The Application of Agent-Based Co-Evolutionary System with Predator-Prey Interactions to Solving Multi-Objective Optimization Problems

Rafał Dreżewski, and Leszek Siwik

Abstract— **The realization of co-evolutionary interactions in evolutionary algorithms results in increased population diversity and speciation. General model of co-evolution in multi-agent system allows for modeling and realization of agent-based co-evolutionary systems in which many species and sexes may exist and interact. In this paper one exemplary agent-based system with predator-prey mechanism is presented. The results from experiments with various multi-objective test problems conclude the paper.**

I. Motivation

In evolutionary algorithms (EAs) techniques based on models of co-operative or competitive interactions between species are primarily used when there arises difficulties with explicit formulation of fitness function—such cases include for example games. Such co-evolutionary techniques also help to improve adaptive capabilities of EAs, introducing open-ended evolution and maintaining useful population diversity through speciation (formation of species—sub-populations—within the search space).

In the case of multi-objective optimization the loss of population diversity in EA (which limits the applicability of EAs in the case of some problems) can result not only in the stagnation of evolutionary process and locating the population in areas located faraway from the ideal Pareto frontier but also in locating only selected parts of Pareto frontier or locating local Pareto frontier instead of a global one when we have to deal with multi-modal multi-objective problems ([1]).

The basic model of *evolutionary multi-agent system (EMAS)* results from the attempts to decentralize the process of simulated evolution and to formulate evolutionary computation models which are closer to real evolutionary processes [2]. Such systems are consisted of environment, agents which are able to reproduce and die and resources for which agents compete. The research on speciation mechanisms for such systems resulted in the formulation of general model of co-evolution in multi agent system (CoEMAS) [3], [4], [5]. This model includes also the possibility of co-existing of several species and sexes and to define co-evolutionary relations between them.

The paper is organized as follows. First the short introduction to multi-criteria decision making processes followed by the previous work on the application of co-evolutionary algorithms to such problems are presented. Then the proposed system is presented. Results of experiments with different kinds of multi-criteria optimization problems conclude the paper.

Rafał Dreżewski works at the Department of Computer Science, The School of Banking and Management, Kraków, Poland; email: drezew@wszib.edu.pl

Leszek Siwik works at the Department of Computer Science, The School of Banking and Management, Kraków, Poland; email: siwik@wszib.edu.pl

II. Multi-Objective Optimization

The most natural process of decision making for human being consists in analyzing many—often contradictory—factors and searching for peculiar compromise among them. Such decisive process is known as a *multi-criteria decision making (MCDM)*. Obviously, human being is equipped with natural abilities for making multi-criteria decisions. If such natural gifts are—as the matter of fact—sufficient in everyday life they are not sufficient in more complex technical, business or scientific decisive processes. In such cases *decision maker*—to make a proper decision has to be supported with appropriate mathematical apparatus and efficient computing units and algorithm built on the basis of this very apparatus. The most frequently, *MCDM* process is based on appropriately defined *multi-objective optimization problem (MOOP)*. Following [1]—*multi-objective optimization problem* in its general form is being defined as minimizing/maximizing the set of objectives $f_m(\bar{x}), \quad where \quad m = 1, 2 \ldots, M$.

The set of constraints—both constraint functions (equalities $h_k(\bar{x})$) defined as $h_k(\bar{x}) = 0, \; where \; k = 1, 2 \ldots, K$, inequalities $g_j(\bar{x})$) defined as $g_j(\bar{x}) \geq 0, \; where \; j = 1, 2 \ldots, J$ and decision variable bounds (lower bounds $x_i^{(L)}$ and upper bounds $x_i^{(U)}$) defined as $x_i^{(L)} \leq x_i \leq x_i^{(U)}$, *where* $i = 1, 2 \ldots, N$—define all possible (feasible) decision alternatives ($\mathcal{D}$).

Because there are many criteria—to indicate which solution is better than the other—specialized ordering relation has to be introduced. To avoid problems with converting minimization to maximization problems (and vice versa of course) additional operator $\lhd$ can be defined. Then, notation $\bar{x}_1 \lhd \bar{x}_2$ indicates that solution $\bar{x}_1$ is simply better than solution $\bar{x}_2$ for particular objective. Now, the crucial concept of Pareto optimality i.e. so called dominance relation can be defined. It is said that solution $\bar{x}_A$ dominates solution $\bar{x}_B$ ($\bar{x}_A \prec \bar{x}_B$) if and only if:

$$\bar{x}_A \prec \bar{x}_B \Leftrightarrow \begin{cases} f_j(\bar{x}_A) \ntriangleright f_j(\bar{x}_B) \quad for \quad j = 1, 2 \ldots, M \\ \exists i \in \{1, 2, \ldots, M\} : \quad f_i(\bar{x}_A) \lhd f_i(\bar{x}_B) \end{cases}$$

A solution in the Pareto sense of the multi-objective optimization problem means determination of all non-dominated alternatives from the set $\mathcal{D}$. The Pareto-optimal set consists of globally optimal solutions, however there may also exist locally optimal solutions, which constitute locally non-dominated set (*local Pareto-optimal set*) [1]. The set $\mathcal{P}_{local} \subseteq D$ is local Pareto-optimal set if [6]:

$$\forall \vec{x}^a \in \mathcal{P}_{local} : \nexists \vec{x}^b \in D \text{ such that } \vec{x}^b \geq \vec{x}^a \wedge \left\|\vec{x}^b - \vec{x}^a\right\| < \varepsilon \wedge \left\|F(\vec{x}^b) - F(\vec{x}^a)\right\| < \delta$$

where $\|\cdot\|$ is a distance metric and $\varepsilon > 0$, $\delta > 0$.

