Novel Parallel Algorithm for Object Recognition with the Ensemble of Classifiers based on the Higher-Order Singular Value Decomposition of Prototype Pattern Tensors

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Abstract: In this paper a novel parallel algorithm for the tensor based classifiers for object recognition in digital images is presented. Classification is performed with an ensemble of base classifiers, each operating in the orthogonal subspaces obtained with the Higher-Order Singular Value Decomposition (HOSVD) of the prototype pattern tensors. Parallelism of the system is realized through the functional and data decompositions on different levels of computations. First, the parallel implementation of the HOSVD is presented. Then, the second level of parallelism is gained by partitioning the input dataset. Each of the partitions is used to train a separate tensor classifiers of the ensemble. Despite the computational speed-up and lower memory requirements, also accuracy of the ensemble showed to be higher compared to a single classifier. The method was tested in the context of object recognition in computer vision. The experiments show high accuracy and accelerated performance both in the training and classification stages.

1 INTRODUCTION

Tensor based methods found great interest in pattern recognition domain. In computer vision these were also shown to provide excellent results in object recognition (Vasillescu and Terzopoulos, 2002; Savas, 2007; Cyganek, 2010). Tensor based methods account for multidimensional nature of processed data. However, the price for tensor processing and decomposition, necessary for object recognition, is high memory and computation time. Thus, important is development of new parallel algorithms for tensor processing which allow full exploitation of the contemporary multi-core microprocessor. In this paper we propose such a new parallel algorithm, as will be discussed.

For different object recognition problems there are many examples of classifiers which can achieve either high accuracy or fast response (Duda, 2000). However, the goal of reaching high accuracy *and* response factors is easier with classifiers which architecture naturally allows parallel processing. For many real data classification tasks such requirements can be accomplished with *ensembles of classifiers*, which recently gained much attention (Kuncheva,

2005; Polikar, 2006; Cyganek, 2010). Such ensembles usually rely on operation of a group of cooperating classifiers. When cooperating in an ensemble, despite their moderate individual classification abilities, such a group frequently shows superior accuracy compared to the more complex but single classifiers (Kuncheva, 2005; Polikar, 2006).

In this paper the problem of parallel implementation of the ensemble composed of tensor classifiers operating with the multi-dimensional data is discussed. The system extends our previous work on the problem of handwritten digits classification, as well as road signs recognition. These showed high accuracy using a serial software implementation (Cyganek, 2010; Cyganek, 2012). In this paper we present parallel versions of the mentioned implementation.

In the proposed system, the member classifiers perform subspace classification in the spaces spanned by the bases obtained from the Higher-Order Singular Value Decomposition (HOSVD) of the prototype pattern tensors. The HOSVD classifier shows good results when applied to multi dimensional data, such as images (Vasilescu and

Terzopoulos, 2002; Cyganek, 2013). This is due to tensor processing which allows separate control of all intrinsic dimensions of data. Let us recall that in the classical PCA-based classification method, images are first vectorized and, in the result, the obtained subspaces are spanned by vector bases (Turk, 1991). Contrary to this, in the HOSVD method the bases of the orthogonal pattern subspace are spanned by the two-dimensional tensors (images). In both methods, when classifying an unknown pattern, these are projected onto the subspaces of each of the trained class and the best fitting projection is returned. However, in the tensor case the bases are multidimensional. Nevertheless, computation of tensor decompositions, such as the HOSVD, is both time and memory demanding.

In the proposed system parallelism is obtained through the functional and data decompositions on different levels of computations. First, the parallel implementation of the HOSVD is presented. Then, the second level of parallelism relies on data decomposition. For this purpose, the training dataset is partitioned into smaller chunks, either by clustering or bagging, as discussed in our previous works (Cyganek, 2013). Each of the training data partitions is then used to concurrently train a corresponding separate tensor classifier of the ensemble. Thanks to this, also memory requirements for the training stage are greatly reduced. Last but not least, the third possible level of parallelism is obtained on the multi-class level since each of the training classes can be trained independently of the others. Parallel operation is also possible at the response time of the system, since each subspace projection can be also computed independently. The experiments with the proposed system applied to the problem of image recognition show high accuracy, adjustable memory requirements, and accelerated performance both in the training and response stages.

