QUANTIZATION ERROR FOR VECTOCARDIOGRAM SPHERICAL COORDINATES

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Abstract
The vectocardiography (VCG) is the methodological extension of electrocardiography (ECG) that allows three-dimensional imaging of the cardiac electrical field. For the lack of the sufficient technological support it was underestimated for years, and currently comes back to the clinical practice thanks to the use of numerically performed spatial transforms. The VCG signal is usually acquired with use of the pseudoorthogonal Frank leads and stored as three simultaneous signals corresponding to three Cartesian coordinates XYZ. This paper addresses the issue of the alternative format for the VCG storage: the Spherical Coordinates. The Spherical Transform is fully reversible in theory, but due to the use of floating-point functions the subsequent quantization affects the perfect reconstruction property. The most important question investigated by the author is the signal distortion level as a function of the wordlength used. With use of a numerical experiment, the reconstruction error was estimated globally for the whole heartbeat and locally in P, QRS and T waves. As it were expected beforehand, the spherical transform features the data compression because, in general, the angular variables do not need to be represented as precisely as the magnitude.

1. Introduction

The discussion about the signal quantization error seems to be out-of-date now, at the beginning of the 21-st century, when all biomedical signals are recorded exclusively with use of digital equipment. In case of electrocardiography and vectocardiography the appropriate considerations on quantization error and bit resolution were made several years ago and depending on application have to comply with specific rules. However, the discretization concerns usually the analog-to-digital conversion of recorded signals (orthogonal X, Y and Z or provided by the 12 chest and limb leads), the topic we focus on in this paper is rather uncommon: quantization of the VCG representation in the spherical coordinates.
The interest of vectocardiogram quantization in the spherical domain is justified by the research on the optimal data reduction technique [Fayn 1990]. In general approach, the spherical representation of the VCG fully corresponds to the Cartesian coordinates. Instead of three orthogonal X, Y and Z signals representing the voltage of the recorded potential, it uses one voltage variable representing the cardiac vector length (A – magnitude) and two angular variables for the description of the vector position in the space (ϕ - latitude, λ - longitude). Unlike the orthogonal representation, the values of ϕ and λ belong to infinite but limited set $\langle -\Pi/2, \Pi/2)$ for ϕ, and $\langle 0, 2*\Pi)$ for λ. As long as the floating-point representation is considered, the $XYZ \rightarrow A\phi\lambda$ transform also called the Spherical Transform (ST) is fully

reversible, the inverse transform (IST) reconstructs the XYZ representation perfectly but there is no chance for data reduction (1).

$$\text{XYZ} \rightarrow \text{A}\phi\lambda: \quad \begin{cases} A = \sqrt{X^2 + Y^2 + Z^2} \\ \phi = atan(\frac{-Y}{\sqrt{X^2 + Z^2}}) \\ \lambda = atan(\frac{Z}{X}) \end{cases} \qquad (1)$$

$$\text{A}\phi\lambda \rightarrow \text{XYZ}: \quad \begin{cases} X = -A \cdot \cos(\lambda) \cdot \cos(\phi) \\ Y = -A \cdot \sin(\lambda) \\ Z = -A \cdot \cos(\lambda) \cdot \sin(\phi) \end{cases}$$

Quantization of spherical coordinates is not so common as for Cartesian signals sampled directly in the A/D converter or derived mathematically from the other leads configuration. On the other hand, due to the use of trigonometric functions, the ST output is floating-point valued and the subsequent quantization is necessary to return to the fixed-point data format. The use of the fixed-point format allows applying any discrete signal data reduction technique at the cost of slight difference of the original XYZ signal representation and the reconstructed X'Y'Z' representation. In no way could be estimated now, how that affects the medical information carried in the signal. Even an original discrete XYZ signal is only the approximation of the real electrical field complying to given accuracy expectations, and in this aspect some medical data may already be lost. Except for easy acquisition, the Cartesian coordinates have no further advantages over the spherical coordinates for VCG applications. If some discretization error is acceptable for XYZ signals, it should be similarly in the spherical domain.

The hope for suitability of the Aϕλ coordinates for data reduction techniques is motivated not only by limitation of angular values range. Another advantage is higher regularity of angular values during the P, QRS and T loops that carry all the medical information expected by doctors in a VCG recording [Moss 1996] (figure 1).

