Corrections in *Measure, Integral and Probability* by M. Capiński and P.E. Kopp 1st printing, 1999

page 6, line 17,
`Replace`
For example, if $f : [a, b] \to \mathbb{R}$,
`by`
For example, if $f : [a, b] \to \mathbb{R}$ is continuous,

page 16, line 8, Definition 2.1
`Replace`

$$A \subseteq \bigcup_{k=1}^{\infty} I_n$$

`by`

$$A \subseteq \bigcup_{n=1}^{\infty} I_n$$

page 17, line 1
`Replace`

$$I_3 = (x_3 - \frac{\varepsilon}{16}, x_3 - \frac{\varepsilon}{16})$$

`by`

$$I_3 = (x_3 - \frac{\varepsilon}{32}, x_3 - \frac{\varepsilon}{32})$$

page 18, line 10, Step 2
`Replace`

$$N = \bigcup_{n=1}^{\infty} N_n \subseteq \bigcup_{j=1}^{\infty} J_k^n.$$

`by`

$$N = \bigcup_{n=1}^{\infty} N_n \subseteq \bigcup_{n=1}^{\infty} \bigcup_{k=1}^{\infty} I_k^n \subseteq \bigcup_{j=1}^{\infty} J_j.$$

page 21, lines 3-4
`Replace`
This shows that Z_A is the interval $(x, +\infty]$ or $[x, +\infty]$ for some real number x.
`by`
This shows that Z_A is either $\{+\infty\}$ or the interval $(x, +\infty]$ or $[x, +\infty]$ for some real number x. (2.1)

page 23, line -11, after (2.5)
`Replace` (2.2) `by` (2.1)

page 23, line -8
`Replace` (2.2) `by` (2.1)

page 30, line -6
`Replace`

$$\begin{aligned} m^*(A \cap E_1^c) &= m^*((A \cap E_1^c) \cap E_2) + m^*((A \cap E_1)^c \cap E_2^c). \\ &= m^*(A \cap (E_1^c \cap E_2)) + m^*(A \cap (E_1^c \cap E_2^c)) \end{aligned}$$

`by`

$$\begin{aligned} m^*(A \cap E_1^c) &= m^*((A \cap E_1^c) \cap E_2) + m^*((A \cap E_1^c) \cap E_2^c). \\ &= m^*(A \cap (E_1^c \cap E_2)) + m^*(A \cap (E_1^c \cap E_2^c)) \end{aligned}$$

page 37, line 7
`Replace`
$\sum_{n=1}^{\infty} I_n - \frac{\varepsilon}{2} \leq m^*(A)$
`by`
$\sum_{n=1}^{\infty} l(I_n) - \frac{\varepsilon}{2} \leq m^*(A)$

page 37, line 11
`Replace`

$$m(O) \ \leq \sum_{n=1}^{\infty} J_n \leq \sum_{n=1}^{\infty} I_n + \frac{\varepsilon}{2} \leq m^*(A) + \varepsilon.$$

`by`

$$m(O) \ \leq \sum_{n=1}^{\infty} l(J_n) \leq \sum_{n=1}^{\infty} l(I_n) + \frac{\varepsilon}{2} \leq m^*(A) + \varepsilon.$$

page 39, line -9
`Replace`

$$m(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{\infty} m(A_i)$$

`by`

$$m(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} m(A_i)$$

page 63, line -6
`Replace`
$\mathbf{1}_A f_n$
`by`
$\mathbf{1}_{A^c} f_n$

page 63, line -5
`Replace`
$g = \mathbf{1}_A f$
`by`
$g = \mathbf{1}_{A^c} f$

page 94, line -9,
`Replace`
To prove (ii), note simply that if Riemann-integrable,
`by`
To prove (ii), note simply that if f is Riemann-integrable,

page 103, line 3,
`Replace` $x = m$
`by` $x = \mu$

page 106, line -5,
`Replace`
$$\{x : F(x) < \omega\} \subset \{x : F(x) < F(y)\} \subset \{x :\leq y\}$$
`by`
$$\{x : F(x) < \omega\} \subset \{x : F(x) < F(y)\} \subset \{x : x \leq y\}$$

page 107, line 16, Theorem 4.19,
`Replace`
$g : \mathbb{R}^n \to \mathbb{R}$ is integrable with respect to P
`by`
$g : \mathbb{R}^n \to \mathbb{R}$ is integrable with respect to P_X

page 108, line -5,
`Replace` $mu = 0$
`by` $\mu = 0$

page 109, line -13, Example 4.10
`Replace`
$$E(X) = \frac{1}{2} \cdot 0 + \frac{1}{2}\frac{1}{25}\int_0^{25} x\,dx = 12.5.$$
`by`
$$E(X) = \frac{1}{2} \cdot 0 + \frac{1}{2}\frac{1}{25}\int_0^{25} x\,dx = 6.25.$$

page 113, line -2,
`Replace` $h_n \leq |F|$
`by` $h_n \leq |f|$

page 114, line 9, Proof of Proposition 4.10
`Replace`
Continuous functions are measurable, hence $f \in \mathcal{L}^1[a,b]$.
`by`
Continuous functions are measurable, and f is bounded on $[a,b]$, hence $f \in \mathcal{L}^1[a,b]$.

