Agent-Based Multi-Objective Evolutionary Algorithm with Sexual Selection

Rafał Dreżewski, and Leszek Siwik

***Abstract*— Evolutionary algorithms are (meta-)heuristic techniques used in the case of search, optimization, and adaptation problems, which cannot be solved with the use of traditional methods. Sexual selection mechanism helps to maintain the population diversity in evolutionary algorithms. In this paper the agent-based realization of multi-objective evolutionary algorithm with sexual selection mechanism is presented. The system is evaluated with the use of Zitzler's test problems and compared to "classical" multi-objective evolutionary algorithms.**

I. Introduction

Evolutionary algorithms (EAs) are heuristic technique based on analogies to Darwinian model of evolutionary processes [1]. They can be applied in the case of problems for which traditional techniques cannot be used. EAs can be applied to global search, optimization, and adaptation problems.

In evolutionary biology *sexual selection* is considered as the mechanism resulting from the co-evolutionary "arms races" between females and males, in which usually females (because of their higher reproduction costs) try to decrease the rate of reproduction and males try to increase it [2]. Such co-evolution results in the appearance of some males' features (displayed trait), which try to attract females to mating as the response to some other features of females (female mate choice), which try to keep the reproduction rate at the optimal level. Sexual selection is one of the mechanisms that cause speciation [3], and create and maintain bio-diversity [4].

Sexual selection is used in EAs mainly as the mechanism that helps to maintain the population diversity, which is a very important issue in the case of multi-modal optimization problems, multi-objective optimization problems, optimization of non-stationary functions, and adaptation to changing conditions of the environment. Sexual selection was applied, for example, in the evolutionary algorithms for multi-modal optimization [5], [6], and multi-objective optimization problems [7], [8], [9].

The paradigm of evolutionary multi-agent systems (EMAS) results from the research on decentralized models of evolutionary algorithms. Such decentralization, as the result of realization of the evolutionary processes in multi-agent system, leads to the appearance of some distinct features, which are not present in the "classical" evolutionary algorithms. Such features include, for example, the autonomous decision making of individuals (agents), decentralization of evolutionary computations and the relaxation of the synchronization constraints, auto-adaptation of some parameters of the system to the difficulty level of the problem being solved, the possibility of constructing hybrid systems (on the basis of agent-based architecture) using different techniques of computational intelligence, and so on. The model of co-evolutionary multi-agent systems (CoEMAS) additionally extends the basic EMAS model, and introduces some new features like: the notion of species, sexes, and co-evolutionary interactions between them [10]. On the basis of the co-evolutionary interactions it was possible to propose niching mechanisms for CoEMAS systems and apply them to multi-modal optimization problems [11]. Co-evolutionary interactions were also used in systems for multi-objective optimization [12], [13]. The sexual selection mechanism for co-evolutionary multi-agent systems was described in [14]. Also, the first attempt to apply the sexual selection mechanism in CoEMAS for multi-objective optimization was made ([15]).

The investigations presented in this paper include the proposition of new agent-based realization of multi-objective evolutionary algorithm with sexual selection mechanism (SCoEMAS). The described system is then experimentally verified with the use of Zitzler's multi-objective test problems and compared to "classical" multi-objective evolutionary algorithms.

II. Multi-Objective Optimization

Multi-objective optimization techniques (both population-based —evolutionary-based in particular—as well as "classical ones") are, and should be, based on a well defined mathematical apparatus. Nowadays, the most frequently used theory is a so-called *Pareto optimality theory*. According to this theory—from the mathematical point of view—a multi-objective (or multi-criteria) optimization problem can be formulated as follows ([16], [17], [18]):

$$MOOP \equiv \begin{cases} Min/Max \quad f_m(\bar{x}), \quad & m = 1, 2 \ldots, M \\ where: \quad g_j(\bar{x}) \geq 0, & j = 1, 2 \ldots, J \\ \qquad\qquad h_k(\bar{x}) = 0, & k = 1, 2 \ldots, K \\ \qquad\qquad x_i^{(L)} \leq x_i \leq x_i^{(U)}, & i = 1, 2 \ldots, N \end{cases}$$

The set of constraints, both equalities ($h_k(\bar{x})$), as well as inequalities ($g_j(\bar{x})$), and constraints related to the decision

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variables, i.e. lower bounds ($x_i^{(L)}$) and upper bounds ($x_i^{(U)}$), define so called searching space—feasible alternatives ($\mathcal{D}$).

