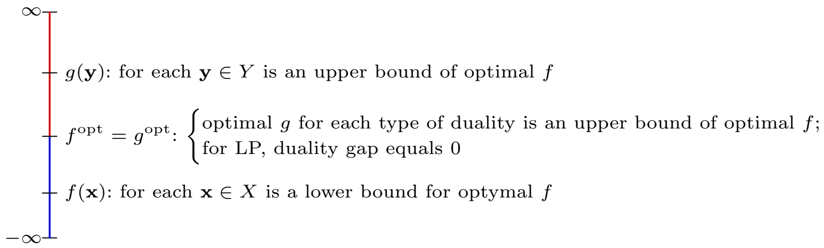
LP duality:

Primal problem		**Dual problem**	
max	$f = \mathbf{cx}$	min	$g = \mathbf{by}$
s.t.:	$X = \{\mathbf{Ax} \le \mathbf{b}, \mathbf{x} \ge \mathbf{0}\}$	s.t.:	$Y = \{\mathbf{A}^T\mathbf{y} \ge \mathbf{c}, \mathbf{y} \ge \mathbf{0}\}$
Goal function (max $\mathbf{cx}$)		Right-hand side $\mathbf{c}$	
Right-hand side $\mathbf{b}$		Goal function (min $\mathbf{by}$)	
$\mathbf{A}$: matrix of constraints		$\mathbf{A}^T$: matrix of constraints	
Unbounded		Infeasible	
Infeasible		Unbounded or infeasible	

Weak duality: $\forall_{\mathbf{x}\in X}\, \forall_{\mathbf{y}\in Y}\; \mathbf{cx} \le \mathbf{by}$

Strong duality (duality principle): $f^{\text{opt}} = g^{\text{opt}}$

Bounds (primal problem is neither infeasible nor unbounded):



Lagrangian dual:

Primal problem: max $z = F(\mathbf{x})$, $h_i(\mathbf{x}) = 0, i = 1, \dots, k, g_j(\mathbf{x}) \ge 0, j = 1, \dots, m, \mathbf{x} \in X$ (X can be itself defined with a set of equalities and inequalities).

Lagrangian function: $L(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\lambda}) = F(\mathbf{x}) + \sum_i \mu_i h_i(\mathbf{x}) + \sum_j \lambda_j g_j(\mathbf{x})$

Dualization of constraints: equalities $h_i(\mathbf{x}) = 0, i = 1, \dots, k$ and inequalities $g_j(\mathbf{x}) \ge 0, j = 1, \dots, m$

Dual variables (defined so that a proper relaxation is obtained): $\mu_i \in \mathbb{R}$, $\lambda_j \in \mathbb{R}, \lambda_j \ge 0$

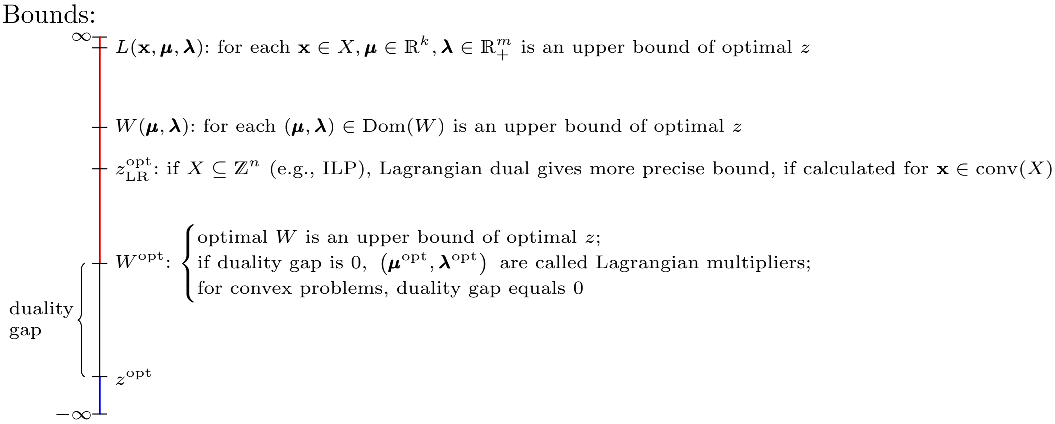
Lagrangian relaxation: $\max L(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\lambda})$, $\mathbf{x} \in X, \boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\lambda} \in \mathbb{R}^m_+$

Dual function: $W(\boldsymbol{\mu}, \boldsymbol{\lambda}) = \max_{\mathbf{x}\in\mathbf{X}} L(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\lambda})$

Domain of a dual function: $\text{Dom}(W) = \{(\boldsymbol{\mu}, \boldsymbol{\lambda})\, |\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\lambda} \in \mathbb{R}^m_+, W(\boldsymbol{\mu}, \boldsymbol{\lambda}) < \infty\}$

Lagrangian dual: $\min W(\boldsymbol{\mu}, \boldsymbol{\lambda})$, $(\boldsymbol{\mu}, \boldsymbol{\lambda}) \in \text{Dom}(W)$

Bounds:



For a convex problem:

- X: convex set,
- h_i: linear functions,
- g_j: functions convex on X,
- min F (F: convex on X) or max F ($-F$: convex on X).

we have the following properties:

- duality gap equals 0,
- complementary slackness,
- saddle point: $W(\boldsymbol{\lambda}^{\text{opt}}) = L(\mathbf{x}^{\text{opt}}; \boldsymbol{\lambda}^{\text{opt}})$.