The set $\mathcal{P} \subseteq D$ is global Pareto-optimal set if [6]:

$$\forall \vec{x}^a \in \mathcal{P} : \nexists \vec{x}^b \in D \text{ such that } \vec{x}^b \geq \vec{x}^a \tag{1}$$

These locally or globally non-dominated solutions create (in the criteria space) so-called local ($\mathcal{PF}_{local}$) or global ($\mathcal{PF}$) Pareto frontiers that can be defined as follows:

$$\mathcal{PF}_{local} = \left\{\vec{y} = F(\vec{x}) \in \mathbb{R}^M \quad | \quad \vec{x} \in \mathcal{P}_{local}\right\} \tag{2a}$$

$$\mathcal{PF} = \left\{\vec{y} = F(\vec{x}) \in \mathbb{R}^M \quad | \quad \vec{x} \in \mathcal{P}\right\} \tag{2b}$$

Multi-objective problems with one global and many local Pareto frontiers are called *multi-modal multi-objective problems* [1].

During over 20 years of research on evolutionary multi-objective algorithms (EMOAs) quite many techniques have been proposed. Generally all of these techniques and algorithms can be classified as elitist (which give the best individuals the opportunity to be directly carried over to the next generation) or non-elitist ones [1].

III. Co-Evolutionary Multi-Objective Algorithms

Co-evolution is the biological process responsible for speciation, maintaining population diversity, introducing arms races and open-ended evolution. In *co-evolutionary algorithms (CoEAs)* the fitness of each individual depends not only on the quality of solution to the given problem (like in EAs) but also (or solely) on other individuals' fitness [7]. As the result of ongoing research quite many co-evolutionary techniques have been proposed. Generally, each of these techniques belongs to one of two classes: competitive or cooperative.

There were also some attempts of applying co-evolutionary algorithms to multi-objective problems. Laumanns, Rudolph and Schwefel proposed predator-prey evolutionary strategy (PPES) (i.e. competitive co-evolutionary algorithm with predator-prey model and spatial graph-like structure) for multi-objective optimization [8]. In their model prey were placed in the nodes of graph (they could not migrate) and predators could migrate from node to node killing the "weakest" prey (each predator had one criteria assigned to it and evaluated prey on the basis of that criteria).

Deb introduced modified algorithm in which predators eliminated prey not only on the basis of one criteria but on the basis of the weighted sum of all criteria [1]. Li proposed other modifications to Deb's algorithm [9]. The main difference was that not only predators were allowed to migrate within the graph but also prey could do it. The model of cooperative co-evolution was also applied to multi-objective optimization

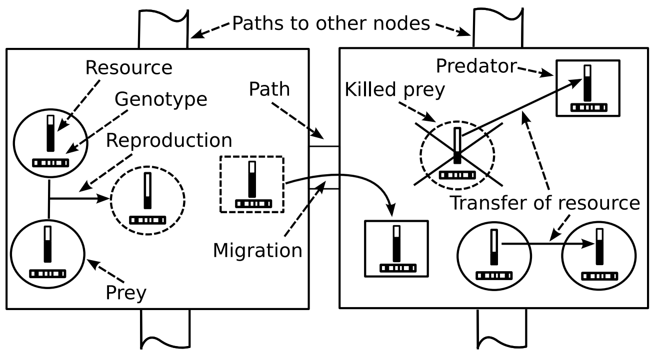

Fig. 1. CoEMAS with predator-prey mechanism

([10]). In this technique each sub-population was responsible for one variable x_i. The complete solution was the group of individuals, each of them chosen from different sub-population. The reproduction and recombination processes took place independently within sub-populations (individuals from different sub-populations interacted with each other only during fitness evaluation).

It seems that co-evolution applied to multi-objective problems should introduce open-ended evolution, improve adaptive capabilities of EA (especially in the case of dynamic environments) and allow speciation (the formation of species located in different areas of Pareto frontier or at different local Pareto frontiers in case of multi-modal multi-objective problems [1]) but this is still an open issue and the subject of ongoing research.

IV. Agent-Based Co-Evolutionary System for Multi-Objective Optimization

In this section the agent-based co-evolutionary system used in experiments is presented. The system is composed of the following elements: the environment with graph-like structure, resources, and two interacting species of agents (predators and prey) (see fig. 1). All types of agents live within the environment, can migrate between nodes, and try to get resources which are used for all kinds of activities, like reproduction and migration. Agents which amount of resource is below the minimal level die and are removed from system. Agents of prey species—which represent solutions of the multi-objective problem—can reproduce when the amount of the possessed resource is above the given level. When two such agents meet within the same node of the environment the new agent is created with the use of recombination and mutation operators. Parents also give to the newly created offspring some of their resources.

The role of predators is to remove from the system dominated prey. Each of the predators is associated with one criteria and seeks for the worst prey—located within the same node as the given predator—according to its criteria. Then such a prey is killed and all of its resources are transferred to the predator. Below, more formal definition of the system is presented.

