# Finding Location Using a Particle Filter and Histogram Matching * 

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#### Abstract

This paper considers the problem of mobile robot localization. The localization is done using a particle filter built on a highly accurate probabilistic model of laser scan and a histogram based representation of sensor readings. A histogram matching exploits sensor data coming from the laser and data obtained from the existing map. Experimental results indicate feasibility of the proposed approach for navigation.


## 1 Introduction

The problem of determining the position that is occupied by the robot is a central issue in robotics and has been deeply investigated in the literature. Mobile robots cannot rely solely on dead-reckoning to determine their location because of cumulative nature of errors in odometry readings. Self-localization techniques are utilized as a way to compensate the errors that accumulate during the robot movement by comparing the acquired sensor data with the pre-stored model of the environment in form of a map. For this reason the mobile robot should be equipped with sensors that allow determining the location. The most commonly used sensors are sonar, laser range finders and CCD. Recently Monte Carlo based algorithms have become a very popular framework to cope with the self-localization of mobile robots. A family of probabilistic algorithms known as Monte Carlo Localization [2][4] is one of the very few methods capable of localizing the robot globally. Global position estimation is the ability to determine the robot's position in a prepared in advance map, given no other information than that the robot is somewhere on the map. Once the robot has been localized in the map within some certainty, a local tracking is performed during maneuvering with aim to keep the track of the robot position over time. Monte Carlo based algorithms represent a robot's belief by a set of weighted particles to approximate the posterior probability of robot location by using a recursive Bayesian filter. The key idea of Bayes filtering is to estimate a probability density over the state space conditioned on the sensor data. Algorithms that deal with the global localization are relatively recent, although the idea of estimating the state recursively using particles is not new. The application of particle filters to mobile robot localization [2][4] was motivated by the Condensation algorithm [5], a particle filter that has been applied with a remarkable success to visual tracking problems [5][6].

[^0]The basic MCL algorithm performs poorly if the proposal distribution contains not enough samples in the right target location. MCL also performs poorly when the sensor noise level is too small taking into account uncertainty coming from the discretized map of the environment. A simple strategy based on adding artificial noise to the sensor readings has been applied in work [4]. The approach presented in [7] overcomes the degradation to small sample sets by integrating two complementary ways of generating samples in the estimation and using a kernel density tree in fast sampling. The approach we present in this paper utilizes the histogram based techniques to compare a laser scan with the scan representation obtained from an existing map. It is based on the concept of correlation between the scan extracted from the map taking into account the location of the considered particle and the sensor scan. A high similarity of histograms indicates a good match between laser readings and scans which represent considered map pose. Due to the statistical nature, a histogram based representation holds sufficient statistics for the sensor distributions and introduces desirable uncertainty. The histogram can be pre-computed and stored for every possible robot orientation and position. In a slower version of the algorithm the histogram at the considered map pose can be computed on-line. We show that histogram based map representation has powerful capability and can be used to distinguish sensor scans in a fast manner. Our experimental results indicate feasibility of the algorithm in which the highly accurate observation model from work [7] and histogram based one are combined within a particle filter in order to perform localization of the robot.

## 2 Monte Carlo Localization and Bayesian filtering

The typical problem in partially observable Markov chains is to obtain a posterior distribution over the state $x_{t}$ at any time $t$ taking into account all available sensor measurements $z_{0}, \ldots, z_{t}$ and controls $u_{0}, \ldots, u_{t}$. The state $x_{t}$ depends on the previous state $x_{t-1}$ according to stochastic transition model $p\left(x_{t} \mid x_{t-1}, u_{t-1}\right)$ for a control signal $u_{t-1}$ which moves the robot from state $x_{t-1}$ to state $x_{t}$. Such a motion model generalizes exact mobile robot kinematics by a probabilistic component and expresses the probability for certain actions to move the robot to certain relative positions. The state in the Markov chain is not observable. At each time step $t$ a robot makes observation $z_{t}$ which is a probabilistic projection of the robot state $x_{t}$ through a stochastic observation model $p\left(z_{t} \mid x_{t}\right)$. The observation model describes the probability for taking certain measurements at certain locations. We assume that observations $z_{t}$ are conditionally independent given the states $x_{t}$ and that the initial distribution at time $t=0$ is $p\left(x_{0}\right)$.

