Controlling the Distortions Distribution in a Wavelet Packet-Based ECG Compression

Piotr Augustyniak
AGH University of Technology, Institute of Automatics,
PL-30-059 Kraków, Mickiewicza 30, POLAND
e_mail: august@biocyb.ia.agh.edu.pl

Abstract

The paper presents a new approach to the ECG compression algorithms. The idea of non-equal density of medical information in the electrocardiogram yields the new design guidelines for a compression system. The distortions of reconstructed signal, considered as unavoidable result of data reduction in other traditional systems, are now subject to control. The temporal distribution of distortions, within the confines of a heart evolution, may be designed as signal-feature dependant, application-dependant or even controlled by the doctor. Being similar in technical aspect, the resulting compression algorithms differ in favoring a particular zone in the signal accordingly to the physiological expectations. The compression ratio, the usual key feature for assessment of the compression efficiency, gives priority to the signal quality that is indisputably essential for electrodiagnostic applications. Controlling the distortions temporal distribution is achieved in time-frequency domain by means of a "function of interest" containing information on the local diagnostic importance of the ECG. A wavelet packet-based compression algorithm is also proposed and tested in the paper.

1. Introduction

1.1. Motivation

The compression of electrocardiogram data is of great practical significance and is widely used in clinical practice. Electrocardiogram is the most frequently performed electrophysiological test, thanks to its accessibility but also because of high mortality of cardiovascular diseases that are implied by the common life style. Various ECG applications usually need the compression, among of them three are considered as most important: management of database for reference purpose, transmission of the ECG over telecommunication networks and ambulatory long term recording (Holter systems).
Several papers were devoted to reviewing and classifying of ECG-compression methods [1] [2], proving the attention received by this issue in scientific world. Detailed study of some of the proposed compression methods (particularly those using time-frequency signal representation [3] [4]) led to formulate the following general remarks:

A. Lossy and lossless methods. Existing methods are usually described as lossy or lossless. The lossless methods guarantee the identity of digital representations for original signal and its reconstructed copy. Despite the common belief, the lossless compression methods represent the continuous real signal only as close as it results from the digitizing parameters. The lossless compression is featured at a price of considerably lower compression efficiency and in many countries is the only legal way of medical data storage.

B. Medical interpretation of distortion. The compression efficiency is usually the main feature for the compression assessments. For lossy compression methods, the distortion coefficient is seen as the necessary evil being a monotone function of compression ratio. The commonly used distortion estimators (like Percent Root-mean-square Difference PRD) do not reflect the variability of signal importance in medical aspect, thus such a technical parameter is hardly interpretable in terms of medical diagnostability.

C. Signal-specific or general-purpose algorithms. The cyclic behavior of heart muscle represented in the structure of ECG offers a rarely spotted opportunity to relate the local density of diagnostic information to easy detectable points on the ECG curve. Some epochs in the hearts cycle are diagnostically more important because most informative to the doctor, than the remaining part (i. e. the ventricle’s contraction). In consequence, the corresponding sections in an ECG have to be handle with special care, while the others may be simplified without altering the medical interpretation of data. For the lack of expressed medical knowledge on local ECG signal importance, this time function is to be assumed in an application-specific way.

Considering all the remarks above, we found reasonable to take a new position facing to the compression issue, and to propose new designing guidelines, favorable to the signal quality and to the maximum reproducibility of medical data. Instead of hesitating in applying a lossy or lossless compression, we prompt the doctor to manage of distortions temporal distribution. In consequence, distortions concentrate in the zones indicated as less important, while the zones of particular interest remain unaffected. Our method joins the advantage of high efficiency typical to the lossy compression (it is a lossy compression indeed) and the control on the medical aspects of the signal. Additionally, in medical aspect, the local distortion tolerance borders or signal importance function are easier to interpret and manage than the global distortion estimate.

1.2. Aspects of local ECG importance

The medical knowledge about the importance of particular sections in the ECG signal is not easy to express. Despite the lack of guidelines, we carry on the experiments and polling with doctors investigating the process of examining an electrocardiogram. Although every doctor looks at the ECG traces differently, the preliminary results are quite converging to the following conclusions:

- A huge percentage of ECG examination is made with a goal to answer a particular question, and only few are “general purpose” recordings with no presumptions. The fashion of ECG interpretation depends on the a priori knowledge about the patient health. In consequence, the local importance of an ECG is patient-specific.
- Accordingly to the medical goal of an examination, the specialized recording equipment is used. Stress-test systems, long term Holter recorders and bedside cardiomonitors are expected to comply with different requirements and to issue different kind of information from the ECG. The function of local importance of an ECG depends in some way on the application it is implemented in.
- The doctors’ experience, in spite of tendencies for “normalization”, plays an important role in assessing of the ECG trace. Cardiologists, gathering their experience over the years of work, become more or less specialized in diagnoses and treatment of particular heart diseases. In that aspect, the local importance of an ECG is also doctor-specific.