2 MULTI-LEVEL PARALLEL ARCHITECTURE OF THE ENSEMBLE OF MULTI-CLASS CLASSIFIERS

As already mentioned, tensor processing usually results in high computational and memory demands. The former can be alleviated by parallel implementation of the specifically chosen parts of the system, as will be discussed. In this approach we exploit both strategies for parallel decompositions:

- Data decomposition.
- Functional decomposition.

However, in many real situations, parallel processing of some software modules leads also to higher memory demands. Therefore both aspects, i.e. computational speed-up due to concurrency, as well as memory requirements, need to be considered together. These issues are addresses in the proposed classification system.

Figure 1 shows an architecture of a single multi-class HOSVD based classifier. Each training dataset of each class is used to build a separate tensor subspace. During classification, a test pattern is projected onto each of these subspaces to check the closest class. Parallel operation of this module is obtained due to the data decomposition, as well as parallel implementation of the HOSVD algorithm, as will be described.
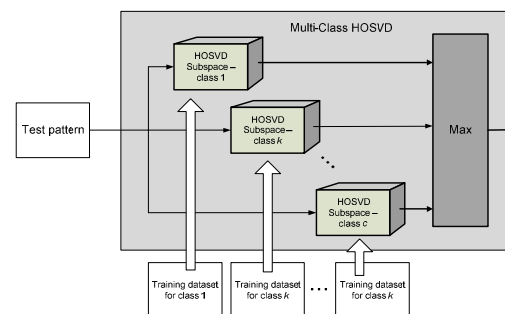

Figure 1: Architecture of a HOSVD based single classifier for multi-class classification. Each training dataset of each class is used to build a separate tensor subspace. A test pattern is projected onto each of the subspaces to check the best fit. Parallel operation of this module can be obtained due to data decomposition, as well as parallel implementation of the HOSVD algorithm.

Figure 2 presents an extension to the system in Figure 1. It is an ensemble composed of the multi-class HOSVD classifiers. Each grayed block has a structure as shown in Figure 1.

3 PARALLELIZATION OF THE HOSVD SUBSPACE CLASSIFIER

Tensors in data mining can be interpreted as multidimensional arrays. Processing and analysis of multi-dimensional data, such as images, builds well into this framework. However, an analysis of data content requires proper decomposition of pattern tensors. One of the most popular decomposition

method is the HOSVD (Cichocki, 2009; Lathauwer, 1997; Lathauwer, 2000; Cyganek, 2013). It can be used to build orthogonal spaces which can be then used for pattern recognition in a way similar to the subspace projection methods (Duda, 2000)(Turk, 1991). This procedure is briefly outlined in this section. More information on tensors in signal processing can be found in literature, e.g. (Cichocki, 2009; Lathauwer, 1997; Lathauwer, 2000; Cyganek, 2013; Cichocki, 2009).
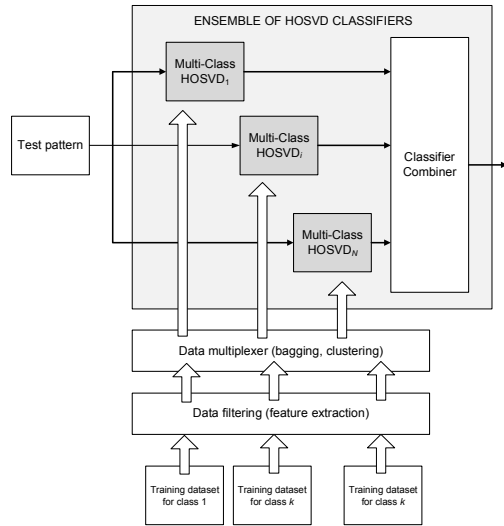

Figure 2: Architecture of the ensemble composed of the multi-class HOSVD classifiers. Each base classifier has a structure shown in Figure 1 (grayed blocks). Each member multi-class classifier is trained with its partition of training data from each class. Data partitions are obtained due to a clustering or a bagging process.

