# DECIMATION AND QUANTIZATION OF VECTOCARDIOGRAMS IN SPHERICAL COORDINATES DOMAIN

Piotr Augustyniak

AGH University of Mining and Metallurgy, Institute of Automatics, Krakow, Poland

email: august@biocyb.ia.agh.edu.pl

**Abstract: The paper discusses two issues involved in reduction of the vectocardiogram's data volume. The fundamental novelty of our approach is the processing of the spherical coordinates (magnitude, latitude and longitude) instead of Cartesian (XYZ) signal representation. The quantization of the angular values at different levels and the resulting distortions of main recording segments are studied at the beginning. Next, the spectral representations of the angular values were investigated with a special attention to the energy distribution. Finally, two decimation stages were applied together with the quantization aiming at finding the compromise between the resulting data rate and the distortion coefficients for particular signal segments. Every our outcome is supported by the numerical experiment carried out with use of a real VCG data. Further considerations for the implementation of the compression algorithm terminate the paper. Main advantage of the method is the possible use of custom-defined compression profiles. An example profile yields the compression ratio 3,2 and distortion level of 17,2% (in loops 5,8%).**

## Introduction

The vectocardiography (VCG), however less popular than the conventional electrocardiography (ECG), returns currently to the clinical practice thanks to the use of numerically supported spatial presentation and diagnostic methods. The medical significance of long-term recordings, first highlighted by N. Holter [1], is applicable to the VCG as well. The only difference is that the ECG may consist of one or more simultaneously recorded signals, whilst in case of VCG the three-channel recording is compulsory.

The common and unconstrained access to the cardiovascular medical aid is one of the most crucial assumptions of the modern societies. That justifies the need for low cost and effective solutions for data storage and transmission. The compression of the ECG signal attracts the attention of searchers all over the world and is one of the hottest topics of research in biomedical signal processing.

The VCG recordings exhibit some very interesting features not present in the conventional ECG. Two of them constitute the background of our approach:

- In vectocardiograms, all diagnostic results are derived from the analysis of the spatial loops representing the P, QRS and T waves. The remaining part of the signal may be thus processed without the extreme care about distortions.
- Vectocardiograms, although recorded and commonly represented in Cartesian coordinates system XYZ, may be alternatively described by other set of time variables, the sphericals coordinates (1). Only one of them, the magnitude (A), has an "electrical" dimension (μV) and the remaining two, the latitude $\phi$ and the longitude $\lambda$, are called "angular" because of their dimension expressed in radians (or degrees).

$$\mathrm{XYZ} \rightarrow \mathrm{A}\phi\lambda : \quad \begin{cases} A = \sqrt{X^2 + Y^2 + Z^2} \\ \phi = atan(\dfrac{-Y}{\sqrt{X^2+Z^2}}) \\ \lambda = atan(\dfrac{Z}{X}) \end{cases} \tag{1}$$

$$\mathrm{A}\phi\lambda \rightarrow \mathrm{XYZ} : \quad \begin{cases} X = -A \cdot \cos(\lambda) \cdot \cos(\phi) \\ Y = -A \cdot \sin(\lambda) \\ Z = -A \cdot \cos(\lambda) \cdot \sin(\phi) \end{cases}$$

Unlike the orthogonal representation, the values of $\phi$ and $\lambda$ belong to infinite but limited set of values: $\langle -\Pi/2, \Pi/2)$ for $\phi$, and $\langle 0, 2\Pi)$ for $\lambda$. As long as the floating-point representation is considered, the Spherical Transform XYZ ↔ A$\phi\lambda$ (ST) is fully reversible, the inverse transform (IST) reconstructs the XYZ representation perfectly but there is no chance for data reduction. Since the natural way of signal acquisition (i. e. discretization and quantization) is performed in XYZ domain and the ST uses floating-point data representation, additional quantization is necessary in A$\phi\lambda$ domain to represent the signal with use of fixed-point values. Obviously, this affects the perfect reconstruction property of the ST to the extent inversely proportional to the wordlength used.

In aspect of data volume reduction, the question of appropriate sampling frequency for the discrete fixed-point representation of VCG angular values has to be reconsidered. In the Cartesian domain, every three traces: X, Y and Z contain the same kind of information and no reason was found to consider one of them more important or more informative than the others. Similarly in the spherical domain, all three traces: A, $\phi$ and $\lambda$ are of the same importance, bearing the complementary information, but for the reason of smoothness expected inside the P, QRS and T loops the angular values may be probably 'acquired' at lower sampling rate. In the remaining parts of the signal, featuring high variability of $\phi$ and $\lambda$ values, the acceptable distortion level is higher for the assumption of lack of diagnostic parameters derived from there.