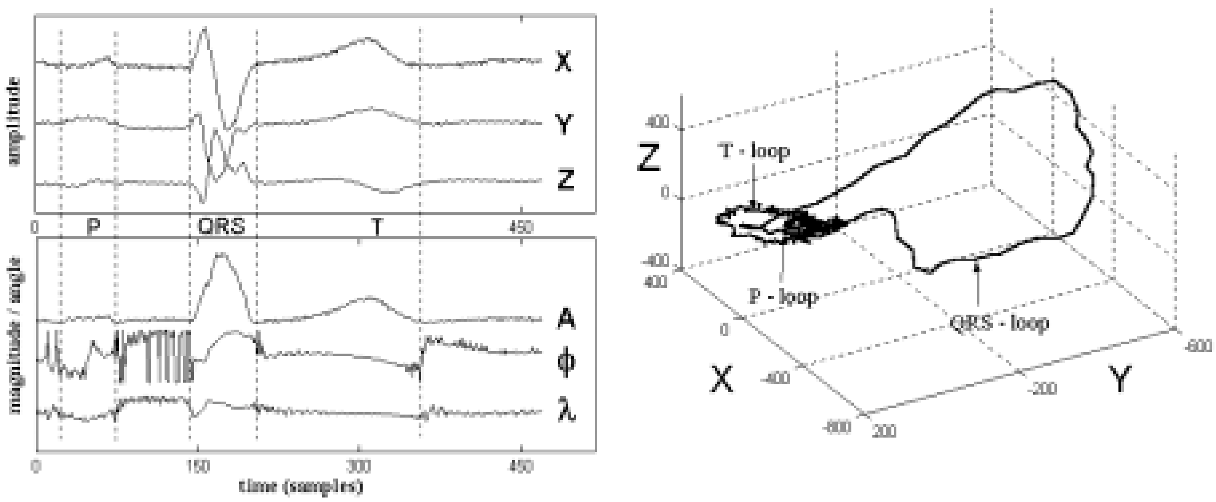

Figure 1. The VCG representation in Carthesian and spherical coordinates
and the spatial plot of the loops representing the whole heartbeat (CSE file 'mo_001')

Thanks to the regularity, a lossless differential coding is supposed to be particularly efficient [Augustyniak 2000], and the reconstructed signal is expected to be very close to the original what is crucial for preserving the integrity of medical information. On the other hand the regularity of angular values drops when the amplitude is relatively low, mainly on the baseline sections or when the signal-to-noise ratio decreases. For these sections, the reconstruction error is expected to be significantly higher, but without affecting the signal contents in diagnostic aspect.
Some extent of similarity may be noticed between the approach presented here and the color image coding applied in the consumer TV and video equipment. The three-color signal is converted to the luminance and two chrominance components and the requirements for chrominance throughput are much lower than for the luminance what economizes the transmission or storage resources.

2. Materials and methods

In our research the source of test signals was the CSE-Multilead Database (data set 3) [Willems 1990] providing a set of 125 recordings containing simultaneous 12-lead ECG + XYZ VCG accompanied with P-QRS-T segmentation points. The amplitude resolution is 12 bits and sampling frequency is 500 Hz. From each file the segment containing data for one heart evolution was extracted accordingly to the start- and endpoint from the database. Only the vectocardiogram traces were used in the numerical experiment concerned hereby.
All parts of the numerical experiment including the ST and error assessment procedures were implemented in Matlab. Since the Matlab environment uses double precision floating point representation by default, the custom-written subroutine rounds the continuous domain to the specified discretization level.
The numerical experiment consisted of the following steps:
- the forward XYZ $\rightarrow$ A$\phi\lambda$ transformation (ST);
- computation of the differential versions for angular signals
- discretization of angular signals in their absolute and differential versions at various levels corresponding to 2...11 bits resolution;
- computation of absolute signals from discrete differential angular signals;
- the inverse A$\phi\lambda$ $\rightarrow$ XYZ transformation (IST);
- error assessment in linear and spatial domains