The above presented different aspects of local signal importance yields the conclusion that a unified approach to the local data density in the ECG should not be expected soon. The custom-defined importance function, however, may be successfully applied instead.

2. Deriving an importance function for a 12-lead recorder

2.1. Vulnerability of the diagnostic parameters to the signal distortion

To estimate the importance function for a 12-lead recorder we studied first the diagnostic parameters issued by that recorder and they vulnerability to distortions. The appropriate discussion is summarized in Table 1.

2.2. Estimating the global quality of ECG diagnostic parameters

The global estimator of ECG diagnostic parameters’ quality bases on the values that are important in electrodiagnostic aspect and that are monotonically less precise when the data stream decreases. As the most reasonable, the positioning of P-onset, P-end, QRS-onset, QRS-end and T-end segmentation points are selected to contribute in the global estimator value. They are medically very meaningful, because of representing the function of stimulus conduction, and are not fully recoverable by mathematical tools if lost in the ECG signal.

To compute the individual contribution of each positioning difference to the global parameters’ quality estimator, we processed the CSE guidelines for the expected precision of the segmentation points positioning given in the Table 2, accordingly to the formulae (1).

The maximal values of difference recommended by CSE *dl* and the contribution coefficients of each point are summarized in the table 2. The formulae (2) defines the global estimator of diagnostic parameters' quality *D*.

Table 1. Discussion on the vulnerability of ECG diagnostic parameters to signal distortion

parameter	diagnostic meaning	vulnerability to the signal quality
precision of the R-wave peak positioning	The fundamental parameter for Heart Rate (HR) and Heart Rate Variability (HRV) computation. Commonly used also for arrhythmia, premature beat detection and many others.	The correct position of the R-wave peak may result from the parabola fitting to the sparse data. The lost of data can be compensated with the precision greater than sampling interval.
precision of the segmentation points positioning	Basic parameters for computation of all temporal relationship between the cardiac events. The precise segmentation is the key of assessing the correctness of functionality of the heart conducting system	For the complexity of the phenomena, the wave borders mathematical models are not precise enough for diagnostic purposes, hence the data loss cannot be recovered with use of mathematical tools
measure of Ventricular Late Potentials (VLP)	Ventricular Late Potentials represent the susceptibility of ventricular muscle fibrils to the spontaneous contraction out of the control of the heart conductive system. Life-critical if failed in detection.	Limiting the signal bandwidth below a value of 250 Hz causes the detection of VLP not possible
measurement of the level and the slope of ST segment (ischemia)	Depression or elevation of the ST segment, exceeding given limits, are considered as symptoms of ischemia, the most frequent disease of the cardiovascular system with the highest mortality in the developed countries. Risk factors are stress, physical overload, overweight and bad diet.	The correct assessment of the ST-segment changes is conditioned by correct acquisition of low frequency components of the ECG signal. The data reduction in the high frequency range do not affects the performance of ST-segment measurement

Table 2. CSE recommendations on precision of P-QRS-T segmentation and the calculated weight coefficients.

i	**1** **P-onset**	**2** **P-end**	**3** **QRS-onset**	**4** **QRS-end**	**5** **T-end**
maximum accepted difference [ms] dl_i	8.0	7.8	4.4	5.8	15.6
calculated weight coefficient w_i	0.174	0.179	0.317	0.240	0.090

$$w_i = \frac{\frac{1}{dl_i}}{\sum_{i=1}^{5} \frac{1}{dl_i}} \tag{1}$$

$$D = d_{P-onset} \cdot w_1 + d_{P-end} \cdot w_2 + d_{QRS-onset} \cdot w_3 + d_{QRS-end} \cdot w_4 + d_{T-end} \cdot w_5 \tag{2}$$

2.3. Testing the estimator of signal quality

The principal assumption on the proposed quality estimator is that the parameters' quality is expected to monotonic decrease with decrease of data rate. We tested the signal quality for a set of 125 12-lead signals from the CSE Multilead Database [5]. The segmentation designed for 500 Hz/12bit signals was obtained as an executable file from a commercial equipment manufacturer. The segmentation was performed for a whole set of signals and compared with the reference points provided by the database. In the subsequent steps, the signal bandwidth was truncated with a low pass filter (FIR, 12 dB/oct) to 200, 150, 100 and 50 Hz. For each version of truncated bandwidth data, segmentation was performed by the same subroutine and compared with the reference. Figure 1 displays the mean inaccuracy of P-onset, P-end, QRS-onset, QRS-end and T-end segmentation points positioning for the original and truncated bandwidth data. The value of global diagnostic parameters' quality *D* is displayed as well.