The crucial notion of the optimality in the Pareto sense is a so-called domination relation which is defined as follows. To avoid potential inconveniences with translating minimization into maximization problem (and vice versa of course) it can be useful to define additional operator $\lhd$. Notation $\bar{x}_1 \lhd \bar{x}_2$ means that solution $\bar{x}_1$ is simply better than solution $\bar{x}_2$ with respect to the selected objective. Keeping in mind defined above operator, it can be said that solution $\bar{x}_A$ dominates solution $\bar{x}_B$ ($\bar{x}_A \succ \bar{x}_B$) if and only if:

$$\bar{x}_A \succ \bar{x}_B \Leftrightarrow \begin{cases} f_j(\bar{x}_A) \ntriangleright f_j(\bar{x}_B) \quad for \quad j = 1, 2 \ldots, M \\ \exists i \in \{1, 2, \ldots, M\}: \quad f_i(\bar{x}_A) \lhd f_i(\bar{x}_B) \end{cases}$$

The solution in the Pareto sense of the multi-objective optimization problem means determining all non-dominated alternatives from the set $\mathcal{D}$.

Apart from the global Pareto-optimal solutions, also local solutions of the multi-objective optimization problem can be distinguished (so-called local Pareto sets $\mathcal{P}_{local}$). The set $\mathcal{P}_{local}$ is a local set of Pareto-optimal solutions (local Pareto set) if and only if ([17]):

$$\forall \bar{x}_A \in \mathcal{P}_{local}: \ \nexists \bar{x}_B) \in D \ \text{ such as} \\ \bar{x}_B \succeq \bar{x}_A \wedge \|\bar{x}_B - \bar{x}_A\| < \varepsilon \wedge \|F(\bar{x}_B) - F(\bar{x}_A)\| < \delta$$

where $\|\cdot\|$ is assumed distance metrics and $\varepsilon > 0$, $\delta > 0$.

The set $\mathcal{P} \subseteq D$ is a global set of Pareto-optimal solutions (it is a global Pareto set) if and only if ([17]):

$$\forall \bar{x}_A \in \mathcal{P}: \ \nexists \bar{x}_B \in D \text{ such, as } \bar{x}_B \succeq \bar{x}_A \tag{1}$$

Local and global Pareto sets define, in the space of objectives, local ($\mathcal{PF}_{local}$) and global ($\mathcal{PF}$) Pareto frontiers, respectively. Their formal definition is:

$$\mathcal{PF}_{local} = \left\{ \bar{y} = F(\bar{x}) \in \mathbb{R}^M \ \mid \ \bar{x} \in \mathcal{P}_{local} \right\} \tag{2a}$$

$$\mathcal{PF} = \left\{ \bar{y} = F(\bar{x}) \in \mathbb{R}^M \ \mid \ \bar{x} \in \mathcal{P} \right\} \tag{2b}$$

Multi-objective optimization problems with distinguished local and global Pareto frontiers are called *multi-modal multi-objective problems* [19].

III. Co-Evolutionary Multi-Agent System with Sexual Selection Mechanism

The system presented in this paper is based on the general model of co-evolution in multi-agent system [10]. The formal model of the first attempt to introduce the sexual selection mechanism into co-evolutionary multi-agent system for multi-objective optimization was presented in [15]. In this section the new approach to the agent-based realization of multi-objective co-evolutionary algorithm with sexual selection is presented.