page 122, line 8,
`Replace`
If $f, g \in L^2(E)$ then $fg \in L^1(E)$
`by`
If $f, g \in L^2(E, \mathbb{C})$ then $fg \in L^1(E, \mathbb{C})$

page 122, lines -5,-6,-7,
`Replace`

$$\begin{aligned}(2\int_{E_k} f_n g_n \,\mathrm{d}m)^2 &\leq 4(\int_{E_k} f_n^2 \,\mathrm{d}m)(\int_{E_k} g_n^2 \,\mathrm{d}m)^2 \\ &\leq 4(\int_E^2 |f|^2 \,\mathrm{d}m)(\int_E^2 |g|^2 \,\mathrm{d}m) \\ &= \|f\|_2^2 \|g\|_2^2.\end{aligned}$$

`by`

$$\begin{aligned}(2\int_{E_k} f_n g_n \,\mathrm{d}m)^2 &\leq 4(\int_{E_k} f_n^2 \,\mathrm{d}m)(\int_{E_k} g_n^2 \,\mathrm{d}m) \\ &= 4\|f\|_2^2 \|g\|_2^2.\end{aligned}$$

page 123, line 4,
`Replace`

$$\|f+g\|_2^2 = \int_E (f+g)^2 \,\mathrm{d}m = \int_E (f+g)\overline{(f+g)}\mathrm{d}m = \int_E (f+g)(\overline{f}+\overline{g}) \,\mathrm{d}m.$$

`by`

$$\|f+g\|_2^2 = \int_E |f+g|^2 \,\mathrm{d}m = \int_E (f+g)\overline{(f+g)}\mathrm{d}m = \int_E (f+g)(\overline{f}+\overline{g}) \,\mathrm{d}m.$$

page 131, line -8, Corollary 5.1
`Replace`

$$|(f,g)|^2 \leq \|f\|_p \|g\|_p$$

`by`

$$|(f,g)| \leq \|f\|_p \|g\|_p$$

page 133, line 3
`Replace`

$$\|f_n - f_{n_k}\|_p < \frac{1}{2^n}$$

`by`

$$\|f_n - f_{n_k}\|_p < \frac{1}{2^k}$$

page 149, line -11
`Replace`

$$\mathcal{G} = \{A \in \mathcal{F} : \text{ for all } \omega_2,\ A_{\omega_2} \in \mathcal{F}_2\}.$$

`by`

$$\mathcal{G} = \{A \in \mathcal{F} : \text{ for all } \omega_2,\ A_{\omega_2} \in \mathcal{F}_1\}.$$

page 149, line -5
`Replace` $\mathcal{F}_2$
`by` $\mathcal{F}_1$

page 150, line 8
`Replace`

$$P(A) = P_1(A_1)P_2(A_2) = \int_{\Omega_1} P(A_{\omega_2})\,\mathrm{d}P_2(\omega_2).$$

`by`

$$P(A) = P_1(A_1)P_2(A_2) = \int_{\Omega_2} P(A_{\omega_2})\,\mathrm{d}P_2(\omega_2).$$

page 150, line 10
`Replace`

$$P(A) = \int_{\Omega_1} P_1(A_{\omega_2})\,\mathrm{d}P_2(\omega_2)$$

`by`

$$P(A) = \int_{\Omega_2} P_1(A_{\omega_2})\,\mathrm{d}P_2(\omega_2)$$

page 150, line -7
`Replace`

$$\int_{\Omega_1} P_1(A_{\omega_2})\,\mathrm{d}P_2(\omega_2) = \int_{\Omega_2} P_2(A_{\omega_1})\,\mathrm{d}P_1(\omega_1)$$

`by`

$$\int_{\Omega_2} P_1(A_{\omega_2})\,\mathrm{d}P_2(\omega_2) = \int_{\Omega_1} P_2(A_{\omega_1})\,\mathrm{d}P_1(\omega_1).$$

page 150, line -5
`Replace`

This family contains $\mathcal{A}$ (if $B \in \mathcal{A}$, then $A \cup B \in \mathcal{A} \subset \mathcal{G}_{\mathcal{A}}$) and is a monotone class. For, let $A_1 \subset A_2 \subset \ldots$ be such that $A_i \cup B \in \mathcal{G}_{\mathcal{A}}$. Then $A_1 \cup B \subset A_2 \cup B \subset \ldots$ hence $\bigcup(A_i \cup B) \in \mathcal{G}_{\mathcal{A}}$, thus $B \cup \bigcup A_i \in \mathcal{G}_{\mathcal{A}}$. Similar arguments work for the intersection of a decreasing chain of sets so for this fixed A
`by`