The linear domain measure of dissimilarity between the discretized signal and its reference was computed as Percent Root-mean-square Difference (PRD) (2) [Levkov 1987]. Although this measure does not reflect the variability of importance of particular ECG sections and is contested as a reliable distortion estimate, it is widely used and applied here for the reason of compatibility. Another measure known as three-dimensional-difference (3DD) (3) was computed in order to estimate the fidelity of reconstructed P, QRS and T loops in the spatial domain. The PRD treats three leads as they were independent signals, while the 3DD is focused on the distortions of spatial information contained in the vectocardiograms.

$$PRD = \left\{ \frac{\sum_{i=1}^{n} \left[x_1(i) - x_2(i) \right]^2}{\sum_{i=1}^{n} \left[x_1(i) \right]^2} \right\}^{\frac{1}{2}} \cdot 100\% \qquad (2)$$

$$3DD = \left\{ \frac{\sum_{i=1}^{n} \left[(x_1(i) - x_2(i))^2 + (y_1(i) - y_2(i))^2 + (z_1(i) - z_2(i))^2 \right]}{\sum_{i=1}^{n} \left[(x_1(i))^2 + (y_1(i))^2 + (z_1(i))^2 \right]} \right\}^{\frac{1}{2}} \quad (3)$$

In order to assess the temporal distribution of distortions more accurately and prove the theoretical assumption that the loops are less affected than the baseline segments by the quantization of angular variables, an alternative experiment was carried out. All the experiment conditions were similar to the previous attempt, except for the use of segments extracted from the P, QRS and T waves and for the baseline, instead of the continuous signal representing the whole heartbeat. The lengths of extracted segments were standardized to the given duration: 64 ms for P and QRS waves and 128 ms for T wave and the baseline. This secondary part of the experiment did not consider the differential versions for angular signals. In order to make the interpretation more straightforward, only the PRD was used for the assessment of signal dissimilarities.

3. Results

In the main part of the experiment signals representing the whole heartbeat were subject to processing. Results of this part experiment are summarized in the Table 1. All values are averaged over whole set of 125 signals, and the standard deviation is computed for express the stability of error estimators. The results for differential signals are put together in a colon with the results of absolute value signals for the comparative purpose.

mean value of error		discretization level (bits per sample)									
		2	3	4	5	6	7	8	9	10	11
absolute signals	**PRD** %	47.0	25.79	12.98	6.627	3.315	1.656	0.833	0.416	0.206	0.104
	3DD	0.40	0.21	0.107	0.054	0.027	0.013	0.006	0.003	0.001	0.000
differen'l signals	**PRD** %	156.	142.7	130.9	98.40	59.89	29.25	15.35	7.018	3.564	1.735
	3DD	1.39	1.29	1.240	0.965	0.588	0.284	0.144	0.064	0.032	0.015
standard deviation of error		**discretization level (bits per sample)**									
		2	3	4	5	6	7	8	9	10	11
absolute signals	**PRD** %	9.353	6.837	3.641	1.866	1.021	0.474	0.225	0.122	0.058	0.029
	3DD	0.086	0.039	0.015	0.006	0.003	0.001	0.000	0.000	0.000	0.000
differen'l signals	**PRD** %	50.84	52.82	42.53	44.45	28.10	15.78	8.822	4.040	2.541	1.030
	3DD	0.313	0.331	0.360	0.424	0.291	0.160	0.081	0.036	0.020	0.008

Table 1. Results of the numerical experiment for signals representing the whole heartbeat

The supplementary part of the experiment concerned the signal segments extracted from the P, QRS and T waves and from the baseline. The PRD values are displayed in the Table 2 for each segment separately in order to measure how the discretization of angular values influences particular components of the VCG. The same results are displayed in the Figure 2. As expected, the QRS wave is the most vulnerable component of the VCG. Limiting the set of possible values for P wave results in distortions coefficient very close to the value gathered at the baseline. Finally, the T wave is the less sensitive to low-wordlength quantization.

quantization level (bits per sample)	mean value of PRD error [%] in the wave P	QRS	T	in the baseline
2	32.3	53.7	22.3	28.0
3	16.2	27.9	11.8	15.3
4	8.14	13.9	6.21	8.00
5	3.99	7.19	3.23	4.13
6	2.02	3.62	1.56	2.10
7	1.01	1.75	0.78	1.06
8	0.51	0.89	0.39	0.51
9	0.25	0.44	0.19	0.26
10	0.12	0.22	0.09	0.12
11	0.06	0.11	0.04	0.06