A. CoEMAS

The co-evolutionary multi-agent system with predator-prey interactions (*CoEMAS*) is defined as follows [3]:

$$CoEMAS = \langle E, S, \Gamma, \Omega \rangle \tag{3}$$

E is the environment of the $CoEMAS$ system, S is the set of species ($s \in S$) that exist and co-evolve in $CoEMAS$, Γ is the set of resource types (the amount of type γ resource which is possessed by the given element of the system will be denoted by r^{γ}), Ω is the set of information types (the information of type ω, which can be used or possessed by the given element of the system is denoted by i^{ω}). Two information types ($\Omega = \{\omega_1, \omega_2\}$) and one resource type ($\Gamma = \{\gamma\}$) are used. Informations of type ω_1 denote nodes to which agent can migrate. Informations of type ω_2 denote such prey that are located within the particular node in time t.

The selection mechanism is based on the closed circulation of resource within the system. The whole amount of resource is constant, the resource can be possessed by the agents, and is transferred from dominated prey to dominating prey, and from prey to predators during killing prey.

The environment E is defined in the following way:

$$E = \left\langle T^E, \Gamma^E = \emptyset, \Omega^E = \Omega \right\rangle \tag{4}$$

T^E is the topography of the environment E. Γ^E is the set of resource types that exist within the environment. Ω^E is the set of information types that exist within the environment. The topography of the environment $T^E = \langle H, l \rangle$, where H is directed graph with the cost function c defined ($H = \langle V, B, c \rangle$, V is the set of vertices, B is the set of arches). In the case of the presented system every node is connected with its four neighbors, which results in the torus-like environment. The $l: \quad A \to V$ (A is the set of agents) function makes it possible to locate particular agent in the environment space.

Vertice v is given by:

$$v = \left\langle A^v, \Gamma^v = \Gamma^E, \Omega^v = \Omega^E, \varphi \right\rangle \tag{5}$$

A^v is the set of agents that are located within the vertice v. There are two types of informations in the vertice. The first one includes all vertices that are connected with the vertice v:

$$i^{\omega_1, v} = \{u : u \in V \wedge \langle v, u \rangle \in B\} \tag{6}$$

The second one includes all agents of species *prey* that are located within the vertice v:

$$i^{\omega_2, v} = \{a^{prey} : a^{prey} \in A^v\} \tag{7}$$

B. Species

The set of species $S = \{prey, pred\}$. The prey species (*prey*) is defined as follows:

$$prey = \langle A^{prey}, SX^{prey} = \{sx\}, Z^{prey}, C^{prey} \rangle \tag{8}$$

where SX^{prey} is the set of sexes which exist within the *prey* species, Z^{prey} is the set of actions that agents of species *prey* can perform, and C^{prey} is the set of relations of *prey* species with other species that exist in the $CoEMAS$.

The set of actions Z^{prey} is defined as follows:

$$Z^{prey} = \{die, get, give, accept, seek, clone, rec, mut, migr\} \tag{9}$$

die is the action of death (prey dies when it is out of resources). *get* action gets some resource from another a^{prey} agent located within the same node, which is dominated by the agent that performs *get* action or is too close to it in the criteria space, *give* action gives some resource to another agent (which performs *get* action), *accept* action accepts partner for reproduction when the amount of resource possessed by the prey agent is above the given level, *seek* action seeks for another prey agent that is dominated by the prey performing this action or is too close to it in criteria space. This action is also used in order to find the partner for reproduction when the amount of resource is above the given level and agent can reproduce. *clone* is the action of producing offspring (parents give some of their resources to the offspring during this action), *rec* is the recombination operator (intermediate recombination is used [11]), *mut* is the mutation operator (mutation with self-adaptation is used [11]). The *migr* is the action of migrating from one node to another. During this action agent loses some of its resource.

The set of relations of *prey* species with other species that exist within the system is defined as follows:

$$C^{prey} = \left\{ \xrightarrow{prey,get-}, \xrightarrow{pred,give+} \right\} \tag{10a}$$

The first relation models intra species competition for limited resources ("-" denotes that as a result of performing *get* action the fitness of another prey is decreased):

$$\xrightarrow{prey,get-} = \{\langle prey, prey \rangle\} \tag{10b}$$

The second one models predator-prey interactions ("+" denotes that when prey gives all its resources to the predator, the predator fitness is increased):

$$\xrightarrow{pred,give+} = \{\langle prey, pred \rangle\} \tag{10c}$$

The predator species (*pred*) is defined as follows:

$$pred = \left\langle A^{pred}, SX^{pred} = \{sx\}, Z^{pred}, C^{pred} \right\rangle \tag{11}$$

All the symbols used have analogical meaning as in the case of *prey* species—see eq. (8). The set of actions Z^{pred} is defined as follows:

$$Z^{pred} = \{seek, get, migr\} \tag{12}$$

The *seek* action allows finding the "worst" (according to the criteria associated with the given predator) prey located within the same node as the predator, *get* action gets all resources from the chosen prey, *migr* action allows predator to migrate between nodes of the graph H—this results in losing some of the resources.

The set of relations of *pred* species with other species that exist within the system are defined as follows:

$$C^{pred} = \left\{ \xrightarrow{prey,get-} \right\} \tag{13a}$$

This relation models predator-prey interactions:

$$\xrightarrow{prey,get-} = \{\langle pred, prey \rangle\} \tag{13b}$$

As a result of performing *get* action and taking all resources from selected prey, it dies.