This family contains $\mathcal{A}$ (if $B \in \mathcal{A}$, then $A \cup B \in \mathcal{A} \subset \mathcal{G}_{\mathcal{A}}$) and is a monotone class. For, let $B_1 \subset B_2 \subset \ldots$ be such that $A \cup B_i \in \mathcal{G}_{\mathcal{A}}$. Then $A \cup B_1 \subset A \cup B_2 \subset \ldots$ hence $\bigcup(A \cup B_i) \in \mathcal{G}_{\mathcal{A}}$, thus $A \cup \bigcup B_i \in \mathcal{G}_{\mathcal{A}}$. Similar arguments work for the intersection of a decreasing chain of sets so for this fixed A

page 152, line -14
`Replace`

$$\mathcal{G} = \Big\{ A : \omega_2 \mapsto P_1(A_{\omega_2}), \quad \omega_1 \mapsto P_2(A_{\omega_1}) \text{ are measurable and}$$
$$\int_{\Omega_1} P_1(A_{\omega_2})\,\mathrm{d}P_2(\omega_2) = \int_{\Omega_2} P_2(A_{\omega_1})\,\mathrm{d}P_1(\omega_1) \Big\}.$$

`by`

$$\mathcal{G} = \Big\{ A : \omega_2 \mapsto P_1(A_{\omega_2}), \quad \omega_1 \mapsto P_2(A_{\omega_1}) \text{ are measurable and}$$
$$\int_{\Omega_2} P_1(A_{\omega_2})\,\mathrm{d}P_2(\omega_2) = \int_{\Omega_1} P_2(A_{\omega_1})\,\mathrm{d}P_1(\omega_1) \Big\}.$$

page 155, line 9
`Replace`

$$P(A) = \int_{\Omega_2} P_2(A_{\omega_1})\,\mathrm{d}P_1(\omega_1).$$

`by`

$$P(A) = \int_{\Omega_1} P_2(A_{\omega_1})\,\mathrm{d}P_1(\omega_1).$$

page 156, line -1
`Replace`
$\int_{\Omega_1} f_n(\omega_1, \omega_2)\,\mathrm{d}\omega_1$
`by`
$\int_{\Omega_1} f_n(\omega_1, \omega_2)\,\mathrm{d}P_1(\omega_1)$

page 157, line 1
`Replace`
$\int_{\Omega_2} f_n(\omega_1, \omega_2)\,\mathrm{d}\omega_2$
`by`
$\int_{\Omega_2} f_n(\omega_1, \omega_2)\,\mathrm{d}P_2(\omega_2)$

page 171, line -1

Replace

$$\|f_n - f\|_\infty = \sup_{x\in E}(\|f_n(x) - f(x)\|) < \varepsilon.$$

by

$$\|f_n - f\|_\infty = \sup_{x\in E}(|f_n(x) - f(x)|) < \varepsilon.$$

page 172, line 2
Replace $||f_n(x) - f(x)|| < \varepsilon$
by $|f_n(x) - f(x)| < \varepsilon$

page 173, line 9
Replace

$$\int_0^1 |h_n(x) - h(x)|^p\,dx = \int_0^1 x^{pn}\,dx = \frac{1}{pn}x^{pn-1}\Big|_0^1 \to 0.$$

by

$$\int_0^1 |h_n(x) - h(x)|^p\,dx = \int_0^1 x^{pn}\,dx = \frac{1}{pn+1}x^{pn+1}\Big|_0^1 \to 0.$$

page 179, Exercise 7.4
Replace 1000
by 100

page 183, line 11
Replace $\leq \frac{E(X_1(n))}{n\varepsilon^2}$
by $\leq \frac{E(X_1^2(n))}{n\varepsilon^2}$

page 191, line 12
Replace

$$E(S_n) \geq E(\sum_{k=1}^n \varphi_k \cdot S_n)$$

by

$$E(S_n^2) \geq E(\sum_{k=1}^n \varphi_k \cdot S_n^2)$$

page 204, line 4
Replace

$$= \prod_{k=1}^n E(\mathrm{e}^{\mathrm{i}\frac{u}{c_n}X_k})$$

by

$$= \prod_{k=1}^n E(\mathrm{e}^{\mathrm{i}\frac{u}{c_n}X_k})$$

page 205, line 5

`Replace`

$$\sum_{k=1}^{n} \frac{1}{c_n} \alpha_{nk} \to 0$$

`by`

$$\sum_{k=1}^{n} \frac{1}{c_n^2} \alpha_{nk} \to 0$$

page 205, line 9

`Replace`

$$\sum_{k=1}^{n} \frac{1}{c_n} \beta_{nk} \to 1$$

`by`

$$\sum_{k=1}^{n} \frac{1}{c_n^2} \beta_{nk} \to 1$$

page 206, line -9

`Replace`

$$\leq \Big| \sum_{k=1}^{n} \gamma_{nk} + \frac{1}{2} u^2 - \frac{1}{6} |u| \theta_3' \varepsilon \Big| + \sum_{k=1}^{n} |\gamma_{nk}|^2 + |u|^3 \varepsilon |\theta_3'|.$$