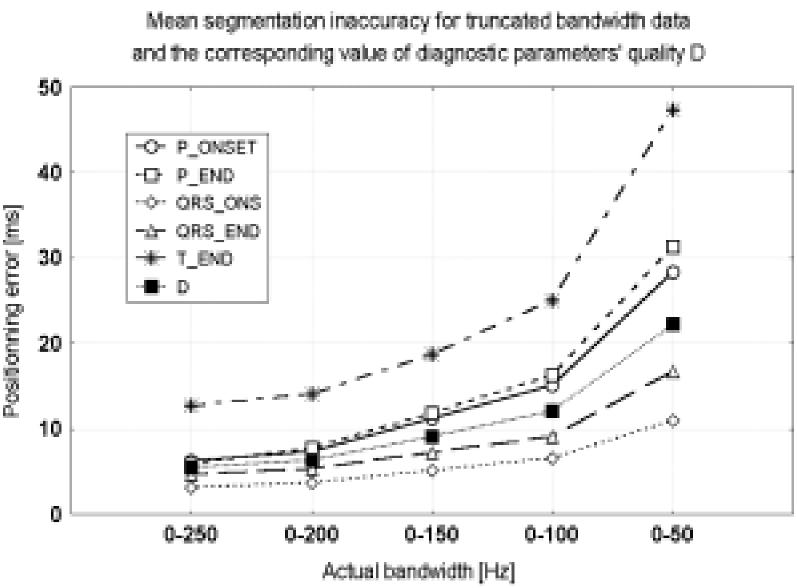

Figure 1. Mean inaccuracy of segmentation points positionning for the original and truncated bandwidth data and the corresponding value of global diagnostic parameters' quality D

The results of testing global diagnostic parameters' quality *D* with the truncated bandwidth signal set support the further use of *D* as reliable representation of data quality. Obtained in an additional test value of correlation (Pearson) between *D* and the bandwidth 91,7 % prove the close relation of *D* and the bit rate.

2.4. Computing the importance function from the parameters quality loss in distorted signal

Another numerical experiment was designed and carried out aiming at estimating the average ECG importance function. The experiment was expected to proof that removing a precise amount of information causes different results, in meaning of affecting the diagnostic parameters' quality, depending on the part of the heart's evolution being subject to the removal. Additionally, moving the removal window along the time in the heart evolution indicates the zones in the ECG signal where the resulting diagnostic parameters are more susceptible to distortions than in the surroundings. Aiming at maintaining the diagnostic parameters as less affected as possible (in terms of maximum value of diagnostic parameters' quality *D*) during the signal data reduction, the importance function should have the values proportional to the local signal vulnerability to distortion. The data reduction will then be subtler on the zones described as particularly vulnerable.

A. Experimental data set. Like in the tests of the global diagnostic parameters' quality estimator *D*, for a set of 125 12-lead signals from the CSE Multilead Database [5] was used for testing the average local vulnerability to distortions. Thirty-seven signals containing ventricular, pacemaker stimulated and low heart rate beats were discarded from the test signal set. The segmentation was performed by the previously described external procedure for a whole set of signals and compared with the reference points provided by the database. In the subsequent steps, each signal was first affected in the time-frequency domain, by removal of 6 randomly selected coefficients out of 14 coefficients in the octaves 1-3 (32...250 Hz) and time window of 32 ms. Five trials of removal were performed for each of 89 signals and for each of 27 window's position.

B. Time-frequency transform. To represent the ECG signal equivalently in the time and the time-frequency domains, the wavelet transform have to meet the following conditions:

- The perfect reconstruction property – in order to guarantee that every changes in the time domain signal are due to the coefficients removal in the time-frequency domain,
- The compact and relative short support – to represent the heart beat components in a temporal selective way,
- Good separation of adjacent frequency octaves to maintain the frequency bands crosstalk as low as possible.

Our final choice was the Daubechies 5-th order compactly supported wavelet (fig. 2) [6].

