

Draft of the lecture

Piotr Cholda

January 3, 2018

1 Duality Theory

1.1 Basics of duality theory in linear programming

1. Dual problem to a linear programming problem (primal problem) — LP dual.
2. Relationships between primal and dual problems.
3. Weak duality.
4. Duality principle (strong duality).
5. Applications of duality:
 - finding bounds on the optimal solution of the primal problem,
 - sensitivity analysis for a solution,
 - complementary slackness property.
6. Mutual duality of the max flow and min cut problems.

1.2 Lagrangian duality

7. Lagrangian relaxation: Lagrangian function, dual variables.
8. Lagrangian duality¹: dual function, dual problem based on Lagrangian relaxation (Lagrangian dual), weak duality theorem, duality gap, Lagrangian multipliers.
9. Identity of Lagrangian dual and LP dual.
10. Lagrangian dual for a convex problem and its properties.
11. Lagrangian dual for MILP and its properties.

¹If we tell about duality for problems other than LP, we are likely to mean the Lagrangian duality, i.e., the one enabling us to remove some constraints and put them to the goal function. There are other types of duality — the obligatory reading presents some of them.

Course: Telecommunication Network Design
Teacher: Piotr Cholda piotr.cholda@agh.edu.pl
Studies: Electronics and Telecommunications
Speciality: Networks and Services
Semester: 2nd sem. MSc stud., Fall
.....

1.3 Reading

1.3.1 Contents of the lecture

Problems described in this lecture are generally deal with in the following positions:

- Michał Pióro and Deepankar Medhi. *Routing, Flow and Capacity Design in Communication and Computer Networks*. Morgan Kaufmann Publishers—Elsevier, San Francisco, CA, 2004: chapter 5.1.2, 5.4.1, appendix A.5-A.7.
- Poompat Saengudomlert. *Optimization for Communications and Networks*. CRC Press/Science Publishers, Boca Raton, FL, 2012: chapter 2.4-2.7, 3.4, 4.4.

1.3.2 Auxiliary references

- John N. Hooker. Integer Programming Duality. In Christodoulos A. Floudas and Panos M. Pardalos, editors, *Encyclopedia of Optimization*, pages 1657–1667. Springer Science+Business Media, LLC., New York, NY, 2009: various types of relaxations and duality.
- Michał Pióro and Deepankar Medhi. *Routing, Flow and Capacity Design in Communication and Computer Networks*. Morgan Kaufmann Publishers—Elsevier, San Francisco, CA, 2004: introduction to duality theory and its use in network design.
- Poompat Saengudomlert. *Optimization for Communications and Networks*. CRC Press/Science Publishers, Boca Raton, FL, 2012: duality theory in network design problems.