C. Agents

Agent a of species *prey* ($a \equiv a^{prey}$) is defined as follows:

$$a = \langle gn^a, Z^a = Z^{prey}, \Gamma^a = \Gamma, \Omega^a = \Omega, PR^a \rangle \tag{14}$$

Genotype of agent a is consisted of two vectors (chromosomes): $\vec{x}$ of real-coded decision parameters' values and $\vec{\sigma}$ of standard deviations' values, which are used during mutation with self-adaptation. $Z^a = Z^{prey}$ (see eq. (9)) is the set of actions which agent a can perform. Γ^a is the set of resource types used by the agent, and Ω^a is the set of information types.

The partially ordered set of profiles includes resource profile (pr_1), reproduction profile (pr_2), interaction profile (pr_3), and migration profile (pr_4):

$$PR^a = \{pr_1, pr_2, pr_3, pr_4\} \tag{15a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \trianglelefteq pr_4 \tag{15b}$$

Each profile pr is defined as follows:

$$pr = \langle \Gamma^{pr}, \Omega^{pr}, M^{pr}, ST^{pr}, GL^{pr} \rangle \tag{16}$$

Γ^{pr} is the set of resource types used in the pr profile ($\Gamma^{pr} \subseteq \Gamma^a$). Ω^{pr} is the set of information types ($\Omega^{pr} \subseteq \Omega^a$). M^{pr} is the set of informations (the model) which represent the agent's knowledge about the environment and other agents. ST^{pr} is the partially ordered set of strategies which agent can use in order to realize the active goal of the given profile. GL^{pr} is the partially ordered set of goals.

Each time step agent tries to realize active goals (goals which should be realized) of the profiles taking into account the priorities of the profiles (pr_1 has the highest priority—see eq. (15)) and also the priorities of the active goals. In order to realize goals of the given profile agent uses strategies (consisted of simple actions) which can be realized within this profile. In this process also the priorities of strategies are considered.

The goal of the pr_1 profile is to keep the amount of resources above the minimal level or to die. In order to realize such goal agent can use the following strategies: $\langle die \rangle$, $\langle seek, get \rangle$. This profile uses the model $M^{pr_1} = \{i^{\omega_2}\}$ (see eq. (7).) The only goal of the pr_2 profile is to reproduce. In order to realize this goal agent can use strategy of reproduction: $\langle seek, clone, rec, mut \rangle$. The model is defined in the following way: $M^{pr_2} = \{i^{\omega_2}\}$. The goal of the pr_3 profile is to interact with predators with the use of strategy $\langle give \rangle$. The goal of the pr_4 profile is to migrate within the environment. In order to realize such goal the migration strategy is used: $\langle migr \rangle$. The model used is defined as follows: $M^{pr_4} = \{i^{\omega_1}\}$ (see eq. (6).) As a result of migrating prey loses some resource.

Agent a of species *pred* is defined analogically to *prey* agent (see eq. (14)). There exist two main differences. Genotype of predator agent is consisted of the information about the criterion associated with the given agent. The set of profiles is consisted only of two profiles, resource profile (pr_1), and migration profile (pr_2): $PR^a = \{pr_1, pr_2\}$, where $pr_1 \trianglelefteq pr_2$.

The goal of the pr_1 profile is to keep the amount of resource above the minimal level with the use of strategy $\langle seek, get \rangle$. The model used within this profile is defined as follows: $M^{pr_1} = \{i^{\omega_2}\}$. The goal of pr_2 profile is to migrate within the environment. In order to realize this goal the migration strategy $\langle migr \rangle$) is used. The model of the environment is defined in the following way: $M^{pr_2} = \{i^{\omega_1}\}$. The realization of the migration strategy results in losing some of the resource possessed by the agent.

V. Multi-Objective Test Problems

In order to investigate whether the proposed system properly solves the multi-objective problems and to compare it to "classical" evolutionary multi-objective algorithms, two standard test problems and one hard, real-life problem were used.

Firstly, slightly modified so-called *Laumanns* multi-objective problem was used, which is defined as follows ([8], [12]):

$$Laumanns = \begin{cases} f_1(x) = x_1^2 + x_2^2 \\ f_2(x) = (x_1 + 2)^2 + x_2^2 \\ -5 \le x_1, x_2 \le 5 \end{cases}$$

It is a test problem with convex and coherent Pareto frontier, which has some interesting properties and in consequence becomes a suitable candidate for initial testing and comparative studies. Visualization of the Pareto frontier for this problem is presented in fig. 2.

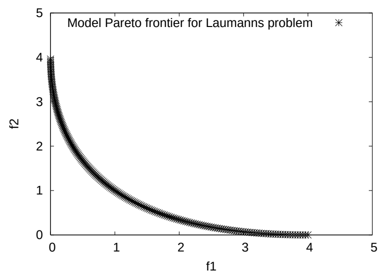

Fig. 2. *Laumanns test problem*: visualization of the Pareto frontier

Secondly the so-called *Kursawe* problem was used. Its definition is as follows ([12]):

$$Kursawe = \begin{cases} f_1(x) = \sum_{i=0}^{n-1}\left(-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right) \\ f_2(x) = \sum_{i=1}^{n}|x_i|^{0.8} + 5\sin x_i^3 \\ n = 3 \quad -5 \le x_1, x_2, x_3 \le 5 \end{cases}$$

Visualization of the Pareto set and Pareto frontier for the *Kursawe* problem is here omitted because of space limitation. This is quite difficult multi-objective problem. Its characteristic features include: disconnected two-dimensional Pareto frontier, disconnected three dimensional Pareto set, and the fact that very small changes in the space of decision variables can seriously affect the results in the space of objectives.