Table 2. Quantization-induced distortion for the P, QRS and T waves and the baseline segment

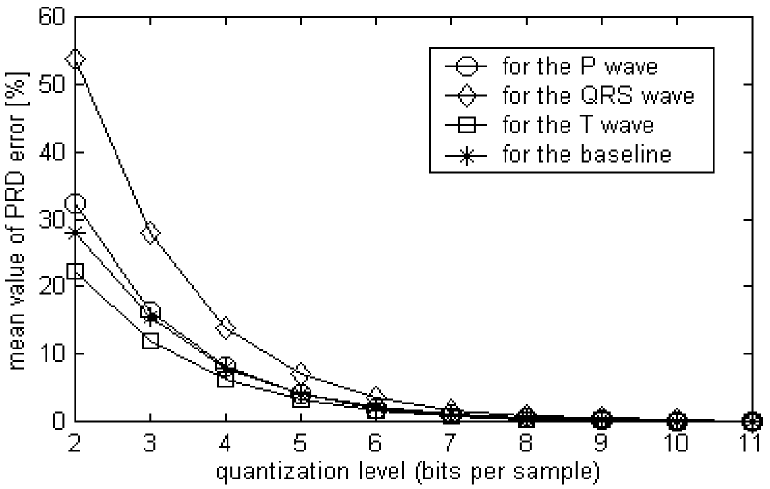

Figure 2. The average PRD value as a function of quantization bits count for the P, QRS and T waves and the baseline segment

4. Discussion

The quantization error for the spherical representation of the VCG was determined in an experimental way. The quantization depth used varied from 2 bits per value that was considered as the lowest reasonable resolution, to 11 bits per value that is only a step lower that the original signal resolution. At each quantization depth, the scale coefficient was adapted to the signal amplitude, in order to optimize the use of all the given dynamic range.

Allowing a slight dissimilarity between the original XYZ signal and the X'Y'Z' representation reconstructed from the discretized spherical domain results in more efficient data storage. The most attractive storage format seems to be the following bit resolution: $A:\phi:\lambda = 8:4:4$, because it provides the full representation of the time sample in one computer word. This resolution corresponds to a bit rate of 1kB/s at the sampling frequency of 500 Hz.

I is worth to notice, that from the $A\phi\lambda$ representation not only a XYZ vectocardiogram, but also a traditional 12-lead electrocardiogram may be derived thanks to the Dower transform [Dover 1980].

The bit resolution of 8:4:4 was initially expected to work at a reasonable low distortions (PRD = 1...2 %) with the differential signals, so it is necessary to briefly comment the worse result.

- The first reason is the differential signal nature that assumes the reconstruction of the current absolute value as the cumulative sum of all the preceding samples. Unfortunately, the quantization error of the current sample influences all the following values and the cumulative growth of round-off error propagates in time. Perhaps the remedy is introducing intra-frames (i.e. values stored in absolute way) equidistantly distributed in the signal. These values make possible to determine the local error value and to distribute it regularly for all the samples occurring between them.
- The second reason is that the PRD error estimator does not make distinction between the sections of heart evolution. The angular coordinates loose their regularity what increases the round-off error at lower bit resolutions. That occurs in the long intra-waves section containing mainly baseline and not relevant from the medical point of view, however the statistical result is bad.

Additional study answering this question may be a comparison of diagnostic parameters such as P-Q interval, Q-T interval and so forth, performed for the original signals and their processed counterparts.

Another observations may be issued from the results of the numerical experiment:

- There is no reason for special consideration for ϕ or λ angular variables; in particular the resolution should be the same for both.
- Applying the different quantization depth for the particular VCG components is worth to be considered; an additional advantage is the easy segmentation of the VCG.
- Applying the lower sampling rate of angular variables for the particular waves is also interesting; probably the latitude and longitude may be sampled at a half of original rate.
- Decreasing the sampling rate usually implied lower output data bit rate. In case of differential signals it may not be true, because decreasing the sampling rate causes increase of differential signal dynamic range.

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