Course: Telecommunication Network Design
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Studies: Electronics and Telecommunications
Speciality: Networks and Services
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Draft of the lecture

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1 Shortest Paths in Graphs

1.1 Shortest path algorithms and their modifications

1. Problem of searching for the shortest path in a graph.

2. Dijkstra's algorithm (the labeling algorithm), assumptions taken to apply this algorithm, algorithm complexity: $\mathcal{O}\left(|V|^2\right)$, looking for a tree of the shortest paths:

procedure DIJKSTRA($G = (V, A, d)$, $r \in V$)
▷ r: root
▷ For non-adjacent vertices, we take $d_{ij} = \infty$: $(i, j) \notin A \Rightarrow d_{ij} = \infty$
▷ Initialization:
  $\mathcal{S} \leftarrow \{r\}$
  ▷ $\mathcal{S}$: a set of labeled vertices (for which the shortes path from r if found)
  $\mathcal{S}' \leftarrow V \setminus \{r\}$
  ▷ $\mathcal{S}'$: a set of unlabeled vertices
  for all $j \in \mathcal{S}'$ **do**
    $D_{rj} \leftarrow d_{rj}$
    if $D_{rj} < \infty$ **then**
      $predecessor(j) = r$
    end if
  end for
  ▷ Main loop:
  while $\mathcal{S}' \neq \emptyset$ **do**
    $k \leftarrow \arg\min_{m \in \mathcal{S}'}\{D_{rm}\}$
    $\mathcal{S} \leftarrow \mathcal{S} \cup \{k\}$
    $\mathcal{S}' \leftarrow \mathcal{S}' \setminus \{k\}$
    ▷ Check improvement of the existing shortest path found so far:
    ▷ $\mathcal{N}_k^-$ is a *forward star* for k, $\mathcal{N}_k^- = \{j \in V : (k, j) \in A\}$
    for all $j \in \mathcal{N}_k^- \cap \mathcal{S}'$ **do**
      if $D_{rk} + d_{kj} < D_{rj}$ **then**
        $D_{rj} \leftarrow D_{rk} + d_{kj}$
        $predecessor(j) = k$
      end if

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        end for
    end while
end procedure
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3. Modified Dijkstra's algorithm (for negative weights, but without negative cycles).

4. Bhandari's algorithm to look for the shortest pair of paths.

1.2 Exercises

- Give an example of a weighted graph with the selected vertex r, so that in this graph: (a) a tree of the shortest paths with root in r, and (b) minimum spanning tree obtained with the Prim's algorithm (when starting with vertex r) are not identical.

1.3 Reading

1.3.1 Contents of the lecture

Problems described in this lecture are generally dealt with in the following positions:

- Wayne D. Grover. *Mesh-Based Survivable Networks. Options and Strategies for Optical, MPLS, SONET, and ATM Networks.* Prentice Hall PTR, Upper Saddle River, NJ, 2004: section 4.10.

- Deepankar Medhi and Karthikeyan Ramasamy. *Network Routing. Algorithms, Protocols, and Architectures.* Morgan Kaufmann Publishers—Elsevier, San Francisco, CA, 2007: chapter 2.

- Michał Pióro and Deepankar Medhi. *Routing, Flow and Capacity Design in Communication and Computer Networks.* Morgan Kaufmann Publishers—Elsevier, San Francisco, CA, 2004: appendix C.1-C.2.

1.3.2 Auxiliary references

- Ramesh Bhandari. *Survivable Networks. Algorithms for Diverse Routing.* Kluwer Academic Publishers, Dordrecht, The Netherlands, 1999: overview of various algorithms useful in network design (mainly for resilient networks).

- David Eppstein. Finding the k Shortest Paths. *SIAM Journal on Computing*, 28(2):652–673, 1998: k-shortest-paths problem (KST).