The posterior density $p\left(x_{t} \mid z_{t}\right)$ over the state space $\mathcal{X}$ characterizes the belief of the subject about its current state at time $t$ given its initial belief and the sequence of observations $z_{0}, \ldots, z_{t}$. Bayes filters estimate the belief recursively. The initial belief characterizes the initial knowledge about the system state, which in global localization corresponds to a uniform distribution reflecting an
unknown initial pose. In the prediction phase the following motion model is used to obtain the predictive density

$$
\begin{equation*}
\operatorname{Bel}\left(x_{t}\right)=\int p\left(x_{t} \mid x_{t-1}, u_{t-1}\right) \operatorname{Bel}\left(x_{t-1}\right) d x_{t-1} \tag{1}
\end{equation*}
$$

The parameter $u$ may be an odometry reading or a control command. In the second phase a measurement model is used to utilize sensor information in order to obtain the posterior

$$
\begin{equation*}
\operatorname{Bel}\left(x_{t}\right) \propto\left(z_{t} \mid x_{t}\right) \operatorname{Bel}\left(x_{t}\right) \tag{2}
\end{equation*}
$$

This term expresses the likelihood of the state $x_{t}$ given that $z_{t}$ was observed. The above two formula describe an iterative scheme for Bayesian filtering.

Monte Carlo Localization relies on the sample based representation of the belief $\operatorname{Bel}\left(x_{t}\right)$ by a set of $N$ weighted samples distributed according to $\operatorname{Bel}\left(x_{t}\right)$

$$
\begin{equation*}
\operatorname{Bel}\left(x_{t}\right) \approx\left\{x_{t}^{[i]}, w_{t}^{[i]}\right\}_{i=1, \ldots, N} \tag{3}
\end{equation*}
$$

and the sampling/importance resampling algorithm. Each particle is represented by a state of the mobile robot $(x, y, \phi)$ and a weight that reflects the contribution of particular particle to belief of the robot. A sample set constitutes a discrete distribution and if the number of samples goes to infinity such distributions approximate the correct posterior density smoothly. From the samples we can always approximately reconstruct the posterior density using a histogram or a kernel based density estimation technique [3]. The population of samples evolves as new action is executed and new sensor observations are obtained. The prediction phase uses the probabilistic motion model to simulate the effect of the action on the set of particles. When the new sensory information is available we use Bayes rule in order to update the probability density function of the moving robot with the latest observation.

One of the practical difficulties that is associated with particle filters is degeneration of the particle population after a few iterations because weights of several particles are negligible to contribute to the probability density function. The aim of resampling is to eliminate particles with low importance weights and multiply particles with high importance weights. The resampling selects with higher probability samples that have a high likelihood associated with them. Without resampling the variance of the weight increases stochastically over time. An algorithm to perform the resampling from a set of particles in $O(N)$ time has been proposed in [1]. The sensor readings are typically incorporated in two phases having on regard the outlined above resampling. In the first phase each importance factor is multiplied by $p\left(z_{t} \mid x_{t}\right)$. In the second one the resampling is conducted and afterwards the importance factors are normalized so that they sum up to 1 and for this reason they constitute a discrete probability distribution. As it was mentioned above the initial pose in the global localization is unknown and therefore the initial prior is uniform over the space of possible poses.

## 3 Scan matching using histogram

A grid based map represents environment by regularly spaced grid cells. Each grid cell indicates the presence of an obstacle in the corresponding region of the environment. If a robot occupies a certain pose in the map we can compute expected laser scan readings using the well known ray-tracing. The scan readings obtained in such a way can be then used in comparison with robot scan readings.

Fig. 1a. illustrates the map of the office environment in which localization experiments have been conducted. This office-like environment is 560 by 460 cm and it has been discretized into 280x230x90 cells. A single scan of the laser range finder which was used in experiments returns a hemicircle of 180 readings with 1 degree incrementation. The distance error of range measurement using this sensor is 1 cm . A sample laser scan is depicted in the Fig. 1b. A reference scan which has been obtained on the basis of the map for the corresponding robot pose from Fig. 1b. is demonstrated in the Fig. 1c.