The basic ideas of the agent-based co-evolutionary computations paradigm are also applied in the case of system presented in this paper. The CoEMAS system with sexual selection (SCoEMAS) is composed of the environment and

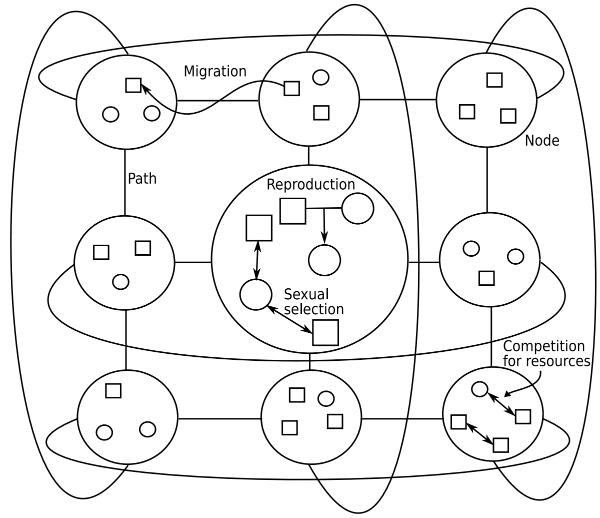


Fig. 1. Co-evolutionary multi-agent system with sexual selection

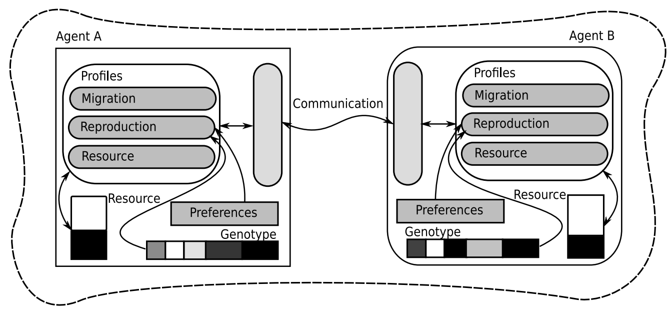


Fig. 2. Agents of different sexes in SCoEMAS

agents, which live within the environment. The environment is composed of computational nodes ("islands") on which agents live (see fig. 1). Nodes are connected with paths, through which agents can migrate from one node to another. Agents make autonomously all their decision concerning reproduction, selection of partner for reproduction, migration within the environment, and so on.

The selection mechanism is based on the resources, which are exchanged between agents, and which are needed for every action, like reproduction and migration. Each agent tries to gain some resources from other agents, which are located within the same node (each agent "sees", can communicate, and interact only with the agents that are located within the same node of the environment). The resources are transferred from dominated agents to agents that dominate them (the relation of Pareto domination is used).

There are two sexes (A and B) within the population (see fig. 1 and 2). The sexual selection mechanism works in the following way. Each agent has its own preferences, which are composed of the vector of weights (each weight for one of the criteria of the problem being solved). These individual preferences are used during the selection of partner for reproduction. When the agent $a_i \in A(t)$ (where $A(t)$ is the set of all agents that exist in the system in time

t) has enough resources, it decides to reproduce. Then it searches for the partner $a_j \in A(t)$ such, that it also has the amount of resources above the given level and it is of the opposite sex. When there are more than one of such agents, then the agent that initiated the process, chooses one of them basing its decision on the vector of preferences and the genotypes of the found candidates for partners. The best partner is then chosen and one offspring is created with the use of intermediate recombination and Gaussian mutation with self-adaptation operators (the real-valued vector is used as the genotype) [1]. Each parent gives the offspring some of its resources. The child's vector of preferences (weights) is randomly determined—it is not the subject of changes during the agent's life. Also the sex of the offspring is determined randomly. The most important agent's activities (of both sexes) are presented in Algorithm 1.

Algorithm 1 Activities of the agent $a_i \in A$ in SCoEMAS

while agent is alive **do**
  if the active goal is from resource profile **then**
    if agent is out of resources **then**
      $\langle die \rangle$ (agent is removed from the system)
    else {the amount of agent's resources is below the minimal level}
      $\langle searchDominated \rangle$ (search for dominated agents located within the same node)
      $\langle get \rangle$ (get some amount of resources from the dominated agents)
    end if
  else if the active goal is from the reproduction profile **then**
    $\langle searchPartner \rangle$ (search for partner from the opposite sex)
    $\langle choose \rangle$ (choose the best partner, according to preferences, from the set of candidates)
    $\langle clone \rangle$ (clone itself—the child is created)
    $\langle rec \rangle$ (recombination)
    $\langle mut \rangle$ (mutation)
    $\langle give \rangle$ (give some resources to the offspring)
  else {the active goal is from the migration profile}
    $\langle selNode \rangle$ (select node of the environment connected with the current node)
    $\langle migr \rangle$ (migrate)
  end if
end while