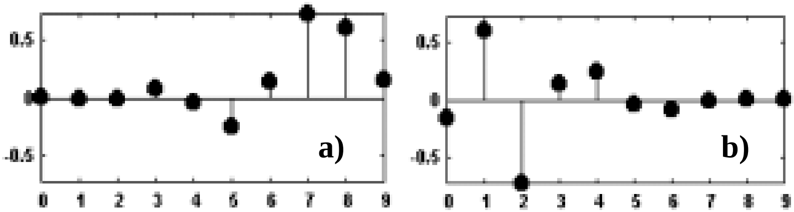

Figure 2. Coefficients of 5-th order Daubechies compactly supported wavelet filters a) lowpass filter, b) highpass filter

C. Eliminating time-frequency coefficients in the sliding window. The time-frequency window contains 14 coefficients for three upper octaves and 32 milliseconds over the heartbeat's representation. From those 6 randomly selected coefficients are set to 0 (eliminated). The altered signal is then reconstructed and processed for segmentation. The values of the difference for P-onset, P-end, QRS-onset, QRS-end and T-end are calculated with respect to the references from the database. Finally, the global diagnostic parameter's quality estimator *D* was computed accordingly to (2). For each particular time-frequency window's position the *D* value was averaged over the five trials of random canceling, over all 89 test signals and over all 12 traces in each signal.

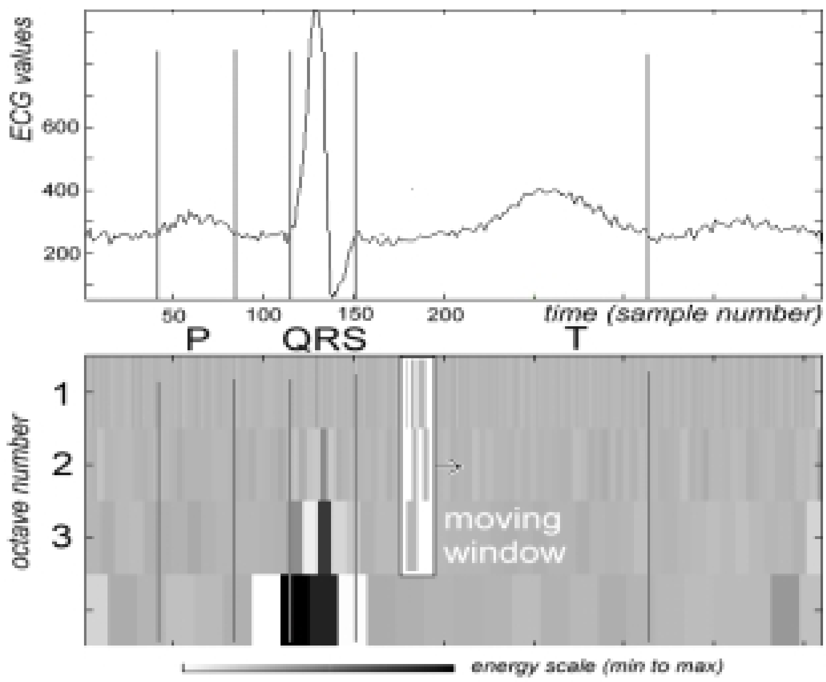

Figure 3. The principle of cancelling the time-frequency coefficients randomly selected in a moving window: octaves 1-3 (32...250 Hz), time span 32 ms

Next the processing was repeated for 27 subsequent time positions of the window, that covers the average length of the heartbeat of 864 ms (fig. 3). Each step resulted in the average *D* estimator value appropriate for the window's position with reference to the segmentation points. Despite the amount of information lost are always the same (6 time-frequency samples), the global quality of medical data, expressed by the value of *D*, changes in function of the time zone position where the removal was performed (fig. 4). That proves the temporally non-equal vulnerability of medical data to the ECG signal distortion. Additionally, the function of local vulnerability *D(t)* quantitatively assess the extend, to that the diagnostic parameters' quality is affected for each position of the distortions zone.

3. Designing an application dependent compression algorithm

3.1. The Wavelet Packet signal compression

Wavelet packets are an extension of wavelets concept. The orthogonal decomposition of the time-domain signal splits its bandwidth into two halfband components: the step-down signal approximation and the detail signal. During the subsequent decomposition step, not only the approximation branch, but also the detail signal, are subject to band splitting, and hence the wavelet packets use the full binary tree decomposition scheme instead of the pyramid decomposition. A signal of length $N = 2^L$ can be expanded in 2^N different ways, making the collection of possible representations very large.

