Chapter 1
A Review of Agent-Based Co-Evolutionary Algorithms for Multi-Objective Optimization

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Abstract Agent-based evolutionary algorithms are a result of mixing two paradigms: multi-agent systems and evolutionary algorithms. Agent-based co-evolutionary algorithms allow for existing many species and sexes of agents within the system as well as for defining co-evolutionary interactions between species and sexes. Algorithms based on the model of co-evolutionary multi-agent system have been already applied in many domains, like multi-modal optimization, generation of investment strategies, portfolio optimization, and multi-objective optimization. In this chapter we present an overview of selected agent-based co-evolutionary algorithms, their formal models, and results of experiments with standard test problems and financial problem, aimed at making comparison of agent-based and "classical" state-of-the-art multi-objective algorithms. Presented results show that, depending on the problem being solved, agent-based algorithms obtain comparable, and sometimes even better, results than "classical" algorithms, however of course they are not the universal solver for all multi-objective optimization problems.

1.1 Introduction

In spite of a huge potential dozing in evolutionary algorithms and a lot of successful applications such algorithms for solving difficult problem of optimization and searching, very frequently such methods have not been able to deal with defined problem and obtained results have not been satisfying. Among the reasons of such a situation the following can be mentioned:

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- centralization of evolutionary process where the process of selection as well as the process of generation of new generations are controlled by one single algorithm;
- reducing of specimen to the (system of) genes without capabilities of exerting of any influence on the process of evolution;
- omitting some crucial—from the evolution and adaptation capabilities point of view—operations and processes observable in the nature. Moreover, in the literature there are opinions that crossover and mutation they are only the kinds of one single—destructive and exploration-oriented—operator and there is no agreement if (and if so—when) they should be used or even if they should be distinguished [17];
- to realize their own goals, during decision-making process, specimens are able neither to gather nor to utilize any kind of information from the environment;
- depriving specimens of such—absolutely natural and obvious in nature—biological and social behaviors like competition, rivalry, cooperation etc.;
- in the consequence of previous point (limited number of operators) it is almost impossible to define in classical evolutionary algorithms more sophisticated (and more effective simultaneously), advanced algorithms and computational methods.

In the consequence, in the literature, there are being raised arguments that classical evolutionary algorithms are methods of adapting and fitting of algorithm's parameters to defined conditions rather than really creative methods of searching and optimization.

It is nothing strange so, that intensive research is being performed on methods utilizing ideas and conceptions of computer models of observable in nature Darwinian evolution but at the same time, on methods that should be devoid of mentioned above shortcomings, and which could be perceived as a full analogy to natural processes.

During the research, decentralization and autonomy have been in the limelight. Proposed, as a result, method called Evolutionary Multi-Agent System—EMAS [2] should be perceived as a new trend among evolutionary algorithms allowing for realization of defined postulates by utilizing advantages simultaneously of both: evolutionary and agent-based approaches.

Proposed paradigm of evolutionary multi-agent system is characterized by the following—crucial, taking the shortcomings of classical evolutionary algorithms into account—features:

- in the process of evolution autonomous agents are taking a part. Agents are able to make decisions to realize their own goals and they are not passive units of global and central evolution which are limited and reduced to the role of (group of) genes;
- the prices of evolution is decentralized and agents taking the part in that are able to create advanced social structures and to realize sophisticated

strategies of cooperation, competition, interactions and reciprocal relations
- agents taking the part in the process of evolution are able to observe the environment (and occurring changes) and to make appropriate decisions and actions what additionally enrich the spectrum of possible for realization complex and effective computational methods and algorithms.

During further research on realizing advanced, complex social and biological mechanisms within the confines of EMAS—general model of so called CoEMAS *Co-evolutionary multi-agent systems (CoEMAS)* [8] has been proposed and it has turned out that with the use of such a model almost any kind of interaction, cooperation or competition among many species or sexes of co-evolving agents is possible what allows for improving the quality of obtained result. Such improvement results mainly from better maintenance of population diversity—what is especially important in the case of applying such systems for solving multi-modal or multi-objective optimization tasks.