Fig. 1. Map, sample laser data from the environment, corresponding reference scan

A histogram is obtained by quantizing the scan distances into $L$ bins and counting the number of times each distance occurs in the single scan. Due to the statistical nature, a scan histogram can only reflect the environment shape in a limited way. Two scan shapes taken at close whereabouts appear very similar to each other and taking the above into account the number of histograms needed for environment representation is reasonably small. If the number of bins $L$ is too high, the histogram is noisy. If $L$ is too low, the density structure of the scan shape is smoothed. The histogram based techniques are effective only when $L$ can be kept relatively low and where sufficient data amounts are available. The reduction of bins makes the comparison of two histograms faster and additionally such a compact representation reduces memory requirements. That aspect is particularly important considering on the one hand a limited computational power of the on-board computer and on the other hand the necessity of work with a rate which enables the mobile robot to utilize localization results during a maneuvering.

In order to compare two histograms we need a similarity or dissimilarity metric. For a given pair of histograms $I$ and $M$ each containing $l$ values, the intersection of the histograms is defined as follows:

$$
\begin{equation*}
H_{\cap}=\frac{1}{\sum_{j=1}^{L} I_{j}} \sum_{j=1}^{L} \min \left(I_{j}, M_{j}\right) \tag{4}
\end{equation*}
$$

The terms $I_{j}, M_{j}$ represent the number of particular scan values inside the $j$-th bucket of the current and the model histogram, respectively, whereas $L$ the total number of buckets. The result of the intersection of two histograms is the percentage of scans which share the same distances in both histograms.

## 4 Robot localization using particles

The probabilistic search for the best pose is realized in the utilized particle filter on the basis of the motion as well as the observation model. Any arbitrary mobile robot motion $[\triangle x, \Delta y]^{T}$ can be carried out as a rotation followed by a translation. The noise is applied separately to each of the two motions because they are independent. When the robot rotates about $\Delta \phi$ the odometry noise can be modeled as a Gaussian with experimentally established mean and standard deviation proportional to $\triangle \phi$. During a forward translation the first error is related to the traveled distance and the second one is associated with changes of the orientation attending the forward translation. The simple way to obtain the translation model is to discretize the motion into $K$ steps and to cumulate the simulated effect of noise from each step. The sensor model describes the probability of obtaining a particular scan shape given the laser's pose and a geometrical map of the environment. In the histogram based version of the particle filter the following observation model $p\left(z_{t} \mid x_{t}\right)=H_{\cap}\left(I_{t}, M\left(x_{t}\right)\right)$ has been utilized.

In order to obtain an estimate of the pose the weighted mean $\left(\sum_{i} w^{[i]} x^{[i]}\right)$, in a small sub-cube around the best particle has been utilized. The orientation of the robot has been determined on the basis of sum of direction vectors of particles from the sub-cube as $\phi=\arctan \left(\sum_{i} \sin \phi^{[i]}, \sum_{i} \cos \phi^{[i]}\right)$. The effect of probabilistic search for the best position has additionally been enhanced via a local move of particles according to their probability. The more probable the particle is, the less it is moved.

## 5 Experimental results

All experiments were carried out in an office environment with our experimental Pioneer [8] based platform which is equipped with a laser range finder as well as an on-board 850 MHz laptop computer. The goal of the first group of tests was experimental verification of efficiency of particle filter utilizing histogram models when the robot is continuously moving. Two 4 -bins histograms representing the $x$ and $y$-components of the scans ensure the high efficiency with relatively low
computational burden. During experiments which typically took about 10 minutes the position has been determined 5 times per sec. and the maximal velocity was $0.8 \mathrm{~m} / \mathrm{s}$. The number of particles used was between 500 and 5000. Assuming stationary particles between consecutive motions, we observed that a cloud consisting of 5000 particles forms a boundary around the robot in which minimum two successive positions of the robot are always contained. The goal of the second group of experiments was to evaluate the precision of determining the position in certain points. In order to record data in known positions the robot has been manually moved several times on a rectangular path of 10 m . Next, the particle filters utilizing the histogram and the accurate probabilistic models of the laser have been compared on recorded data. The histogram based algorithm reports the position of the robot anywhere from ten to twenty iterations from the start of the global localization. In the second algorithm the position is known after a few iterations. The square root of the sum of squared errors of 100 measurements on the mentioned above path was about 1000 cm and 750 cm , respectively. The overall performance of the histogram based algorithm is poorer than that of the conventional one. However, each approach has complimentary strengths and weaknesses. The particle filter which merges both approaches yields superior localization performance. The square root of the sum of squared errors was about 800 cm .

## 6 Conclusion

We have presented a method that robustly localizes a robot in an office. The histogram based representation of the environment is very useful in particle filters relying on laser readings. Initial results show that our combined method outperforms the method using highly accurate probabilistic model of the laser scan.

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