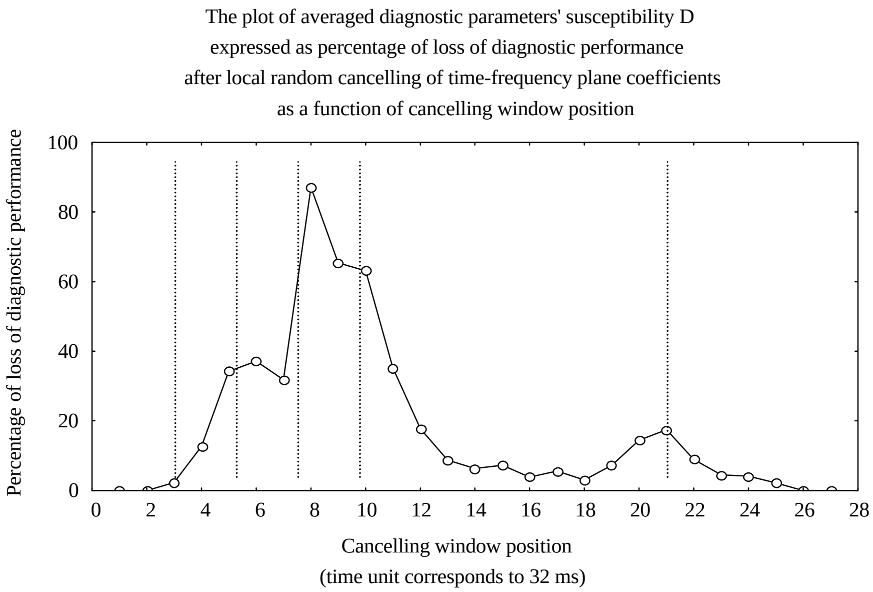

Figure 4. The function of diagnostic parameters' susceptibility to distortion caused by local random canceling of time-frequency coefficients. Additionally, average wave borders are marked.

Aiming at the best result of the compression, there is an interest to find the optimal representation that best preserves the features contained in the signal. At each node of the decomposition tree a decision is to be made, whether the band splitting is interesting. For efficient searching of binary-tree structure, the entropy-like cost functions are commonly used. The compression is achieved by minimizing the value of the cost function at each node. In consequence, some branches of the binary tree are pruned, and the remaining structure, adapted to the time-frequency properties of the analyzed signal, is called the best tree. It contains the most relevant time-frequency atoms of the signal, but some information is lost during the optimization, hence the compression is lossy.

All the cost functions reported in the literature, and also in the papers on wavelet-packet based ECG compression use the values of the wavelet packet coefficients s_i to compute the cost function:

$$E = \sum_i P(s_i) \qquad (3)$$

where P is a functional operator like logarithm, square, absolute value and so forth. The computation of the cost function proceeds identically for all the coefficients with no respect to their temporal position i.

3.2. Choice of the mother wavelet

Since the time-frequency domain coefficients of the compressed signal are to be stored in a fixed-point (integer) format, we considered the use of a transform that maps integers to integers (i.e. Lifting Wavelet Transform LWT [7]). The use of integer decomposition is possible in wavelet packet application as well. It has serious advantages in the aspect of compression:

- Avoiding the time-frequency coefficients round-off error and thus truly lossless processing,
- Advantages for fixed-point DSP implementation and processing speed-up.

After detailed tests of the LWT time-frequency properties and taking into consideration that the best tree-based compression is in principle lossy procedure, we decided to apply the "conservative" Daubechies 5-th order compactly supported wavelet (fig. 2). The reason for being less inventive was to focus the attention on the main topic and to highlighting the advantage of temporal distortion's distribution. For the same, we intentionally avoided the use of the superimposed compression rules as those reported in [3] (residual coding + wavelet packet + Karhunen-Loeve Transform), even at the price of inferior compression ratio.

3.3. Modulating the compression with the ECG importance function

Modulating the local compression parameters and, in result, controlling the temporal distortion's distribution is the main novelty of our approach. In our application the previously derived importance function was used as a source of modulation, since we found it reasonable for a stand-alone 12-lead recorder. It is to point out, however, that any other modulating sequence related to the temporal representation of a heartbeat may be applied alternatively.

Modulating the compression parameters consists in modifying the cost function computation rules, with use of weighting coefficients provided by the importance function. The time-frequency representation contains true values that may be used for decompression without any additional modifiers. The cost function, used for the best tree lookup, have to respect the temporal position i expressed by the weighting coefficient w_i (4).

$$E = \sum_i P(s_i \cdot w_i) \tag{4}$$

The string of consecutive values of w_i is computed for the segmented heartbeat signal. For the convenience of dyadic wavelet decomposition its length is expected to be an integer power of 2. For the lack of assumption on the importance function sampling, it is necessary to calculate each w_i value with respect to its temporal distance to the segmentation points. The interpolation has to consider the temporal variability of each section of a heartbeat. The temporal re-scaling of each section length in the importance function to the actual section length in the segmented signal is used to fit all the waves start- and endpoints.

