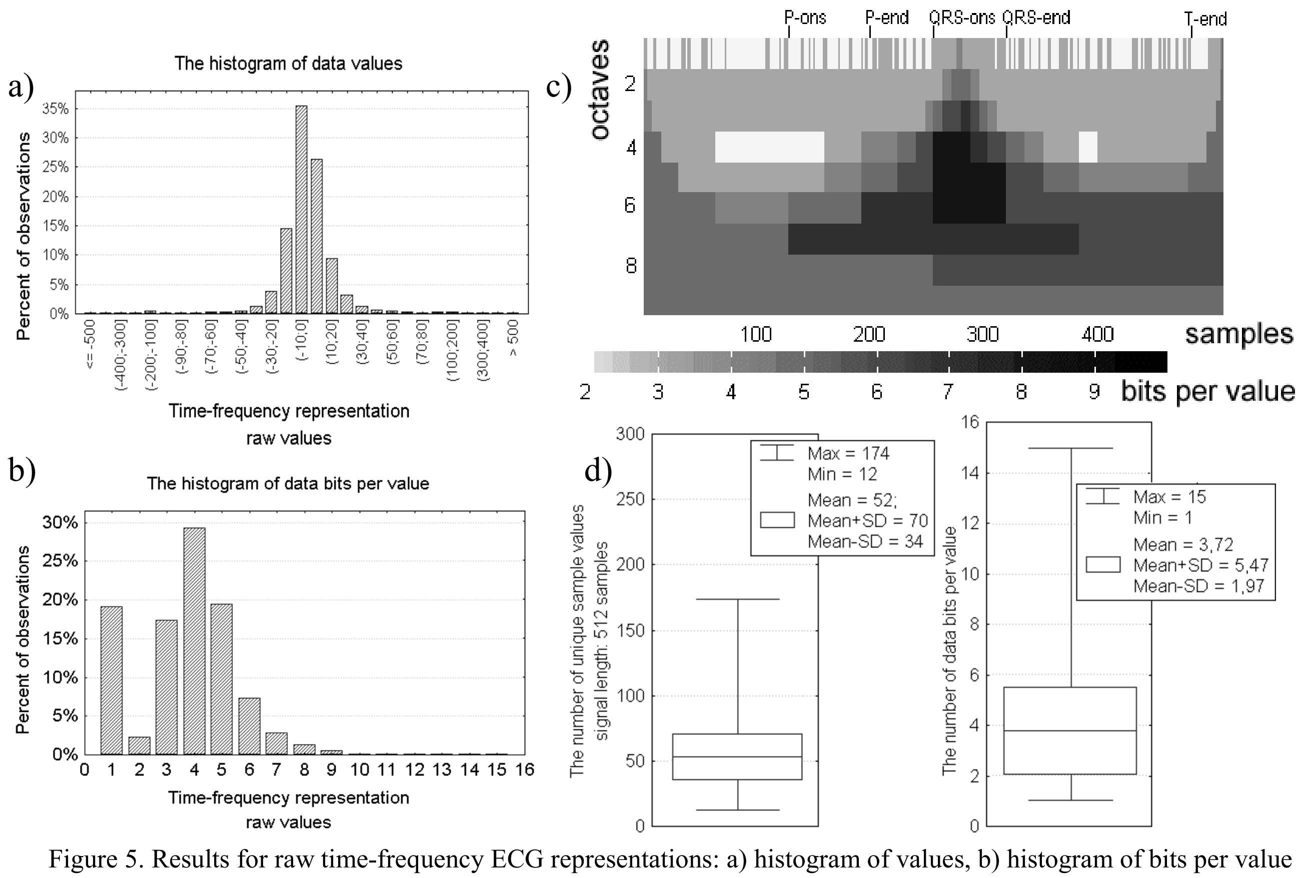

Figure 5. Results for raw time-frequency ECG representations: a) histogram of values, b) histogram of bits per value representation, c) bits per value density distribution d) unique values and bits per value statistic properties