In this section let us briefly present the main concepts of tensors and their decomposition. The first concept is the *k-mode vector* of a *P-th* order tensor $\mathcal{T} \in \Re^{N_1 \times N_2 \times ... N_P}$. It is a vector obtained from the elements of $\mathcal{T}$ by changing only one index n_k, and keeping all other fixed. The second important concept is the operation of the *k*-mode flattening of a tensor. For a tensor $\mathcal{T}$, a result of its *k*-mode flattening is the following matrix (Lathauwer, 1997; Kolda, 2008)

$$\mathbf{T}_{(k)} \in \Re^{N_k \times \left(N_1 N_2 ... N_{k-1} N_{k+1} ... N_P \right)} . \tag{1}$$

Now we can define the HOSVD decomposition. For any *P*-dimensional tensor $\mathcal{T} \in \Re^{N_1 \times N_2 \times ... N_m \times ... N_n \times ... N_P}$, the HOSVD

decomposition allows to equivalently represent $\mathcal{T}$ in the following form (Lathauwer, 1997)

$$\mathcal{T} = \mathcal{Z} \times_1 \mathbf{S}_1 \times_2 \mathbf{S}_2 ... \times_P \mathbf{S}_P . \tag{2}$$

In (2) $\mathbf{S}_k$ denote unitary matrices of dimensions $N_k \times N_k$, which are called *mode matrices*. The tensor $\mathcal{Z} \in \Re^{N_1 \times N_2 \times ... N_m \times ... N_n \times ... N_P}$ is *a core tensor* which fulfills properties of the sub-tensor orthogonality and decreasing energy value (Lathauwer, 1997)(Kolda, 2008).

Lathauwer proposed a method of computation of the HOSVD which is based on successive application of the matrix SVD decompositions to the flattened matrices of a given tensor (Lathauwer, 1997). The HOSVD decomposition algorithm for a *P*-dimensional tensor $\mathcal{T}$ is outlined in Figure 3. It can be easily observed that computation of the HOSVD requires a series of computations of the SVD decompositions on flattened matrices. These are independent versions (different modality) of the input tensor. Therefore it is possible to run all these SVD decompositions concurrently, which must be synchronized on a barrier just before computation of the core tensor in (8), however. Figure 3 shows the algorithm for computation of the HOSVD. Its grayed area can be run concurrently, as discussed.

Let us now observe that, thanks to the commutative properties of the *k*-mode multiplication, for each mode matrix $\mathbf{S}_i$ in (2) the following sum can be constructed

$$\mathcal{T} = \sum_{h=1}^{N_P} \mathcal{T}_h \times_P \mathbf{s}_P^h . \tag{3}$$

Further, it can be shown that tensors

$$\mathcal{T}_h = \mathcal{Z} \times_1 \mathbf{S}_1 \times_2 \mathbf{S}_2 ... \times_{P-1} \mathbf{S}_{P-1} \tag{4}$$

in (3) constitute the basis tensors and $\mathbf{s}_P^h$ are columns of the unitary matrix $\mathbf{S}_P$ (Lathauwer, 1997)(Lathauwer, 2000). Thus, they form an orthogonal basis which spans a subspace. This property is used to construct a HOSVD based classifier (Savas, 2007; Cyganek, 2010).

In each subspace spanned by tensors $\mathcal{T}_h$, pattern recognition can be stated as a testing of a distance of a given test pattern $\mathbf{P}_x$ to its projections in each of the spaces spanned by the set of the bases $\mathcal{T}_h$ in (4). That is, the following optimization process needs to be solved (Savas, 2007):

$$\min_{i,c_h^i} \left\| \mathbf{P}_x - \underbrace{\sum_{h=1}^{H} c_h^i \mathcal{T}_h^i}_{Q_i} \right\|^2, \qquad (5)$$

where the scalars c_h^i denote unknown coordinates of the pattern $\mathbf{P}_x$ in the space spanned by $\mathcal{T}_h^i$, and $H \leq N_P$ denotes a number of chosen dominating components. It can be further shown that to minimize (5) we need to maximize the following value (Savas, 2007; Cyganek, 2013)

$$\hat{\rho}_i = \sum_{h=1}^{H} \left\langle \hat{\mathcal{T}}_h^i, \hat{P}_x \right\rangle^2. \qquad (6)$$