The weighting coefficient's string, computed for the first decomposition level, has to be adapted for the use at the lower nodes of the decomposition tree. Using the dyadic scheme wavelet decomposition simplifies this adaptation – the only necessary processing is the subsequent decimation of weighting coefficients string.

4. Testing the compression algorithm

4.1. Description of the numerical experiment

For testing, the whole compression algorithm was coded in Matlab 5.2. All waves start- and endpoints are provided in the CSE Multilead Database and the segmentation procedure is no longer necessary. All available ECG traces were subject for compression as if they were separate signals. All the traces belonging to the same recording have common segmentation points, and thus the importance function and weighting coefficients string are calculated once per recording. Two recordings containing pacemaker stimulated beats and five recordings with low heart rate were excluded from the test signal set. The remaining 118 beats are of supraventricular and ventricular origin.

A. Extracting the ECG signal. First step of processing is the normalization of the signal length. The most reasonable value is 512 points and considers the condition implied by the dyadic decomposition scheme and the sampling rate of 500 Hz. No padding was applied for shorter beats, after the beat is centered to the half of desired length, the original ECG values were picked from the trace while necessary. The values altered by the border effect are thus far from the P-QRS-T waves.

B. Calculating the weighting coefficients string. Since the segmentation points do not have to be computed when using the CSE Database, the algorithm proceeds directly to the temporal re-scaling of the importance function. For each interval in the heart evolution, the length expressed in importance function samples is compared with the actual interval length. The importance function is then segmentary resampled using the appropriate higher sampling points. For the computation simplicity the linear interpolation was used. The last operation is the normalization of values in the weighting coefficient string. In result the average of all coefficients equals to 1.

C. Searching for the best tree. The wavelet packet decomposition uses the Daubechies 5-th order compactly supported wavelet and the searching for the best tree is performed using the modified function of logarithm of the energy entropy (5)

$$E = \sum_i \log(s_i^2 \cdot w_i) \tag{5}$$

where w_i are elements of weighting coefficients string.

All the nodes at the given decomposition level use the same version of weighting coefficients string. When proceeding to a lower level, all the signal representations are decimated since they contain a half of the previous bandwidth. In order to maintain the temporal compatibility, the decimation is applied to the weighting coefficients string as well.

D. Storage and retrieval of the ECG. The essential components of the compressed signal are:

- Best tree values,
- Position parameters of the best tree components,
- Synchronization and scale parameters.

Following the guidelines given in [3], the positioning of the best tree nodes is encoded in a 16-bit value. The values of decomposition components, expressed in floating-point format in consequence of applying the real-valued wavelet transform and weighting coefficients, are subject to quantization and, similarly to the original signal, are represented in 12 bit integer. The quantization is a source of additional data loss but, considering that the prevalence of the local signal energy is represented in best tree values, the estimated error is comparable with the error of quantization during the analog to digital conversion. Thanks to additional amplitude scaling, the best tree coefficients always use the full dynamic range of the 12 bit coding.

The decompression procedure is very straightforward. The signal samples are retrieved from the best tree values and the nodes position parameters with use of the inverse wavelet transform. The amplitude re-scaling and the temporal synchronization of the heartbeat end the decompression.

4.2. Results for compression efficiency

As it was mentioned before, the compression efficiency is not the most important parameter of our approach, because it is a result of the distortion's distribution strategy. However, for respecting the common habit, the compression ratio results and the global PRD distortion measure are presented at first.

All the traces for each particular file were processed separately and thus the results of efficiency and error, displayed in the Table 3, are averaged over all traces in the recording.

Table 3. Results for CSE Database files

CSE-ID	compression ratio	total distortion PRD [%]
1	4.6390	6.4871
2	7.9102	3.5952
3	6.3116	4.6818
4	5.0881	6.1332
123	6.5532	5.0970
124	4.9714	5.8661
125	5.8650	4.7592
mean value	**5.9006**	**5.1711**

4.3. Results for distortions distribution

The comprehensive presentation of general results for the temporal distortion's distribution is quite difficult. The distortion's distribution is a time-domain function that is expected to show a correlation to the ECG segmentation points. For each signal, however, the segmentation points are set individually with respect to the detected signal features. In order to collect the corresponding information on error distribution from various signals, the appropriate segmentary resampling of individual difference function must be used to match the positions of the segmentation points. Additionally, there is no solution suitable for the ventricular beats without the P-wave.

