Course:	Telecommunication Network Design
Teacher:	Piotr Chołda piotr.cholda@agh.edu.pl
Studies:	Electronics and Telecommunications
Speciality:	Networks and Services
Semester:	2^{nd} sem. MSc stud., Fall

Draft of the lecture

Piotr Chołda

November 14, 2017

1 Network Design based on Mathematical Programming — Introduction

1.1 Mathematical methods used in network design

1. System design with applied optimization.
2. Structure of the formulation of an optimization problem/task in mathematical programming:
   - indices,
   - constants,
   - variables,
   - goal function (objective),
   - constraints.
3. Types of variables:
   - continuous ($\in \mathbb{R}$),
   - integer ($\in \mathbb{Z}$),
   - binary ($\in \mathbb{B} = \{0, 1\}$), typically known as decision variables (e.g., 1: a condition is met, 0: a condition is not met).
4. Two basic approaches to optimization:
   - MIN: cost-centered, loss-centered;
   - MAX: profit-centered, utility-centered.
5. Types of mathematical programming approaches interesting for us (we deal with constraint programming only):
   - Linear programming (LP): continuous variables; the objective and constraints are linear (i.e., given by linear equalities or inequalites).
   - Combinatorial optimization: with finite number of feasible solutions — some problems of discrete programming, including integer programming (IP): integer variables (when only binary: binary integer programming, BIP) — we limit our interest mainly to models with the linear objective and constraints (integer linear programming, I[L]P).

- MI[L]P: mixed integer (linear) programming — continuous and integer variables; the objective and constraints are linear.
- Non-linear programming: many different, typically very complex and difficult to solve problems; we will focus on simple representatives of two groups only — convex programming (CXP) and concave programming (CVP), mainly in the context of the objective linearization (with the linear constraints).

6. Feasible solutions versus optimal solutions.

7. Various options related to the existence of solutions in linear programming problems:

   - there is a single feasible solution,
   - there are feasible solutions, including infinite number of optimal solutions,
   - an infeasible problem (assumptions are contradictory and the solution space is empty),
   - an unbounded problem: for each feasible solution it is possible to find another feasible solution with a better value of the goal function.

1.2 Basics of linear programming

1. Standard form, matrix form, solution space. Standard form:

   - MAX:
     - ⋆ $z = \sum_{j=1,2,\ldots,n} c_j x_j$
   - S.t. (subject to):
     - ⋆ $\sum_{j=1,2,\ldots,n} a_{ij} x_j = b_i \quad i = 1, 2, \ldots, m;$
     - ⋆ $x_j \geq 0 \quad j = 1, 2, \ldots, n;$
     - ⋆ $x_j \in \mathbb{R} \quad j = 1, 2, \ldots, n.$

   and its matrix form:

   - MAX:
     - ⋆ $z = \mathbf{c}\mathbf{x}^T$ (transposition is usually not emphasised, especially when it is obvious from the context; therefore, typically it is written as: $z = \mathbf{cx}$).
   - S.t.:
     - ⋆ $\mathbf{A}\mathbf{x}^T = \mathbf{b}^T$ ($\mathbf{Ax} = \mathbf{b}$);
     - ⋆ $\mathbf{x}^T \geq \mathbf{0}^T$ ($\mathbf{x} \geq \mathbf{0}$).

   and the compact notation: $z = \max\{\mathbf{cx} : \mathbf{Ax} = \mathbf{b} \wedge \mathbf{x} \geq \mathbf{0}\}$.

2. Basic solution method: simplex method (Dantzig), extreme point, simplex, basis matrix, basis solution, feasible basis solution, properties of the optimal solution (a number of non-zero optimal variables).