Two ways of angular data reduction were investigated in this paper:

- The discretization level (i. e. how accurately the angular values have to be represented)
- The sampling frequency (i. e. how frequently the angular values have to be represented)

## Materials and methods

Three numerical experiments were designed and carried out in order to prove main assumptions of the reasoning above. All procedures were implemented in the Matlab 5 environment, since at this stage no special attention was paid to the numerical efficiency of the algorithm. An additional advantage of Matlab is the huge library of ready-to-use functions, containing the coordinates transform, decimation and signal distortions measure.

As a source of annotated reference VCG signal, the CSE Multilead Database was used [2]. The database contains 125 recordings of 10 s duration each (quantization 12 bits, sampling frequency 500 Hz) acquired simultaneously in X, Y and Z channels. Each recording contain a reference heartbeat for that the segmentation points (i. e P-onset, P-end, QRS-onset, QRS-end and T-end) derived with use of 8 world-best software are provided.

The distortion measures used were the percent root-mean-square difference (PRD) (2), the correlation coefficient C (3) and the regression coefficient R (4) [3].

$$PRD = \left\{ \frac{\sum_{i=1}^{n} \left[ x_1(i) - x_2(i) \right]^2}{\sum_{i=1}^{n} \left[ x_1(i) \right]^2} \right\}^{\frac{1}{2}} \cdot 100\% \qquad (2)$$

$$C = \frac{\sum_{i=1}^{n} x_1(i) \cdot x_2(i)}{\sqrt{\sum_{i=1}^{n} [x_1(i)]^2 \cdot \sum_{i=1}^{n} [x_2(i)]^2}} \qquad (3)$$

$$R = \frac{\sum_{i=1}^{n} x_1(i) \cdot x_2(i)}{\sum_{i=1}^{n} [x_1(i)]^2} \qquad (4)$$

Despite commonly questioned, the PRD is still widely used for the purpose of comparing the ECGs. Moreover, the use of PRD for comparing the VCG segments (i. e. loops) seems to be appropriate if the constant distribution of diagnostic information is assumed (what is not true for the longer part of the signal i. e. a whole heartbeat).

The aim of the preliminary investigation was to find the correlation of the angular values quantization depth and the resulting signal distortions. The ST was followed by quantization and the IST, than the reconstructed signals were compared to their original counterparts. The quantization was performed on 10 different levels corresponding to $\langle 2, 11 \rangle$ bits per value or to the values ranges from $\langle 0, 3 \rangle$ to $\langle 0, 2047 \rangle$. The whole heartbeats' representations were initially subject to quantization, but after first revision, for the levels corresponding to $\langle 5, 8 \rangle$ bits per value, segments representing P, QRS and T waves and one baseline segment were quantized separately.

For the investigation of angular values' spectral properties the ST was performed on signal segments representing P, QRS and T waves and on one baseline segment. All signals were cut to the desired length (in ms: 64 for P, 64 for QRS, 128 for T and 128 for the baseline) with respect to the waves' center. The CSE files not providing the segments of desired length were excluded. The ST was followed by trend removal and tempering of 15% of length at each end of the signal. Tempering was achieved with use of sine half-envelope that minimizes distortions induced by the border effect. Finally the power-of-two FFT algorithm was applied and the magnitude of the complex spectrum was taken as an estimator of energy distribution. No additional smoothing was performed due to the low number of samples in the spectrum.

The final experiment involved both decimation and quantization of angular values. Since the output of the ST has floating-point representation, we used the QMF-based decomposition. The relative high energy of the lowest quarter of the spectrum (i. e. $\langle 0$, sf/8)) restrained us from applying more than two stages of decimation. Even so, the signal length drops down to a value of 8 samples and thus it is necessary to apply a compact-supported filter. Our final choice was the $4^{th}$ order Daubechies filter [4] for its support length of 9 samples and good resolution in frequency. The decimated floating-point signals were quantized at the level corresponding to $\langle 5, 8 \rangle$ bits per value and stored in the output data stream. The reconstruction consisted in upsampling of the fixed-point angular values to the initial length followed by the IST. Comparing the reconstructed and original signals and calculating the compression and distortion ratios end the experiment.

**Results**

The averaged distortion values for the whole heartbeats at all considered quantization depth are summarized in the Table 1. For the lack of space, only the PRD values are displayed. The quantization-induced distortion for the waves and baseline at the given quantization depth are displayed in the Table 2. The Figure 1 displays the average PRD value as a function of quantization bits count.