Proposed co-evolutionary agent-based approach has also been preliminarily assessed using the problem of effective portfolio building. The meaning of symbols used in the definitions below, are as follows:

p - the number of shares in the wallet;
n - the number of rate of return;
α_i,β_i - coefficients of the equations;
ω_i - percentage participation of i-th share in the wallet;
e_i - random component of the equation;
R_{it} - the rate of return in the period t;
R_{mt} - the rate of return related to market index in period t;
R_m - the rate of return of market index;
R_i - the rate of return of the i-th share;
R_p - the rate of return of the wallet;
${s_{e_p}}^2$ - the variance of the i-th share;
${s_{e_i}}^2$ - the variance of the random index of the i-th share;
${s_{e_p}}^2$ - the variance of the wallet;
$\overline{R_i}$ - arithmetic mean of rate of return of the i-th share;
$\overline{R_m}$ - arithmetic mean of rate of return of market index;

The algorithm (based on the Sharpe model) of computing the expectation of the risk level and, generally speaking, income expectation related to the wallet of p shares is as follows:

1) Compute the arithmetic means on the basis of rate of returns;
2) Compute the value of α coefficient:
$$\alpha_i = \overline{R_i} - \beta_i\overline{R_m} \tag{17}$$
3) Compute the value of β coefficient:
$$\beta_i = \frac{\sum_{t=1}^{n}(R_{it} - \overline{R_i})(R_{mt} - \overline{R_m})}{\sum_{t=1}^{n}(R_{mt} - \overline{R_m})^2} \tag{18}$$
4) Compute the share expectation:
$$R_i = \alpha_i - \beta_i R_m + e_i \tag{19}$$
5) Compute the variance of random index:
$${s_{e_i}}^2 = \frac{\sum_{t=1}^{n}(R_{it} - \alpha_i - \beta_i R_m)^2}{n-1} \tag{20}$$
6) Compute the variance of market index:
$${s_m}^2 = \frac{\sum_{t=1}^{n}(R_{mt} - \overline{R_m})^2}{n-1} \tag{21}$$
7) Compute the risk level of the investing wallet:
$$\beta_p = \sum_{i=1}^{p}(\omega_i\beta_i) \tag{22}$$
$${s_{e_p}}^2 = \sum_{i=1}^{p}(\omega_i^2 {s_{e_i}}^2) \tag{23}$$
$$risk = \beta_p^2 {s_m}^2 + {s_{e_p}}^2 \tag{24}$$
8) Compute the investing wallet expectation:
$$R_p = \sum_{i=1}^{p}(\omega_i R_i) \tag{25}$$

The goal of the optimization is to maximize the investing wallet expectation and minimize the risk level. Model Pareto frontiers for two cases (three and seventeen stocks set), which are analyzed in the following section, are presented in fig. 3.

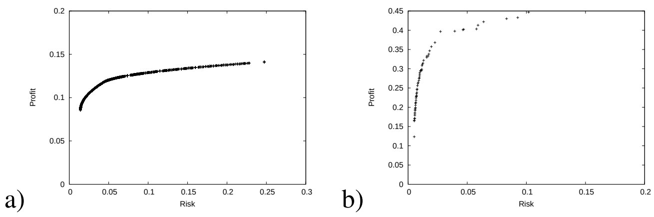

Fig. 3. *Building of effective portfolio*: visualization of the model Pareto frontier obtained using utter review method for a) three and b) seventeen stocks set

VI. Results of Experiments

In this section the results of experiments with different types of multi-objective test problems are presented. Also, the results obtained by proposed system are compared with results obtained by "classical" (i.e. non agent-based) predator prey evolutionary strategy (PPES) [8] and another "classical" evolutionary algorithm for multi-objective optimization: niched pareto genetic algorithm (NPGA) [6]. In order to deeper analyze the results obtained by compared algorithms values of HV and HVR metrics (which can be found in [1]) are also presented.

In the very first experiments with CoEMAS system relatively simple Laumanns test problem was used. In fig. 4 there are presented Pareto frontier approximations obtained by CoEMAS and PPES algorithms and in table I there are presented values of HV and HVR metrics for all three algorithms being compared. As it can be seen the differences between algorithms being analyzed are not so distinct, however proposed CoEMAS system seems to be the best alternative.

The second problem used is quite demanding multi-objective Kursawe problem with disconnected both Pareto set and Pareto frontier. In fig. 5 there are presented approximations of Pareto frontier obtained by CoEMAS and by reference algorithms after 10, 600 and 6000 time steps. As one may notice initially, i.e. after 10 (see fig. 5a,b,c)