Although all the traces are considered as separate ECG signals for processing, the 12 components of a particular recording have common segmentation points. In consequence, the temporal distortion's distribution may be displayed for each recording individually as the average absolute difference of the original and decompressed signal. The distribution plots were studied for all 118 files, but for the lack of space only two examples are given here (fig. 5).

The segmentary values of PRD are summarized in the Table 4. The additional parameter is the correlation coefficient R (Pearson) of the distortion percentage and the values of the weighting coefficient string at the zero level. The data in the table 4 are also averaged over all 12 traces in each file.

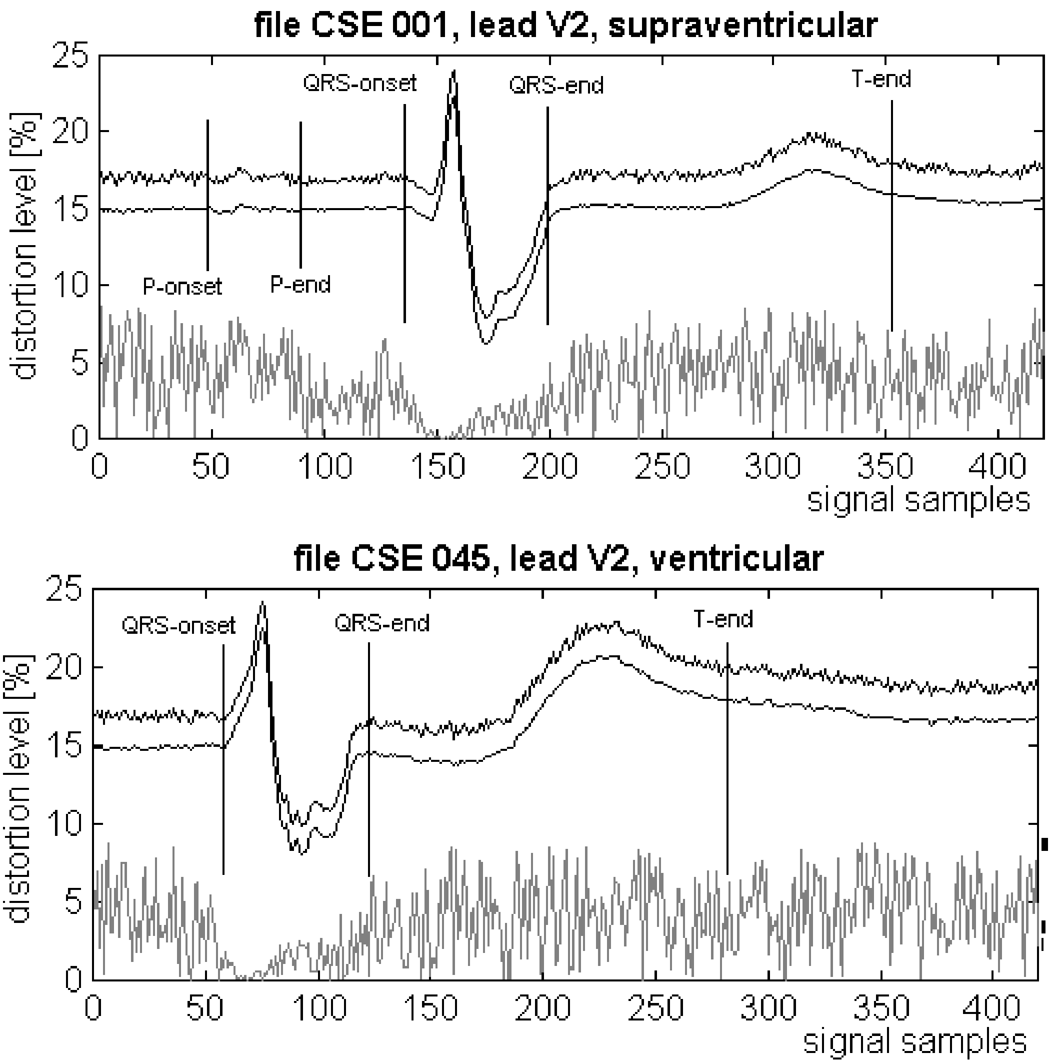

Figure 5. Temporal distortions distribution averaged for all 12 traces of selected file along with the lead V2 reconstructed (the top-first) and original (the top-second) traces. Note that the vertical axes' values apply to distortion plots only.