In other words, the (single) HOSVD based classifier returns a class i for which its ρ_i from (6) is the largest.

```
begin
    for each k=1, …, P do
        1. From Eq.(1) compute k-
        mode flattened matrix T_k of
        tensor T

        2. Compute S_k from the SVD
        decomposition of T_k
```

$$\mathbf{T}_k = \mathbf{S}_k \mathbf{V}_k \mathbf{D}_k^T \qquad (7)$$

```
    end

    Compute the core tensor from all
    matrices S_k
```

$$\mathcal{Z} = \mathcal{T} \times_1 \mathbf{S}_1^T \times_2 \mathbf{S}_2^T \dots \times_P \mathbf{S}_P^T \qquad (8)$$

```
end
```

Figure 3: Algorithm for computation of the Higher-Order Singular Value Decomposition of tensors of any dimensions. The steps in gray can be executed concurrently.

Each of the multi-class HOSVD blocks denotes a single classification sub-system, as presented in Figure 1. Each member multi-class classifier is trained with its partition of training data from each class. In the proposed system data partitions are obtained due to the bagging process which also showed to be superior to the clustering. However, thanks to data partitions which are less numerous than the whole training set, training of each of the HOSVD member classifiers if possible on computers with very limited memory. More details on this can be found in our previous publication

(Cyganek, 2012).

4 IMPLEMENTATION AND EXPERIMENTAL RESULTS

The presented method was implemented in C++, supported by the DeRecLib software from (Cyganek, 2013) and the OpenMP library for the multicore processing (Chapman, 2008; OpenMP, 2013). The experiments were carried out on the computer with 8 GB RAM and the Pentium® Quad Core Q 820 microprocessor (eight cores due to the hyper-threading technology (Intel, 2013)).

For the experiments the USPS dataset was used which contains selected and preprocessed scans of the handwritten digits (Hull, 1994)(LeCun, 1998), as shown in Figure 4. Each test and train pattern is a 16×16 gray level image. The database is divided into the training and testing partitions. The former counts 7291, and the latter 2007 patterns, respectively. However, the bagging process was applied, as described. In effect the training dataset is split into smaller partitions. In this paper two partitions of 64 and 128 elements were used in experiments (Cyganek, 2012).
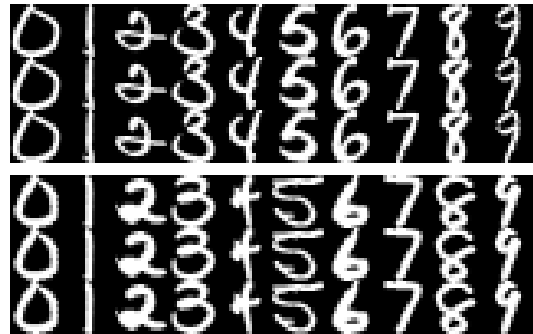

Figure 4: Examples of the training (top) and test (bottom) datasets from the USPS database (from data).

In our experiments parallelism on different levels of computations were measured. Also, each parallel realization was analyzed in the context of memory requirements. The following parallel configurations were built and analyzed:

1. Parallel version of the HOSVD algorithm (see Figure 3).
2. Parallel construction of the HOSVD multi-classifier shown in Figure 1 (i.e. each subspace built concurrently).
3. Parallel training of the ensemble of multi-class

classifiers, shown in Figure 2 (i.e. each member classifier trained concurrently with its data partition).

4. Parallel run-time system response (formula (6)).

From the above, the version no. 1 resulted in a speed-up ratio of 20-30%. This is due to two factors. First reason is a relatively small number of parallel threads since number of SVD computations in the HOSVD algorithm (Figure 3) is equal to the tensor valence P. In our case P=3 since the input tensors consists of a number of 2D image prototypes. In effect, despite a high number of computations, the time overhead related to the thread launch makes the whole operation less effective. The second factor relates to a number of memory blocks allocated by the SVD procedure in our framework. In a serial implementation some of these allocations can be reused which makes computations faster. However, in the cases of higher order tensors, such as ten or above, this way of parallelism can be considered provide higher acceleration level than in the presented experiments. This is also due to the fact that the main tensor does not need to be copied. Instead, it is accessed through the proxy objects, each responsible for a different flattening mode. This feature was implemented in our software framework (Cyganek, 2013).

Because of the above, the two other parallel processing options, no. 2 and 3, were analyzed. These gave the best results, however when used separately. Their concurrent application would result in the nested parallelism which showed to be ineffective in our system due to high thread overhead (there are only eight cores). However, this option can be used in systems with higher number of cores or graphic processing units (GPU).