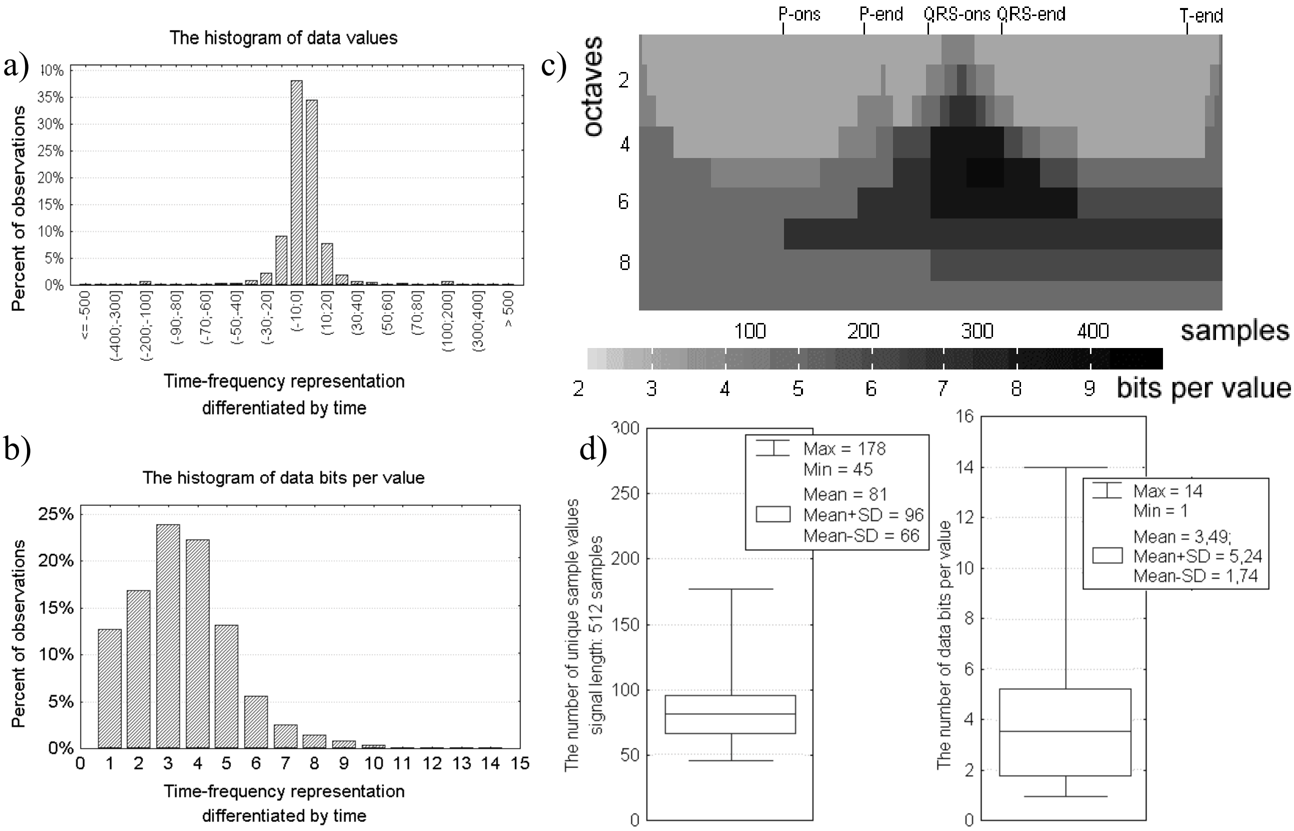

Figure 6. Results for time-frequency ECG representations differentiated by time: a) histogram of values, b) histogram of bits per value representation, c) bits per value density distribution d) unique values and bits per value statistic properties