Table 1. Global distortion values at given quantization levels

| quantization level (bits per sample) | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| mean value of PRD error [%] | 47.0 | 25.7 | 12.9 | 6.62 | 3.31 |
| quantization level (bits per sample) | 7 | 8 | 9 | 10 | 11 |
| mean value of PRD error [%] | 1.65 | 0.83 | 0.41 | 0.21 | 0.10 |

Table 2. Quantization-induced distortion for the P, QRS and T waves and baseline

| quantization level (bits per sample) | mean value of PRD error [%] in the wave | | | in the baseline |
|---|---|---|---|---|
| | P | QRS | T | |
| 2 | 32.3 | 53.7 | 22.3 | 28.0 |
| 3 | 16.2 | 27.9 | 11.8 | 15.3 |
| 4 | 8.14 | 13.9 | 6.21 | 8.00 |
| 5 | 3.99 | 7.19 | 3.23 | 4.13 |
| 6 | 2.02 | 3.62 | 1.56 | 2.10 |
| 7 | 1.01 | 1.75 | 0.78 | 1.06 |
| 8 | 0.51 | 0.89 | 0.39 | 0.51 |
| 9 | 0.25 | 0.44 | 0.19 | 0.26 |
| 10 | 0.12 | 0.22 | 0.09 | 0.12 |
| 11 | 0.06 | 0.11 | 0.04 | 0.06 |

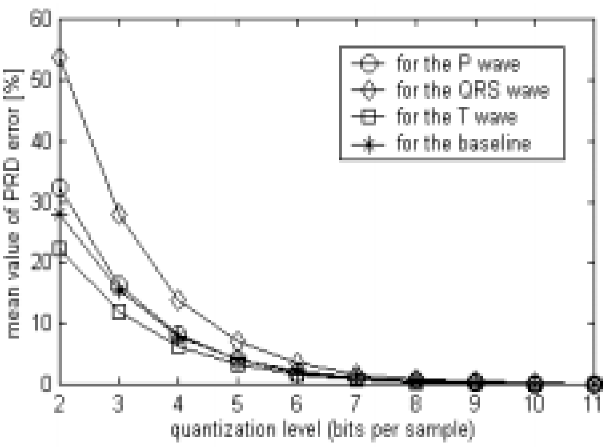


Figure 1. The average PRD value as a function of quantization bits count

Figure 2 displays the spectral energy distribution of the angular values: the latitude ϕ and longitude λ in the P, QRS and T waves and in the baseline segment. This figure may also be used for prediction of decimation consequences. Since no significant difference was found between ϕ and λ spectral features, both angular values are processed in the same way in the subsequent experiment.

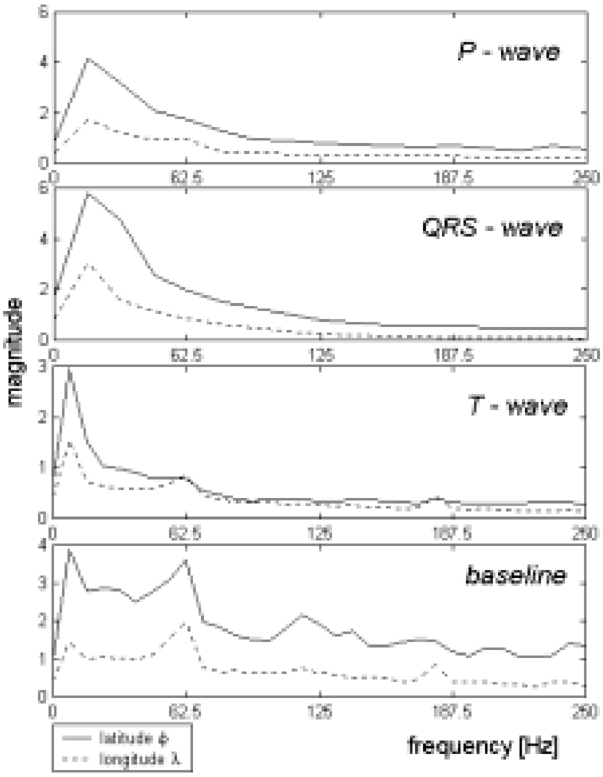


Figure 2. The average spectrum of the latitude ϕ and longitude λ for the P, QRS and T waves and for the baseline segment.

The expected local compression ratio may be derived directly from the resulting data bitrate. The original signal was represented in 12 bit fixed-point values sampled at 500 Hz (i. e. 6000 bps). Decimation and quantization with lower resolution reduce the data volume by the factor displayed in the Table 3. The global compression ratio may be calculated for each individual combination of parameters, with respect to the proportion of waves' duration in a real VCG.

Table 3. Local compression ratio calculated for all combination of decimation and quantization used in the experiment

| decimation steps | quantization level (bits per sample) | | | |
|---|---|---|---|---|
| | 5 | 6 | 7 | 8 |
| 0 | 2.4 | 2 | 1.71 | 1.5 |
| 1 | 4.8 | 4 | 3.42 | 3 |
| 2 | 9.6 | 8 | 6.84 | 6 |

The Table 4 summarizes the distortion (PRD) measurements' results for the P, QRS and T waves and in the baseline segment.