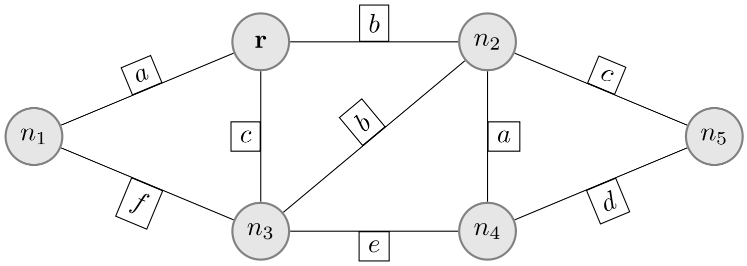

Figure 1: An example of a weighted graph.

1.3 Exercises

- An optimization task is given as below:
  - ⋆ goal function: $\min z = 2x_1 + 2x_2$
  - ⋆ constraints:
    - ∗ $x_1 \leq 5$
    - ∗ $x_2 \leq 5$
    - ∗ $Ax_1 + Bx_2 \leq C$

  Find such A, B, and C ($A, B, C \in \mathbb{R} \setminus \{0\}$), so that this task is infeasible.

- A weighted graph is given in Fig. 1 (values of weights given in rectangles can be replaced with values represented by digits of the number of your student's book, where the number is $fedcba$). Find the tree of the shortest paths starting at root **r**. Then, formulate a linear programming LP problem used to find this tree (indices, constants, variables, goal function and constraints should be selected to describe the given graph). If necessary, you can replace each edge with a pair of inversely directed arcs having the same weight as the transformed edge.

- A weighted graph is given in Fig. 1 (values of weights given in rectangles can be replaced with values represented by digits of the number of your student's book, where the number is $fedcba$). Find the minimum cut between vertices n_1 and n_5. Then, formulate a linear programming LP problem used to find this cut (indices, constants, variables, goal function and constraints should be selected to describe the given graph). If necessary, you can replace each edge with a pair of inversely directed arcs having the same weight as the transformed edge.

- Let x_{ij} be 1 if router i is physically connected to router j (x_{ij} is 0, otherwise). Let there are N routers in a network. How to interpret the following constraints, when explaining them to a network engineer responsible for router configuration?

$$x_{ii} = 0 \qquad i = 1, 2, \ldots, N$$
$$\sum_{j=1}^{N} x_{ij} \leq 5 \qquad i = 1, 2, \ldots, N$$
$$\sum_{j=1}^{N} x_{ij} \geq 1 \qquad i = 1, 2, \ldots, N$$

- Routers are installed in each of N cities where an operator is active. Let x_{ij} be 1 if the operator installs routers produced by vendor i in city j (x_{ij} is 0, otherwise). There are P router vendors in the market. How to interpret the following constraints, when explaining them to a network engineer responsible for router installation?

$$\sum_{i=1}^{P} x_{ij} \geq 2 \qquad j = 1, 2, \ldots, N$$

- Hubs are installed in each of P cities where an operator is active. Let x_{ij} indicate a number of users connected to a hub in city j, but this concerns only the users who subscribe to a QoS class indexed with i. The operator offers N classes in its portfolio. How to interpret the following constraints, when explaining them to a network engineer responsible for hub installation?

$$\sum_{i=2}^{N} x_{ij} = x_{1j} \qquad j = 1, 2, \ldots, P$$

1.4 Reading

1.4.1 Contents of the lecture

Problems described in this lecture are generally dealt with in the following position:

- Michał Pióro and Deepankar Medhi. *Routing, Flow and Capacity Design in Communication and Computer Networks.* Morgan Kaufmann Publishers—Elsevier, San Francisco, CA, 2004: appendix C.3.

1.4.2 Auxiliary references

- Terje Jensen. Network Planning—Introductory Issues. *Telektronikk*, 99(3/4):9–46, 2003: practical side of network design.
- Michał Pióro and Deepankar Medhi. *Routing, Flow and Capacity Design in Communication and Computer Networks.* Morgan Kaufmann Publishers—Elsevier, San Francisco, CA, 2004: basic network design problems.
- Poompat Saengudomlert. *Optimization for Communications and Networks.* CRC Press/Science Publishers, Boca Raton, FL, 2012: overview of various optimization problems relevant to communications/computer networks.