Table 4. Results for CSE Database files (distortions in %)

CSE-ID	compression ratio	total distortion	P-wave distortion	QRS-wave distortion	T-wave distortion	extra-wave distortion	correlation R
1	4.6390	6.4871	0.8756	0.2335	0.9770	4.4010	-0.78
2	7.9102	3.5952	0.7901	0.2195	0.7317	1.8539	-0.84
45	5.9507	5.2612	-	0.3144	0.9810	3.9658	-0.81
124	4.9714	5.8661	1.0644	0.4177	0.8760	3.5080	-0.91
125	5.8650	4.7592	0.9817	0.2914	0.8426	2.6435	-0.86
mean value	**5.9006**	**5.1711**	**0.9457**	**0.3098**	**0.8911**	**3.2782**	**-0.86**

5. Concluding remarks

The main goal of the research described in this paper was to explore the possibility of controlling the distortion in the wavelet packed-based ECG signal compression. Distortions, being unavoidable in lossy but efficient compression algorithms, no longer appear as phenomena of unknown behavior described with use of a meaningless global parameter. From now, the distortions may be put away from the most important zones of the ECG accordingly to the user-definable rules.

Resulting from the efficiency test compression ratio of **5.9** is not outstanding comparing to other published methods. Some of them show significantly higher values of compression ratios, particularly when a superposition of transforms is used. The global PRD value, equal **5.17 %** is fairly low but even for this order of distortion level, significant alterations of medical findings are reported by other authors [3], [8]. The global distortion coefficient does not express the temporal distortion concentration, so it is inadequate to express the main advantage of our algorithm.

More interesting are results for distortion's distribution. A visual inspection of the figure 5 gives an idea on the local distortion's concentration with respect to the medical events detected in the electrocardiogram. The zones defined as more important, that is P, QRS and T-waves are less affected than the remaining part of the signal. Also the results of the segmentary PRD value, being the quantitative comparison of distortion level, led to the similar conclusions. Another very meaningful parameter is the correlation of the values of the weighting coefficient's string at the zero level with the local distortion's amplitude. The value of correlation express the extend, to that the temporal distortion's distribution matches to the local vulnerability of diagnostic parameters quality. The average value of **-0.86** means that the local distortion level values are almost inversely proportional to the signal importance. As it were expected, the distortions are concentrated in less important sections of the signal.

In principle, the future user is practically unlimited in defining the own importance function. Suppressing distortions in a particular section causes the proportional increase of distortions in the neighboring parts of the signal, so the global distortion's level remains unchanged. The local distortion level may be suppress to the very low values, but expecting a segmentary lossless compression is obstructed by the use of the floating-point signal representation in time-frequency domain. A truly lossless compression is possible by replacing the Daubechies QMF decomposition filters by the Lifting Wavelet Transform.

The algorithm was not optimized for speed or computing efficiency. We believe that despite of using the floating-point representation and the Fourier Transform, the real-time application is feasible with use of modern DSP chip.

5. Acknowledgement

This work is supported by Polish State Committee for Scientific Research grant no: 8 T11E 007 17

6. References

[1] JALALEDDINE S. M. S., HUTCHENS C. G., STRATTAN R. D., COBERLY W. A. “ECG data compression techniques – a unified approach” IEEE Trans. Biomed. Eng. vol. 37 pp 329-343, 1990

[2] CARDENAS-BARRERA J. L., LORENZO-GINORI J. V. "Mean Shape Vector Quantizer for ECG Signal Compression" IEEE Trans. Biomed. Eng. vol. 46 pp 62-70, 1999

[3] BRADIE B. "Wavelet Packet-Based Compression of Single Lead ECG" IEEE Trans. Biomed. Eng. vol. 43 pp 493-501, 1996

[4] HILTON M. "Wavelet and Wavelet Packet Compression of Eletrocardiograms" IEEE Trans. Biomed. Eng. vol. 44 pp 394-402, 1997

[5] WILLEMS J. L. "Common Standards for Quantitative Electrocardiography" 10-th CSE Progress Report, 1990. Leuven: ACCO publ., 1990, 384p.

[6] DAUBECHIES I. "Ten lectures on wavelets" CBMS-NSF conference series in applied mathematics. SIAM Ed, 1992

[7] CALDERBANK A. R., DAUBECHIES I., SWELDENS W. and YEO B. "Wavelet transforms that map integers to integers", technical report, Princetown Univ., 1996

[8] ZIGEL Y., COHEN A., KATZ A. "A Diagnostic Meaningful Distortion Measure for ECG Compression" in proc. 19^{th} Conv. of Electrical & Electronic Eng. In Israel, pp. 117-120 1996