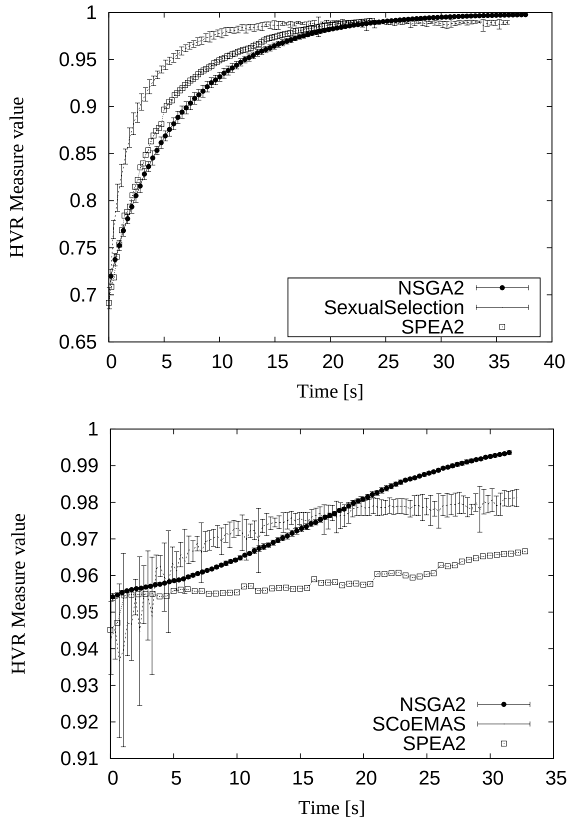
The agent is composed of the profiles, the communication mechanism, the genotype, the preferences, and the resources (see fig. 2). The communication mechanism is used to exchange informations with the other agents. The most important part of the agents is the profile mechanism, which is used to decide what actions should be performed in order to realize agent's goals. Each profile is composed of the goal (which can be active or not active), and actions, which can be realized within the given profile, and which result in the realization of the profile's goal. The agent selects the profile with the active goal and the highest priority and realizes actions of that profile, what eventually lead to the realization of the goal, which then becomes not active. Then the whole process is repeated. In the system presented in this paper there are three profiles: resource (with the highest priority), reproduction, and migration (with the lowest priority). Within the resource profile there are three actions possible to realize: $\langle die \rangle$ (which is performed when the agent is out of resources—the agent is removed from the system), $\langle searchDominated \rangle$ (which finds the agents that are dominated by the given agent), and $\langle get \rangle$ (which is used to get the resources from a dominated agent). Within the reproduction profile there are the following actions: $\langle searchPartner \rangle$ (which is used to find candidates for reproduction partners), $\langle choose \rangle$ (this actions realizes the mechanism of sexual selection—the partner is chosen on the basis of individual preferences), $\langle clone \rangle$ (which is used to make the new agent—offspring), $\langle rec \rangle$ (this action realizes the recombination), $\langle mut \rangle$ (which realizes the mutation), and $\langle give \rangle$ (which is used to give the offspring some amount of the parent's resources). The migration profile is composed of the following actions: $\langle selNode \rangle$, which chooses the node (from the nodes connected with the current node) to which the agent will migrate, $\langle migr \rangle$, which allows the agent to migrate from one node to another node of the environment. The migration causes the lose of some amount of the agent's resources.

IV. Experimental Results

TABLE I
SELECTED CONFIGURATION PARAMETERS

Parameter	Comments	SCoEMAS
Initial-Resources-Per-Agent	Resources possessed initially by individual just after its creation	50
Resources-To-Transfer	Resources transferred in the case of domination	30
Mutation-Probability	—	0.5
Resources-For-Crossover	Resources required for reproduction	50

