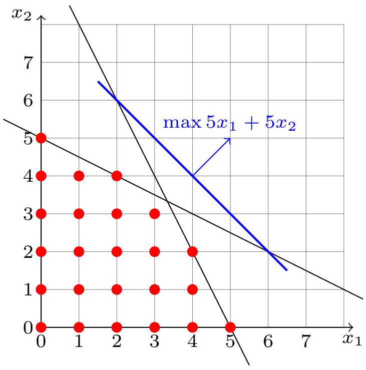
I(L)P and MI(L)P:
Integer (Linear) Program:

- maximize $z = \mathbf{cx}$,
- subject to $\mathbf{Ax} \leq \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$ (linear constraints),
- $\mathbf{x}$: integer (integrality constraints).

Example:

- maximize $z = 5x_1 + 5x_2$,
- subject to $2x_1 + x_2 \leq 10$,
- $x_1 + 2x_2 \leq 10$,
- $x_1 \geq 0$, $x_2 \geq 0$ and integer.

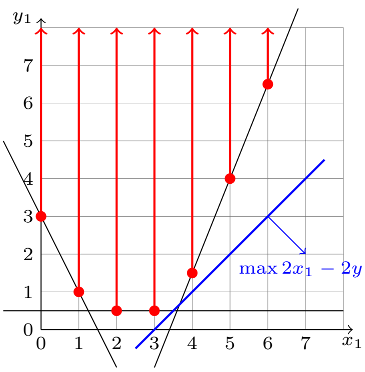


Mixed Integer (Linear) Program:

- maximize $z = \mathbf{cx} + \mathbf{dy}$
- subject to $\mathbf{Ax} + \mathbf{Dy} \leq \mathbf{b}$, $\mathbf{x}, \mathbf{y} \geq \mathbf{0}$ (linear constraints),
- $\mathbf{x}$: integer (integrality constraints).

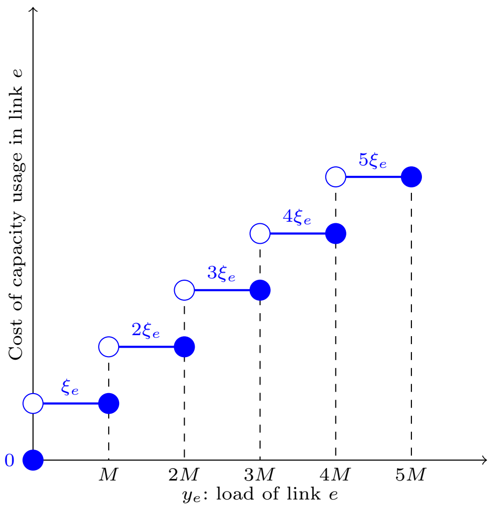
Example:

- maximize $z = 2x_1 - 2y_1$,
- subject to $-2x_1 - y_1 \leq -3$,
- $5x_1 - 2y_1 \leq 17$,
- $-2y_1 \leq -1$,
- $x_1 \geq 0$ and integer, $y_1 \geq 0$.



Flow allocation with modular capacity:

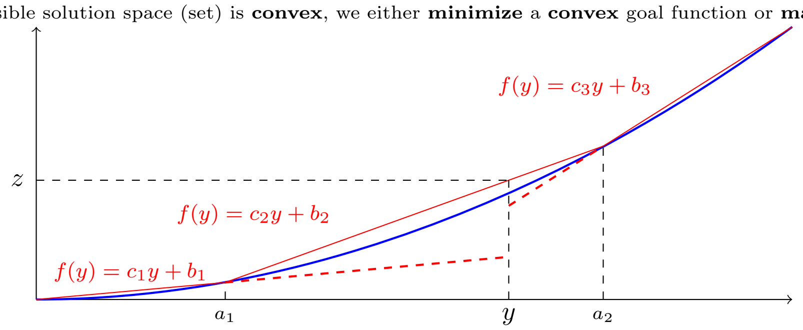
Modular capacity cost (a fixed capacity module M for which we pay ξ_e):



<u>**Linearization of a non-linear goal function $z = f(y)$ with linear constraints:**</u>

<u>Convex problem:</u>

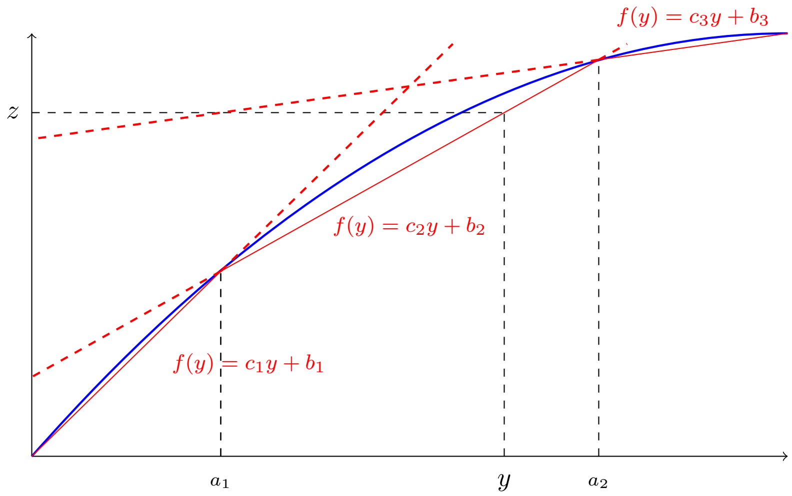
Feasible solution space (set) is **convex**, we either **minimize** a **convex** goal function or **maximize** a **concave** goal function.



- min z;
- constraints:
  - ⋆ $y = \ldots$; [e.g., $y = \sum_d \sum_p \delta_{edp} x_{dp}$]
  - ⋆ $\ldots$; [other constraints]
  - ⋆ $z \geq c_k y + b_k \qquad k = 1, 2, \ldots, n$.

<u>Concave problem:</u>

Feasible solution space (set) is **convex** (!), we either **minimize** a **concave** goal function or **maximize** a **convex** goal function.



- min $z = \sum_k (c_k y_k + b_k u_k)$;
- constraints:
  - ⋆ $y = \ldots$; [e.g., $y = \sum_d \sum_p \delta_{edp} x_{dp}$]
  - ⋆ $\ldots$; [other constraints]
  - ⋆ $\sum_k y_k = y$;
  - ⋆ $\sum_k u_k = 1$;
  - ⋆ $0 \leq y_k \leq W u_k \qquad k = 1, 2, \ldots, n$;
  - ⋆ u_k — binary.
- constant W is greater than any feasible y.