In the course of this chapter we are focusing on applying co-evolutionary multi-agent systems for solving multi-objective optimization tasks.

Following [5]—*multi-objective optimization problem—MOOP* in its general form is being defined as follows:

$$MOOP \equiv \begin{cases} Minimize/Maximize & f_m(\bar{x}), \quad m = 1, 2 \ldots, M \\ Subject\ \ to & g_j(\bar{x}) \geq 0, \quad j = 1, 2 \ldots, J \\ & h_k(\bar{x}) = 0, \quad k = 1, 2 \ldots, K \\ & x_i^{(L)} \leq x_i \leq x_i^{(U)}, \quad i = 1, 2 \ldots, N \end{cases}$$

Authors of this chapter assume that readers are familiar with at least fundamental concepts and notions regarding multi-objective optimization in the Pareto sense (relation of domination, Pareto frontier and Pareto set etc.) and their explanation is omitted in this paper (interested readers can find definitions and deep analysis of all necessary concepts and notions of Pareto multi-objective optimization for instance in [3, 5]).

This chapter is organized as follows:

- in Section 1.2 formal model as well as detailed description of *Co-Evolutionary Multi-Agent System—CoEMAS* is presented;
- in Section 1.3 detailed description and formal model of two realization of CoEMAS applied for solving MOOP is given. In this section *Co-Evolutionary Multi-Agent System with Predator-Prey* interactions (*PP-CoEMAS*) as well as *Co-Evolutionary Multi-Agent System with Cooperation (CCoEMAS)* are discussed;
- in Section 1.4 we discuss shortly test suite and performance metric used during experiments, and next we glance at results obtained by both systems presented in the course of this chapter (PPCoEMAS and CCoEMAS);

- in Section 1.5 the most important remarks, conclusions and comments are given.

1.2 Model of Co-Evolutionary Multi-Agent System

Agent-based models of evolutionary algorithms are the result of mixing two paradigms: multi-agent systems and evolutionary algorithms. The result is decentralized evolutionary system, in which agents "live" within the environment of the system, compete for limited resources, reproduce, die, migrate from one computational node to another, observe the environment and other agents, and can communicate with other agents and change the environment.

Basic model of agent-based evolutionary algorithm (so called *evolutionary multi-agent system—EMAS* model) was proposed in [2]. EMAS model included all the features which were mentioned above. However in the case of some problems, for example multi-modal optimization or multi-objective optimization, it turned out that these mechanisms are not sufficient. Such types of problems require maintaining of population diversity mechanisms, speciation mechanisms and possibilities of introducing additional biologically and socially inspired mechanisms in order to solve a problem and obtain satisfying results.

Mentioned above limitations of the basic EMAS model and research aimed at applying agent-based evolutionary algorithms to multi-modal and multi-objective problems led to the formulation of the model of *co-evolutionary multi-agent system—CoEMAS* [8]. This model included the possibilities of existing different species and sexes in the system and allowed for defining co-evolutionary interactions between them. Below we present basic ideas and notions of CoEMAS model, which we will use in Section 1.3 when the systems used in experiments will be described.

1.2.1 Co-Evolutionary Multi-Agent System

The *CoEMAS* is described as 4-tuple:

$$CoEMAS = \langle E, S, \Gamma, \Omega \rangle \tag{1.1}$$

where E is the environment of the $CoEMAS$, S is the set of species ($s \in S$) that co-evolve in $CoEMAS$, Γ is the set of resource types that exist in the system, the amount of type γ resource will be denoted by r^{γ}, Ω is the set of information types that exist in the system, the information of type ω will be denoted by i^{ω}.