Figure 5 shows a speed-up ratio of the training process in the serial and parallel implementations, respectively. The plots are drawn for a different number of member classifiers E in the ensemble and different chunks of data from bagging. These are 64 chunks in Figure 5a, and 128 chunks in Figure 5b, respectively. Analyzing the plots in Figure 5 we noticed a two-times speed-up ratio in the OpenMP implementation and with 8 cores. The speed-up ratio is higher for larger chunks of data (128 in this case). However, increasing numbers of data in the chunks has its limits due to memory capacity (only 8 GB in our system), as well as time for data transfers. Nevertheless, the proposed data decomposition at this level pave the way for parallel implementation.

Interestingly, application of many classifiers (an ensemble) leads also to higher accuracy of the
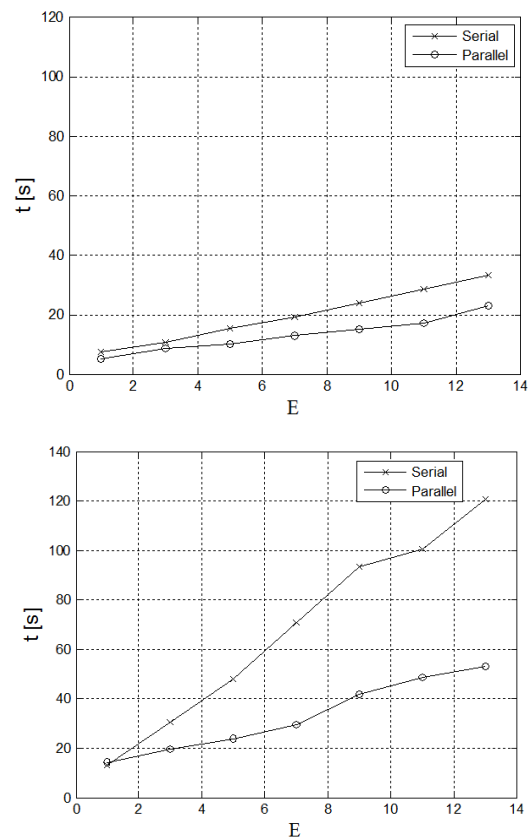

Figure 5: System training speed-up ratio of the serial and parallel implementations for different number of member classifiers E in the ensemble and different chunks of data from bagging: 64 chunks (upper), 128 chunks (lower).

system when compared to a single classifier. The highest accuracy A=95% was obtained for E=13 member classifiers in the ensemble and 128 chunks of data, as reported in our previous work (Cyganek, 2012). Nevertheless, the values of data in chunks and classifiers in the ensemble need to be determined experimentally since they depend on type of used data.

Finally, parallel version no. 4 from the above list was tested. It resulted in speed-up ratio of 10-15% in the OpenMP software implementation. However, after that the GPU implementation was implemented and checked (CUDA platform) which resulted in a speed-up ratio of 30-120 times depending on a size of the base tensors $\mathcal{T}_h^i$ in (6). This option can be used in demanding classification tasks with large base tensors $\mathcal{T}_h^i$, or in brute-force tasks in which patterns are checked in each pixel position of an image, etc.

5 CONCLUSIONS

In this paper the parallel implementation of the ensemble composed of classifiers operating with multi-dimensional data is presented. The classifiers of the ensemble are based on the Higher-Order Singular Value Decomposition of the prototype pattern tensors. Parallelism of the system is obtained through the functional and data decompositions on different levels of computations. As presented, the first level of parallelism can be achieved by functional decomposition of the SVD step in the HOSVD algorithm. The second level of parallelism is obtained by concurrent subspace construction for each of the HOSVD classifiers. The third level of parallelism is due to data decomposition with proper partitioning of the input dataset (in our system this was achieved by data bagging). The proposed method also greatly limits memory requirements. Finally, the response time of the system can be significantly accelerated, which constitutes the fourth level of parallelism in the presented classification system. The experiments conducted on image recognition show high accuracy and the training speed-up ratio in order of 100-150% in the multi-core operation. Despite the computational advantages, also accuracy of the ensemble showed to be higher than in the case of a single classifier.

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