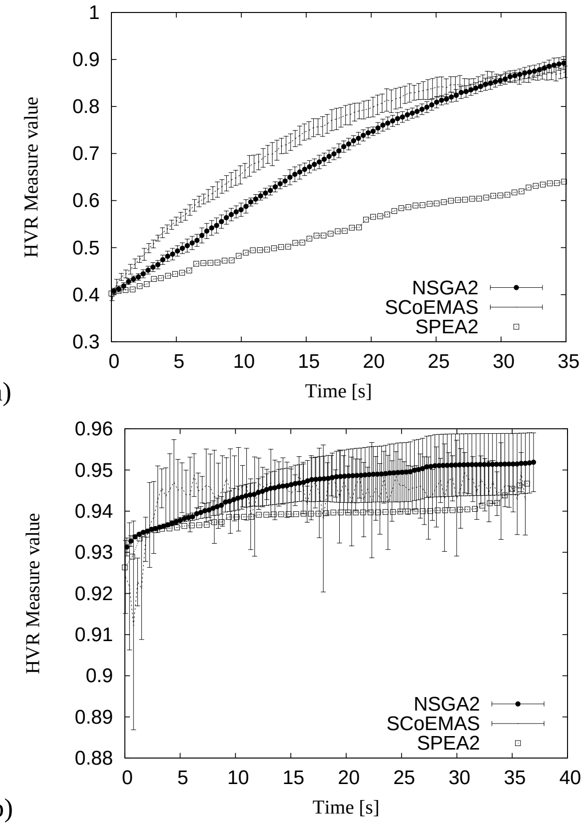
To assess the quality of algorithm for solving multi-objective optimization problems one has to use: well defined, commonly and *worldwide* used benchmarking algorithms, challenging benchmarking test problems and metrics—on the basis of which it can be said that the algorithm being the subject of tests obtains better/worse results (and in the consequence it is a better/worse algorithm) than benchmarking algorithms. Obviously test problems have to really test different aspects of algorithms (in the case of the algorithms for solving multi-objective optimization problems they should test such aspects as easiness/possibility of solving problems with not only connected and convex Pareto



a)

b)

Fig. 3. HVR values obtained by SCoEMAS, NSGA-II and SPEA2 run against Zitzler's problems: ZDT1 (a), and ZDT2 (b)



a)

b)

Fig. 4. HVR values obtained by SCoEMAS, NSGA-II and SPEA2 run against Zitzler's problems: ZDT3 (a), and ZDT4 (b)

frontiers/sets but also problems with concave Pareto frontiers, with disconnected Pareto frontiers, multi-modal problems etc.) To assess presented in the course of this paper agent-based multi-objective co-evolutionary algorithm with sexual selection the following assumptions were made:

- as benchmarking algorithms—NSGA-II and SPEA2 algorithms were used (i.e. algorithms commonly perceived as the most efficient and most frequently used evolutionary multi-objective algorithms);
- as benchmarking problems—Zitzler's problems—ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6 (their definitions can be found in [17]) were used. And again these are one of the most commonly and frequently used test problems for assessing multi-objective optimization algorithms;
- and finally, as the quality measure the HVR metric was used [19]. As the metrics with its well known shortcomings, but on the other hand as the metrics measuring simultaneously both closeness to the model Pareto frontier and dispersing solutions over the whole frontier it is commonly and *worldwide* used.

  Hypervolume or Hypervolume ratio (HVR) [20], describes the area covered by solutions of obtained result set. For each solution, hypercube is evaluated with respect to the fixed reference point. In order to evaluate hypervolume ratio, value of hypervolume for obtained set is normalized with hypervolume value computed for true Pareto frontier. HV and HVR are defined as follows:

$$\mathrm{HV} = v\left(\bigcup_{i=1}^{N} v_i\right) \tag{3a}$$

$$\mathrm{HVR} = \frac{\mathrm{HV}(PF^*)}{\mathrm{HV}(PF)} \tag{3b}$$

  where v_i is hypercube computed for $i-th$ solution, PF^* represents obtained Pareto frontier and PF is the true Pareto frontier.

The size of population of the algorithm that is being assessed and benchmarking algorithms are as follows: SCoEMAS—100, NSGA-II—300 and SPEA2—100. In the table I there are presented selected values of parameters for the co-evolutionary multi-agent system used during experiments, which results are presented in this section.

In the figures 3, 4, and 5 there are presented values of HVR measure obtained with time by co-evolutionary multi-agent system with sexual selection (SCoEMAS) for ZDT1 (fig. 3a), ZDT2 (fig. 3b), ZDT3 (fig. 4a), ZDT4 (fig. 4b) and ZDT6 (fig. 5) problems. For comparison there are presented also results obtained by NSGA-II and SPEA2 algorithms.