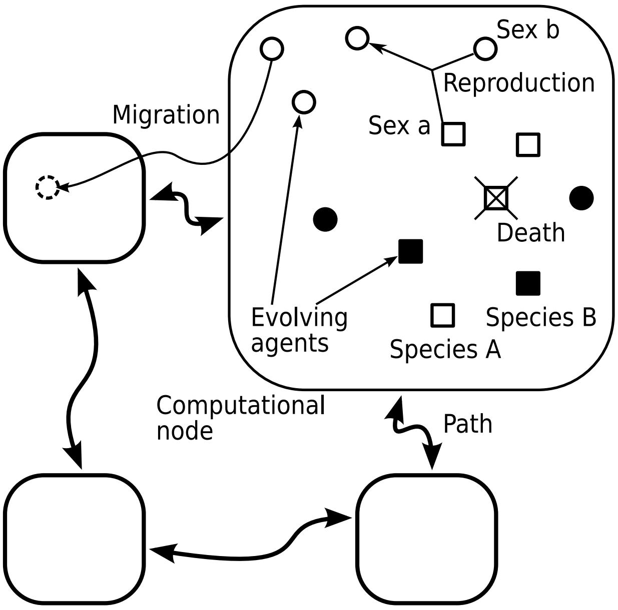


Fig. 1.1: Co-evolutionary multi-agent system

1.2.2 Environment

The environment of *CoEMAS* may be described as 3-tuple:

$$E = \left\langle T^E, \varGamma^E, \varOmega^E \right\rangle \tag{1.2}$$

where T^E is the topography of environment E, $\varGamma^E$ is the set of resource types that exist in the environment, $\varOmega^E$ is the set of information types that exist in the environment. The topography of the environment is given by:

$$T^E = \langle H, l \rangle \tag{1.3}$$

where H is directed graph with the cost function c defined: $H = \langle V, B, c \rangle$, V is the set of vertices, B is the set of arches. The distance between two nodes is defined as the length of the shortest path between them in graph H.

The l function makes it possible to locate particular agent in the environment space:

$$l : \quad A \to V \tag{1.4}$$

where A is the set of agents, that exist in *CoEMAS*.

Vertice v is given by:

$$v = \langle A^v, \varGamma^v, \varOmega^v, \varphi \rangle \tag{1.5}$$

A^v is the set of agents that are located in the vertice v, $\varGamma^v$ is the set of resource types that exist within the v ($\varGamma^v \subseteq \varGamma^E$), $\varOmega^v$ is the set of information types that exist within the v ($\varOmega^v \subseteq \varOmega^E$), φ is the fitness function.

1.2.3 Species

Species $s \in S$ is defined as follows:

$$s = \langle A^s, SX^s, Z^s, C^s \rangle \tag{1.6}$$

where:

- A^s is the of agents of species s (by a^s we will denote the agent, which is of species s, $a^s \in A^s$);
- SX^s is the set of sexes within the s;
- Z^s is the set of actions, which can be performed by the agents of species s ($Z^s = \bigcup\limits_{a \in A^s} Z^a$, where Z^a is the set of actions, which can be performed by the agent a);
- C^s is the set of relations with other species that exist within $CoEMAS$.

The set of relations of s_i with other species (C^{s_i}) is the sum of the following sets of relations:

$$C^{s_i} = \left\{ \xrightarrow{s_i, z-} : z \in Z^{s_i} \right\} \cup \left\{ \xrightarrow{s_i, z+} : z \in Z^{s_i} \right\} \tag{1.7}$$

where $\xrightarrow{s_i, z-}$ and $\xrightarrow{s_i, z+}$ are relations between species, based on some actions $z \in Z^{s_i}$, which can be performed by the agents of species s_i:

$$\begin{aligned}\xrightarrow{s_i, z-} = \big\{ \langle s_i, s_j \rangle \in S \times S : &\text{ agents of species } s_i \text{ can decrease the fitness of} \\ &\text{agents of species } s_j \text{ by performing the action } z \in Z^{s_i} \big\}\end{aligned} \tag{1.8}$$

$$\begin{aligned}\xrightarrow{s_i, z+} = \big\{ \langle s_i, s_j \rangle \in S \times S : &\text{ agents of species } s_i \text{ can increase the fitness of} \\ &\text{agents of species } s_j \text{ by performing the action } z \in Z^{s_i} \big\}\end{aligned} \tag{1.9}$$