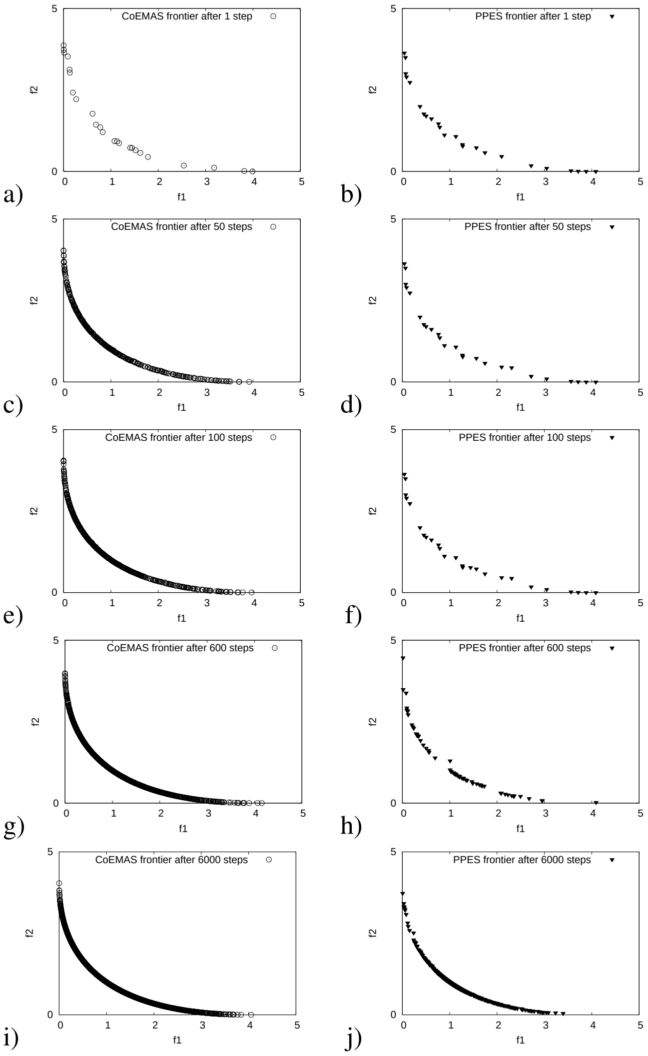

Fig. 4. Pareto frontier approximations in selected consecutive steps obtained by CoEMAS and PPES algorithms for Laumanns problem

steps, Pareto frontiers obtained by all three algorithms are—in fact—quite similar if the number of found non-dominated individuals, their distance to the model Pareto frontier and their dispersing over the whole Pareto frontier are considered. Afterwards yet, definitely higher quality of CoEMAS-based Pareto frontier approximation is more and more distinct (it is enough to compare results obtained by CoEMAS, NPGA and PPES algorithms after 600 and 6000 time steps—see fig. 5d,e,f and fig. 5g,h,i). Generally, there is no doubt that CoEMAS is definitely the best alternative since it is able to obtain Pareto frontier that is located very close to the model solution, that is very well dispersed and what is also very important—it is more numerous than PPES and NPGA-based solutions. The above observations are fully confirmed by the values of HV and HVR metrics presented in table II.

In the case of optimizing investing portfolio each individual in the prey population has p-dimensional vector encoded in its genotype. Each dimension represents the percentage participation of i-th ($i \in 1 \dots p$) share in the whole portfolio. Because of the space limitation in this paper only a kind of summary of two single experiments will be presented (of course during our research a lot of experiments have been conducted and—moreover—we are still working on this demanding problem.) During presented experiment quotations from 2003-01-01 until 2005-12-31 were taken into consideration. Simultaneously the portfolio consists of the following three (in experiment I) or seventeen (in experiment II) stocks quoted on the Warsaw Stock Exchange. In experiment I portfolio is consisted of: RAFAKO, PONARFEH, and PKOBP stocks. In the case of experiment II portfolio is consisted of: KREDYTB, COMPLAND, BETACOM, GRAJEWO, KRUK, COMARCH, ATM, HANDLOWY, BZWBK, HYDROBUD, BORYSZEW, ARKSTEEL, BRE, KGHM, GANT, PROKOM, and BPHPBK stocks. As the market index WIG20 has been taken into consideration.

In fig. 3 there are presented model Pareto frontiers obtained for effective portfolio building problem for three and seventeen stocks obtained using utter review method. Consecutive Pareto frontiers obtained by both—system that is being analyzed and by reference algorithms as well are presented in details in [13]. In this paper authors decided to

TABLE I

Comparison of proposed CoEMAS approach with selected classical EMOAs according to the *HV* and *HVR* metrics obtained during solving Laumanns problem

	HV / HVR		
Step	**CoEMAS**	**PPES**	**NPGA**
1	59.24 / 0.982	58.45 / 0.969	58.41 / 0.968
10	59.57 / 0.987	58.45 / 0.969	59.72 / 0.990
20	59.78 / 0.991	58.45 / 0.969	53.30 / 0.883
30	59.81 / 0.991	58.45 / 0.969	51.02 / 0.846
40	59.79 / 0.991	58.45 / 0.969	49.37 / 0.818
50	59.81 / 0.991	58.45 / 0.969	48.62 / 0.806
100	59.77 / 0.991	58.45 / 0.969	47.25 / 0.783
600	59.79 / 0.991	59.11 / 0.980	47.04 / 0.780
200	59.73 / 0.990	59.59 / 0.988	47.04 / 0.780
4000	59.76 / 0.991	59.64 / 0.988	47.04 / 0.780
6000	59.75 / 0.990	59.34 / 0.983	47.04 / 0.780

TABLE II

Comparison of proposed CoEMAS approach with selected classical EMOAs according to the *HV* and *HVR* metrics obtained during solving Kursawe problem

	HV / HVR		
Step	**CoEMAS**	**PPES**	**NPGA**
1	541.21 / 0.874	530.76 / 0.857	489.34 / 0.790
10	588.38 / 0.950	530.76 / 0.867	563.55 / 0.910
20	594.09 / 0.959	531.41 / 0.858	401.79 / 0.648
30	601.66 / 0.971	531.41 / 0.858	378.78 / 0.611
40	602.55 / 0.973	531.41 / 0.858	378.73 / 0.611
50	594.09 / 0.959	531.41 / 0.858	378.77 / 0.611
100	603.04 / 0.974	531.42 / 0.858	378.80 / 0.6117
600	603.79 / 0.975	577.44 / 0.932	378.80 / 0.611
200	611.43 / 0.987	609.47 / 0.984	378.80 / 0.611
4000	611.44 / 0.987	555.53 / 0.897	378.80 / 0.611
6000	613.10 / 0.990	547.73 / 0.884	378.80 / 0.611

