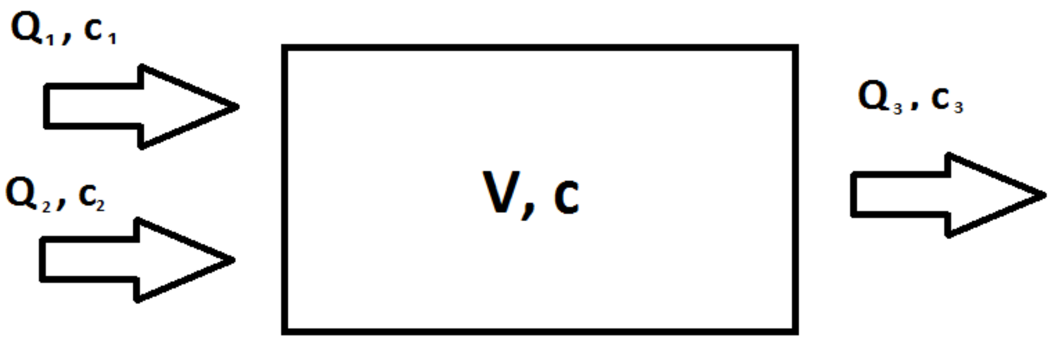
Exercise 1.1. In the picture below, a system with incoming and outgoing fluxes is presented:



Assuming, that incoming fluxes with contamination started at certain moment of time t=0 and that $Q_1+Q_2=Q_3$, determine the concentration of contaminant in the system over time c(t). Our initial condition, is that $c(t=0)=c_0$.

Solution

$$\frac{d(Vc)}{dt} = Q_1c_1 + Q_2c_2 - Q_3c$$

$$V\frac{dc}{dt} = Q_3\left(\frac{Q_1}{Q_3}c_1 + \frac{Q_2}{Q_3}c_2 - c\right)$$

$$\frac{dc}{\left(\frac{Q_1}{Q_3}c_1 + \frac{Q_2}{Q_3}c_2 - c\right)} = \frac{Q_3}{V}dt$$

$$x = \frac{Q_1}{Q_3}c_1 + \frac{Q_2}{Q_3}c_2 - c \quad \Rightarrow \quad dx = -dc$$

$$\frac{dx}{x} = -\frac{Q_3}{V}dt$$

$$x = Ae^{-\frac{Q_3}{V}t}$$

$$c = \frac{Q_1}{Q_3}c_1 + \frac{Q_2}{Q_3}c_2 - Ae^{-\frac{Q_3}{V}t}$$

From initial conditions:

$$A = \frac{Q_1}{Q_3}c_1 + \frac{Q_2}{Q_3}c_2$$

As a result

$$c = \left(\frac{Q_1}{Q_3}c_1 + \frac{Q_2}{Q_3}c_2\right)\left(1 - e^{-\frac{Q_3}{V}t}\right)$$

Exercise 1.2. The inflow to a lake is Q. At a certain point in time t=0, contamination of the inflow to the lake starts. Thereafter, the contaminant concentration in the lake inflow is c_{in}. Our initial condition are $c(t=0)=c_0$ and $v(t=0)=v_0$. **Hint:** integrate mass balance equation when it is in the form of equation (1.2) and formulate function V=V(t).

Solution:

$$\frac{d(Vc)}{dt} = Qc_{in}$$

$$V = V_0 + Qt$$

$$Vc = Qc_{in}t + A$$

$$c = \frac{Qc_{in}t + A}{V}$$

$$c = \frac{Qc_{in}t + A}{V_0 + Qt}$$

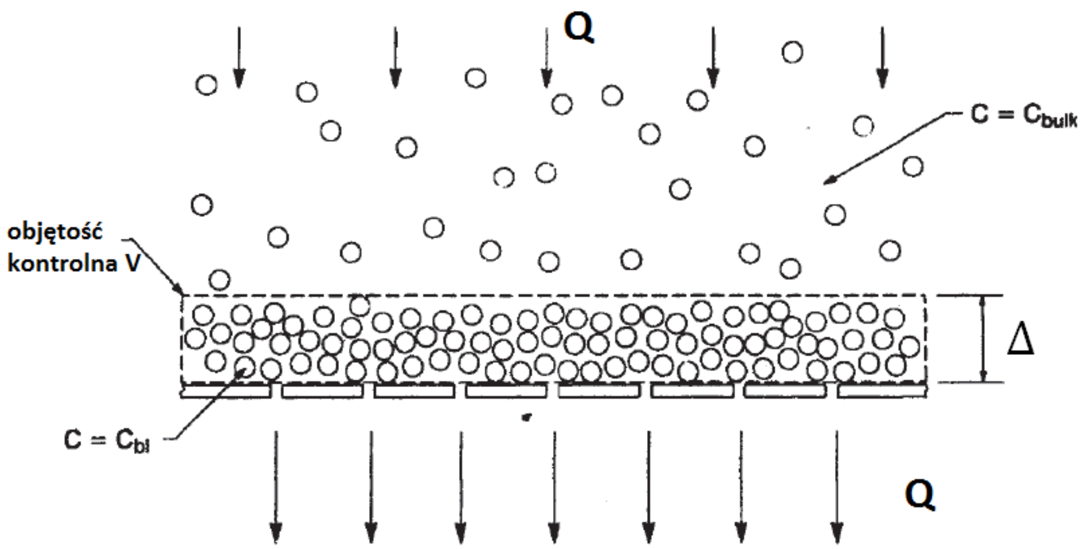
From initial conditions:

$$c_0 = \frac{A}{V_0} \Rightarrow A = c_0V_0$$

Finally:

$$c = \frac{Qc_{in}t + c_0V_0}{V_0 + Qt}$$

Exercise 1.3. Let's consider a ultrafiltration membrane of area A, through which a suspension of particles is flowing. Pores in the membrane are such, that all particles are being stopped on the membrane, while the liquid can pass through with ease. Calculate the thickness of the particle layer on the membrane as a function of time.



Solution:

$$\frac{d(Vc)}{dt} = Qc_{bulk}$$

$$c_{bl}\frac{dV}{dt} = Qc_{bulk}$$

$$c_{bl}\frac{d(A\cdot\Delta)}{dt} = Qc_{bulk}$$

$$c_{bl}A\frac{d\Delta}{dt} = Qc_{bulk}$$

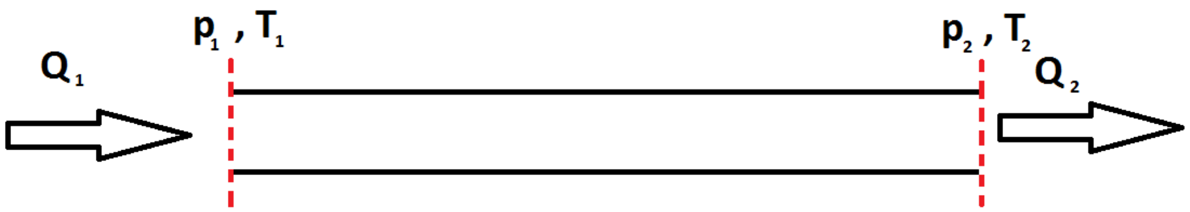
$$\Delta = \frac{Qc_{bulk}}{c_{bl}A}t + B$$

At t=0, Δ=0, so: $$B = 0$$

As a result:

$$\Delta = \frac{Qc_{bulk}}{c_{bl}A}t$$

Exercise 1.4. Let's consider a steady state gas transport through a pipe:



a) Knowing a molar weight of the flowing gas find its density ρ=ρ(p,T)

b) Find Q_2 assuming that Q_1, p_1, T_1, p_2, T_2 are known

Solution:

a)

$$pV = nRT$$

$$\frac{n}{V} = \frac{p}{RT}$$

$$M\frac{n}{V} = \frac{Mp}{RT}$$

$$\frac{m}{V} = \rho = \frac{Mp}{RT}$$

b)

$$Q_1\rho_1 = Q_2\rho_2$$

$$Q_2 = \frac{Q_1\rho_1}{\rho_2} = Q_1\frac{\frac{p_1}{RT_1}}{\frac{p_2}{RT_2}} = Q_1\frac{p_1T_2}{p_2T_1}$$