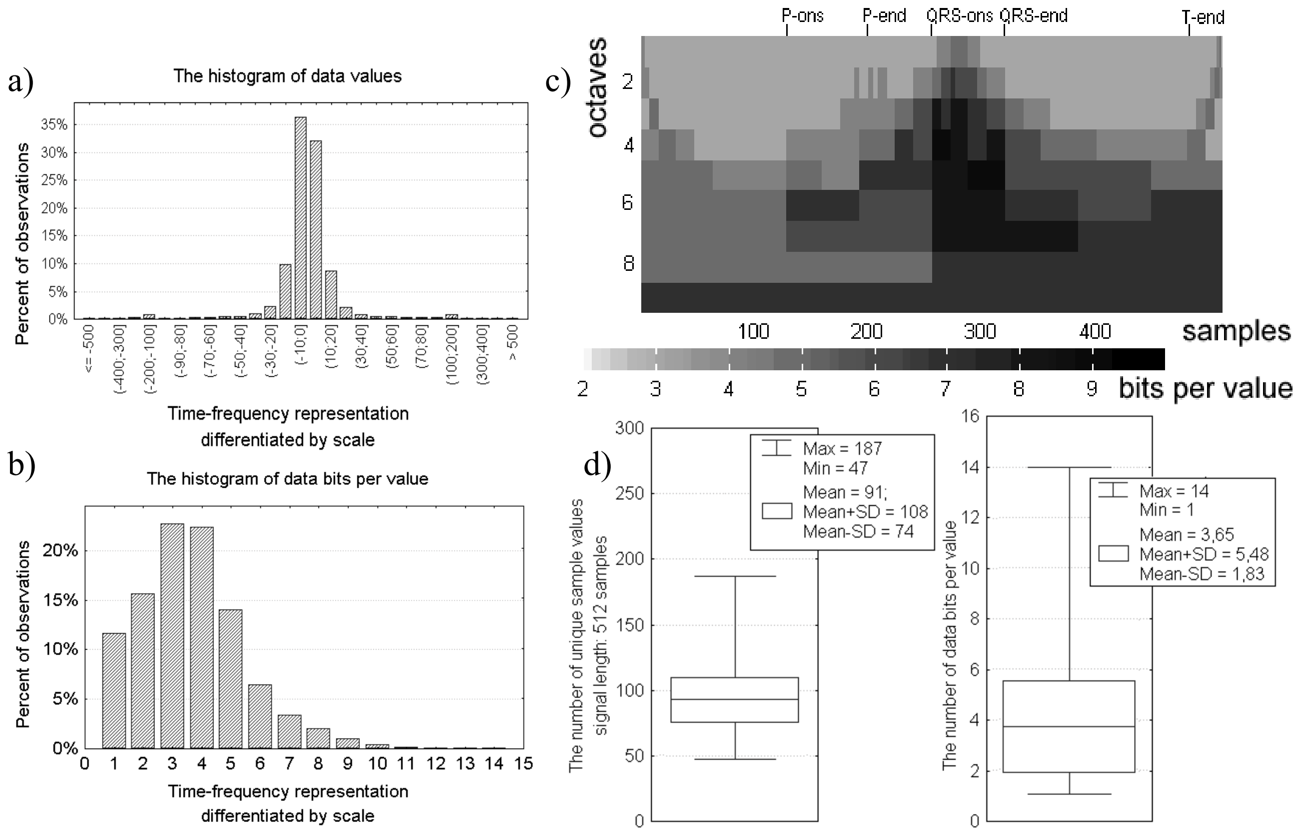

Figure 7. Results for time-frequency ECG representations differentiated by scale: a) histogram of values, b) histogram of bits per value representation, c) bits per value density distribution d) unique values and bits per value statistic properties

The results obtained for raw time-frequency coefficients' dynamics: histograms of values, bits per value, bits per value density distribution on the time-frequency plane as well as unique values and bits per value statistic properties are displayed in figure 5.

3.2. RESULTS FOR DIFFERENTIATED TIME-FREQUENCY COEFFICIENTS

The results obtained for time-frequency coefficients differentiated by time and by scale are displayed in figures 6 and 7 respectively. Like in figure 5, histograms of values, bits per value, bits per value density distribution on the time-frequency plane as well as unique values and bits per value statistic properties were computed for both differentiating approaches.

4. DISCUSSION

The expected average count of bits per value limits the theoretical lossless compression effectiveness. It is obvious that differentiated time-domain uses less of unique values of less dynamic range than the raw signal, and this property is widely used for coding and compression. In our experiment we prove the diminution of average count of unique values from 108 to 63 and the average count of bits per value from 6,20 to 2,99 when storing the differentiated instead of the raw values signal.

For raw time-frequency ECG representations (fig. 5), the average count of unique values is slightly lower than in case of time-domain differentiated signal, but the values used occupy in general highest number of bits in their representation. The look-up table or other coding technique is necessary to achieve the compression effectiveness comparable to those of time-domain differentiated signal. Theoretically expected compression ratio equals: $c = (512/52)*(12/3.72) = 31.7$; but extra bits are always necessary for correct reconstruction.

For time-frequency ECG representations, any differentiating technique does not improve significantly the expected compression ratio. Differentiating by time, however corresponds more to the natural signal smoothness, needs a reference point in every octave that creates the additional unique values. On the other hand, differentiating by scale show, that the dynamics correlation of frequency bands coefficients in a specified time point is not as strong as expected. That results in lower statistics parameters than time-differentiating.

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