On the basis of presented characteristics it can be said that initially co-evolutionary multi-agent system with sexual

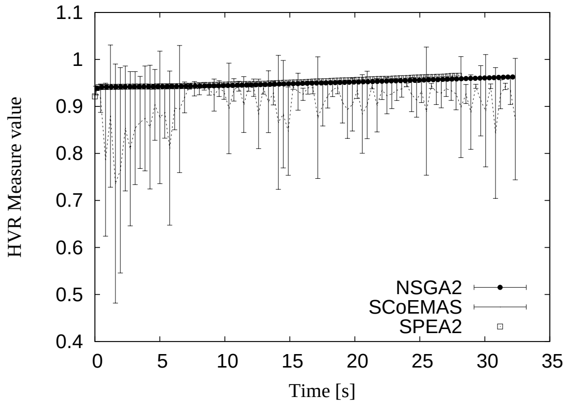


Fig. 5. HVR values obtained by SCoEMAS, NSGA-II and SPEA2 run against Zitzler's ZDT6 problem

selection is faster than two other algorithms, it allows for obtaining better solutions—what can be observed as higher values of HVR(t) metrics but finally best results are obtained by NSGA-II algorithm. A little bit worse alternative than NSGA-II is SCoEMAS and finally SPEA2 is the third alternative—but obviously it depends on the problem that is being solved and differences between analyzed algorithms are not very distinctive.

So, to recapitulate it is worth to underline that in the situation when time is crucial and it is the most important for user to obtain maybe even not the best possible solutions, but it is important to obtain valuable solutions as soon as possible, SCoEMAS is a very interesting alternative. Of course apart from its initial efficiency SCoEMAS possesses also another advantages such as ability for solving multi-objective multi-modal problems, ability for modeling "arms races" and in the consequence the ability for modeling biological environments and economical systems etc.—but those issues are beyond the scope of this paper.

V. Summary and Conclusions

To compare "classical", i.e. non agent-based algorithms, versus proposed in this paper co-evolutionary multi-agent system with sexual selection, at least two aspects have to be taken into consideration, since the effectiveness of optimization algorithm can be analyzed as the function of (algorithm's) step and as the function of time. Decision maker is interested obviously in time aspects (it is important for him how fast it is possible to obtain valuable results) and how precisely given algorithm is able to approximate the model (ideal) solution (in the case of multi-objective optimization it is of course model Pareto frontier). Researchers, during the process of algorithm development should keep in mind also its effectiveness per computational step. There is no doubt that such algorithms as SPEA2 or NSGA2 are much more effective per computational step than the proposed agent-based approach—however, each of their steps is much more complex.

Analyzing characteristics presented in the previous section the following conclusion can be drawn: non-agent algorithms are able to obtain finally better approximation of the model Pareto frontier than the proposed agent-based algorithm. Simultaneously however mentioned "classical" algorithms are computationally much more complex algorithms than the agent-based approach. For instance in NSGA2 algorithm in each step all individuals are sorted according to the consecutive levels of domination, also crowding mechanism used in this algorithm is quite complex and time-consuming, etc. In the consequence, initially (in our experiments during c.a. 15 seconds) solutions proposed by the agent-based algorithm were better than solutions proposed by "classical" (non agent-based) algorithms. So, if time is not crucial and more important is final closeness to the model Pareto frontier then generally speaking "classical" algorithms seem to be better alternative, but if obtaining as valuable results as it is possible and as fast as it is possible is crucial, then agent-based approach can be the very attractive alternative (as it was mentioned, another advantages of agent-based approach, i.e. the fault tolerance, similarity of the model to the real phenomena and in the consequence the ability for modeling different biological, social and economical mechanisms—this was discussed for instance in [21]—are out of the scope of this paper).

The future research could include the additional experimental verification of the described co-evolutionary multi-agent system with sexual selection mechanism. Also, some changes in the sexual selection mechanism are planned. For example the modified mechanism should include the possibility of encoding the agent's preferences of the partner in its genotype—such vector would be then modified by evolutionary operators. Also, some mechanism of elitism would be introduced and verified.

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