Table 4. Distortions (PRD, [%]) for all combination of decimation and quantization used in the experiment

| ***P - wave*** | | | | |
|---|---|---|---|---|
| decimation steps | quantization level (bits per sample) 5 | 6 | 7 | 8 |
| 0 | 3.99 | 2.02 | 1.01 | 0.51 |
| 1 | 16.1 | 15.7 | 15.5 | 15.5 |
| 2 | 25.4 | 25.1 | 25.1 | 25.1 |
| ***QRS – wave*** | | | | |
| decimation steps | quantization level (bits per sample) 5 | 6 | 7 | 8 |
| 0 | 7.19 | 3.62 | 1.75 | 0.89 |
| 1 | 13.9 | 11.8 | 11.0 | 10.8 |
| 2 | 23.2 | 22.2 | 21.8 | 21.8 |
| ***T - wave*** | | | | |
| decimation steps | quantization level (bits per sample) 5 | 6 | 7 | 8 |
| 0 | 3.23 | 1.56 | 0.78 | 0.39 |
| 1 | 8.74 | 8.20 | 8.05 | 8.00 |
| 2 | 13.2 | 12.8 | 12.7 | 12.7 |
| ***baseline*** | | | | |
| decimation steps | quantization level (bits per sample) 5 | 6 | 7 | 8 |
| 0 | 4.13 | 2.10 | 1.06 | 0.51 |
| 1 | 33.1 | 33.0 | 32.9 | 32.9 |
| 2 | 44.8 | 44.8 | 44.7 | 44.7 |

Although the decimation and quantization parameters may be selected individually for each segment, we found interesting to investigate the distortion ratio for the constant values. The very limited subset of these results is displayed in the table 5.

Table 5. Distortions (PRD, [%]) for selected combination of decimation and quantization parameters.

| decim'n steps | quantiz'n level | mean value of PRD error [%] in the wave P | QRS | T | in the baseline |
|---|---|---|---|---|---|
| 0 | 6 | 2.02 | 3.62 | 1.56 | 2.10 |
| | 8 | 0.51 | 0.89 | 0.39 | 0.51 |
| 1 | 6 | 15.7 | 11.8 | 8.20 | 33.0 |
| | 8 | 15.5 | 10.8 | 8.00 | 32.9 |
| 2 | 6 | 25.1 | 22.2 | 12.8 | 44.8 |
| | 8 | 25.1 | 21.8 | 12.7 | 44.7 |

**Discussion**

The representation of the VCG in the spherical domain is fully equivalent to its Cartesian original. The PRD value for 11-bit quantization matches exactly the quantization error in the Cartesian domain. For some applications, however, the 8 or even 5-bit quantization would reproduce the VCG with the tolerable precision.

Angular variables exhibit very interesting features for segmented VCG signal. Segmentation belongs to the intrinsic ECG procedures and is performed automatically by the software with high reliability, thus for further consideration we assume that the segmentation is done.

In the waves, the cardiac vector's end draws a loop in the space. The physiology of depolarization and repolarization of the subsequent fibrils in the cardiac muscle justifies the smoothness of the angular variables in the spherical VCG representation. This smoothness, expressed as low contribution of high frequency components in the total signal energy is in turn the background for applying a lower sampling frequency for the latitude and longitude.

In the baseline segments, devoid of the heart activity, the random values of the dominant noise in the recorded XYZ signal cause high variability of the angular variables and, in consequence, higher energy in the upper part of the spectrum. For this reason, decimation of angular variables results in high distortion level in the baseline. Fortunately, these segments do not contain medical information of high importance and do not have to be reproduced very accurately.

The main advantage of decimation and quantization of the VCG angular variables is the adaptability for defining the custom compression profiles. Depending on current diagnostic requirements, the compromise between the compression efficiency and the distortion ratio may be set individually for each wave and for the baseline separately. The example compression profile uses no decimation for P and QRS waves and one decimation step for T wave and the baseline. The quantization level is 6 bits per sample in each segment. These parameters yield the compression ratio 3,2 and distortion level 17,2% (in loops 5,8%).

**Acknowledgement**

This work is supported by University of Mining and Metallurgy, grant no: 11.11.120.249

**References**

[1] Holter N. ‘New method for heart studies: continuous electrocardiography of active subjects over long periods is now practical’ Science 1961, 134: 1214-20

[2] Willems J. L. 'Common Standards for Quantitative Electrocardiography' 10-th CSE Progress Report, 1990. Leuven: ACCO publ., 1990, 384p.

[3] Levkov Ch. L. 'Orthogonal electrocardiogram derived from the limb and chest electrodes of the conventional 12-lead system' Med. & Biol. Eng. & Comput. 1987, 25, 155-164

[4] Daubechies I. 'Ten lectures on wavelets' Society for Industrial and Applied Mathematics, 1992.