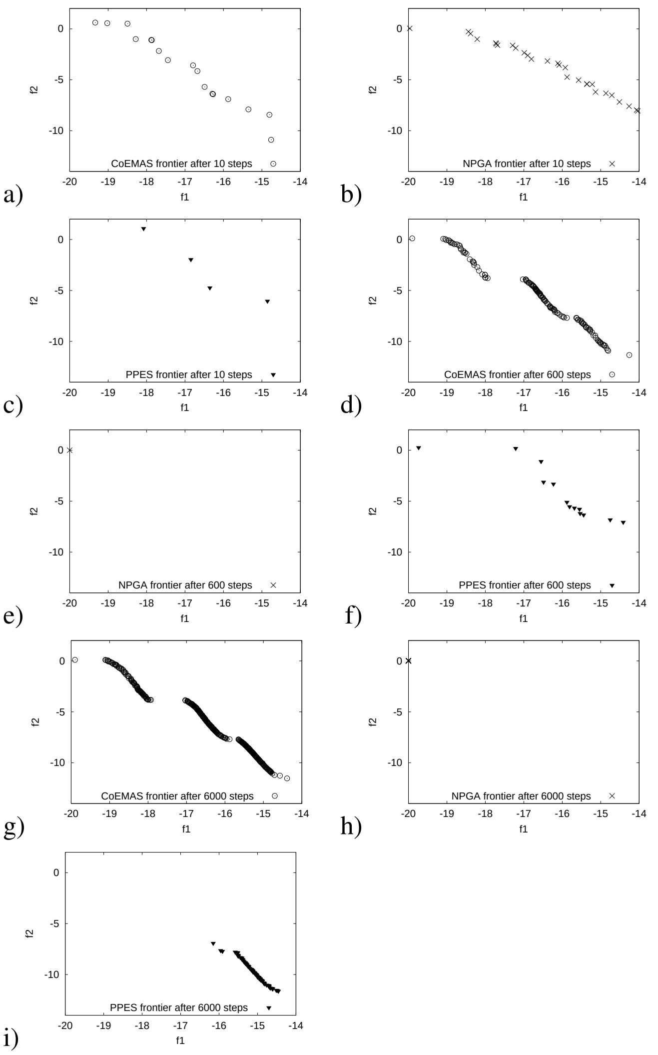

Fig. 5. Pareto frontier approximations for Kursawe problem in selected consecutive steps obtained by CoEMAS, PPES and NPGA

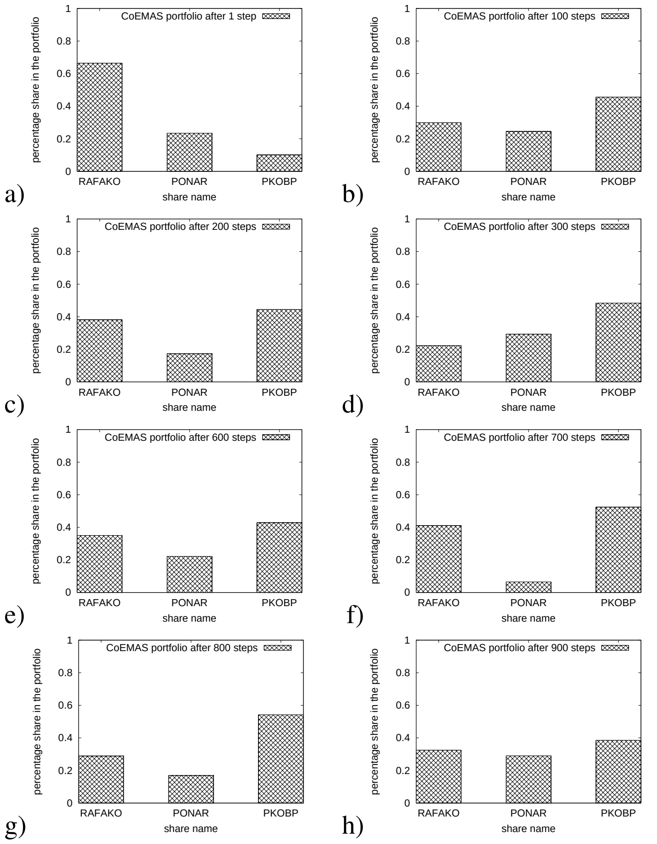

Fig. 6. Effective portfolio in consecutive steps consisting of three stocks proposed by CoEMAS

present rather in details portfolio composition. It is of course impossible in the course of this paper to present consecutive portfolios proposed by all non-dominated solutions—that is why we decided to choose average non-dominated solution in first step and then to follow during consecutive steps solutions proposed by this very solution (or its descendant(s)). Such hypothetical non-dominated average portfolios for experiment I and II are presented in fig. 6 and in fig. 7 respectively (in fig. 7 shares are presented from left to right in the order in which they were mentioned above). Generally, it can be said that during experiment I—average solution proposed by CoEMAS system is a kind of balanced portfolio (percentage share of all three stocks are quite similar), whereas during experiment II there are more important stocks (with given assumptions and parameters of course)—i.e. HANDLOWY, HYDROBUD, ARKSTEEL.