`by`

$$\leq \Big| \sum_{k=1}^{n} \gamma_{nk} + \frac{1}{2} u^2 - \frac{1}{6} |u|^3 \theta_3' \varepsilon \Big| + \sum_{k=1}^{n} |\gamma_{nk}|^2 + |u|^3 \varepsilon |\theta_3'|.$$

page 208, line 13

`Replace`

Conversely, assume that $d(X_n, X) \to 0$. This implies convergence to zero of $\frac{|X_n - X|}{1+|X_n - X|}$ almost surely. For fixed ω, for the fractions to converge to zero, their numerators must converge to zero, which means that $X_n \to X$ almost surely and this implies convergence in probability by Theorem 7.2.

`by`

Conversely, let $E_{\varepsilon,n} = \{\omega : |X_n(\omega) - X(\omega)| > \varepsilon\}$ and assume $0 < \varepsilon < 1$. Additonaly let $A_n = \{\omega : |X_n(\omega - X(\omega)| < 1\}$ and write $d(X_n X) = \int_{A_n} \frac{|X_n - X|}{1+|X_n - X|} dP + \int_{A_n^c} \frac{|X_n - X|}{1+|X_n - X|} dP$. We estimate from below each of the two terms. First,

$$\int_{A_n} \frac{|X_n - X|}{1 + |X_n - X|} dP \geq \int_{A_n \cap E_{\varepsilon,n}} \frac{|X_n - X|}{1 + |X_n - X|} dP \geq \frac{1}{2} \int_{A_n \cap E_\varepsilon} \varepsilon dP = \frac{\varepsilon}{2} P(A_n \cap E_{\varepsilon,n})$$

since $\frac{a}{1+a} > \frac{a}{2}$ if $a < 1$. Second,

$$\int_{A_n^c} \frac{|X_n - X|}{1 + |X_n - X|} dP \geq \int_{A_n^c} \frac{1}{2} dP \geq \int_{A_n^c \cap E_{\varepsilon,n}} \frac{1}{2} dP \geq \frac{\varepsilon}{2} P(A_n \cap E_{\varepsilon,n})$$

since $\varepsilon < 1$. Hence, $d(X_n, X) \geq \frac{\varepsilon}{2} P(E_{\varepsilon,n}) \to 0$, so (X_n) converges to X in probability.

page 210, line 5
`Replace`
$m^*(\bigcup O_n \setminus E) = 0$, $m^*(E \setminus \bigcap F_n) = 0$
`by`
$m^*(\bigcap O_n \setminus E) = 0$, $m^*(E \setminus \bigcup F_n) = 0$

page 210, line 9
`Replace`

$$P(A) = \sum_{i=1}^{\infty} P(A \cap H_i) = \sum_{i=1}^{\infty} P(A|H_i) \cdot P(\cap H_i)$$

`by`

$$P(A) = \sum_{i=1}^{\infty} P(A \cap H_i) = \sum_{i=1}^{\infty} P(A|H_i) \cdot P(H_i)$$

page 212, line 9
`Replace`

$$\lim \int_0^{\infty} f_n \, \mathrm{d}m = \int_0^{\infty} \lim f_n \, \mathrm{d}m = \int_0^{\infty} u\mathrm{e}^{u^2} \, \mathrm{d}u = \frac{1}{2}.$$

`by`

$$\lim \int_0^{\infty} f_n \, \mathrm{d}m = \int_0^{\infty} \lim f_n \, \mathrm{d}m = \int_0^{\infty} u\mathrm{e}^{-u^2} \, \mathrm{d}u = \frac{1}{2}.$$

page 214, line 16, 5.4
`Replace`
(a) $\|f_n - f_m\|_1 = 2$ if $n \neq m$
`by`
(a) $\|f_n - f_m\|_1 = m - n$ if $m > n$

page 215, line 12, 5.13
`Replace`
$\mathrm{Cov}(Y, 2Y+1) = E(Y)E(2Y+1) - E(Y)E(2Y+1) =$
`by`
$\mathrm{Cov}(Y, 2Y+1) = E((Y)(2Y+1)) - E(Y)E(2Y+1) =$