In fig 8 there are presented Pareto frontiers obtained by CoEMAS, NPGA and PPES algorithms after 900 time steps for both experiments. In both cases CoEMAS-based frontier is quite numerous and quite close to the model Pareto frontier—unfortunately diversity of population in CoEMAS system is visibly worse than in the case of NPGA or PPES-based frontiers (it is also confirmed by values of HV and HVR metrics, but because of space limitations these characteristics are omitted in this paper). What is more, with time the tendency of CoEMAS-based solver for focusing solutions around small part of the whole Pareto frontier is more and more distinct (see [13]).

VII. Summary and Conclusions

Growing interest in co-evolutionary algorithms and their application in the area of multi-objective optimization results from the ability of CoEAs to promote the useful population diversity and their improved adaptive capabilities as compared to evolutionary algorithms.

The system presented in this paper is based on the idea of realization of co-evolutionary processes in the multi-agent system what results in the decentralization of evolutionary processes and co-evolutionary interactions. Presented results of experiments with Laumanns and Kursawe problems clearly show that CoEMAS not only properly located Pareto frontiers of these two test problems but also the results of this system were better than in the case of two other "classical" algorithms. The population of CoEMAS was

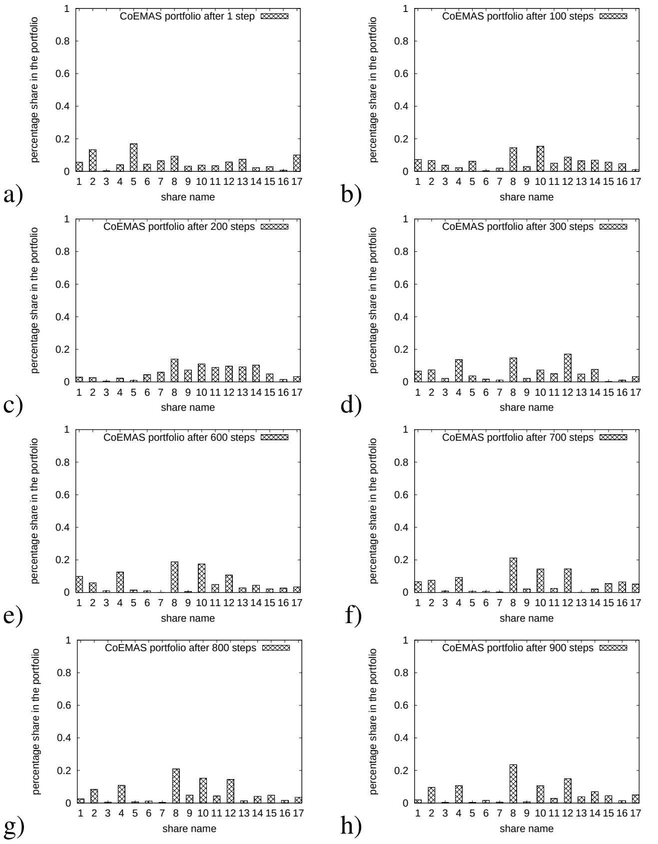

Fig. 7. Effective portfolio in consecutive steps consisting of seventeen stocks proposed by CoEMAS

located very close to the "ideal" Pareto frontier. Also, the Pareto frontier was very well "covered" by individuals, and solutions were more numerous than in the case of PPES and NPGA algorithms. This was the result of the tendency to maintain high useful population diversity.

The results of experiments with the effective portfolio building problem shows however that in the case of some problems the proposed co-evolutionary mechanism is not fully sufficient. It turned out that, in spite of the fact that the Pareto frontier formed by the proposed system was more numerous than in the case of "classical" multi-objective evolutionary algorithms, the tendency to lose population diversity appeared. This resulted in the fact that in the case of this problem PPES and NPGA algorithms were able to form frontiers better "covered" with individuals.

The results of experiments show that still more research is needed on the proposed co-evolutionary mechanism—especially when we consider the stable maintaining of useful population diversity.

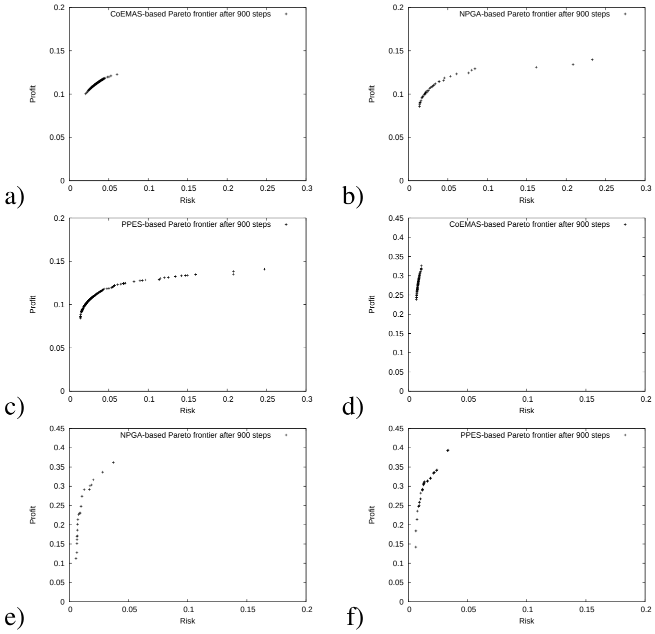

Fig. 8. Pareto frontier approximations after 900 steps obtained by CoEMAS, PPES and NPGA for building effective portfolio consisting of three and seventeen stocks

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