If $s_i \xrightarrow{s_i, z-} s_i$ then we are dealing with the intra-species competition, for example the competition for limited resources, and if $s_i \xrightarrow{s_i, z+} s_i$ then there is some form of co-operation within the species s_i.

With the use of the above relations we can define many different co-evolutionary interactions, e.g., mutualism, predator-prey, host-parasite, etc.

For example *mutualism* between two species s_i and s_j $(i \neq j)$ takes place if and only if $\exists z_k \in Z^{s_i}$ $\exists z_l \in Z^{s_j}$, such that $s_i \xrightarrow{s_i, z_k +} s_j$ and $s_j \xrightarrow{s_j, z_l +} s_i$ and these two species live in tight co-operation.

Predator-prey interactions between two species, s_i (predators) and s_j (preys) $(i \neq j)$, takes place if and only if $\exists z_k \in Z^{s_i}$ $\exists z_l \in Z^{s_j}$, such that $s_i \xrightarrow{s_i, z_k -} s_j$ and $s_j \xrightarrow{s_j, z_l +} s_i$, where z_k is the action of killing the prey (*kill*), and z_l is the action of death (*die*).

1.2.4 Sex

The sex $sx \in SX^s$ which is within the species s is defined as follows:

$$sx = \langle A^{sx}, Z^{sx}, C^{sx} \rangle \tag{1.10}$$

where A^{sx} is the set of agents of sex sx and species s $(A^{sx} \subseteq A^s)$:

$$A^{sx} = \{a : a \in A^s \wedge a \text{ is the agent of sex } sx\} \tag{1.11}$$

With a^{sx} we will denote the agent of sex sx $(a^{sx} \in A^{sx})$. Z^{sx} is the set of actions which can be performed by the agents of sex sx, $Z^{sx} = \bigcup_{a \in A^{sx}} Z^a$, where Z^a is the set of actions which can be performed by the agent a. And finally C^{sx} is the set of relations between the sx and other sexes of the species s.

Analogically as in the case of species, we can define the relations between the sexes of the same species. The set of all relations of the sex $sx_i \in SX^s$ with other sexes of species s (C^{sx_i}) is the sum of the following sets of relations:

$$C^{sx_i} = \left\{ \xrightarrow{sx_i, z-} : z \in Z^{sx_i} \right\} \cup \left\{ \xrightarrow{sx_i, z+} : z \in Z^{sx_i} \right\} \tag{1.12}$$

where $\xrightarrow{sx_i, z-}$ and $\xrightarrow{sx_i, z+}$ are the relations between sexes, in which some actions $z \in Z^{sx_i}$ are used:

$$\xrightarrow{sx_i, z-} = \big\{ \langle sx_i, sx_j \rangle \in SX^s \times SX^s : \text{agents of sex } sx_i \text{ can decrease the fitness of agents of sex } sx_j \text{ by performing the action } z \in Z^{sx_i} \big\} \tag{1.13}$$

$$\xrightarrow{sx_i, z+} = \big\{ \langle sx_i, sx_j \rangle \in SX^s \times SX^s : \text{agents of sex } sx_i \text{ can increase the fitness of agents of sex } sx_j \text{ by performing the action } z \in Z^{sx_i} \big\} \tag{1.14}$$

With the use of presented relations between sexes we can model for example sexual selection interactions, in which agents of one sex choose partners for reproduction from agents of the other sex within the same species, taking into account some preferred features (see [10]).

1.2.5 Agent

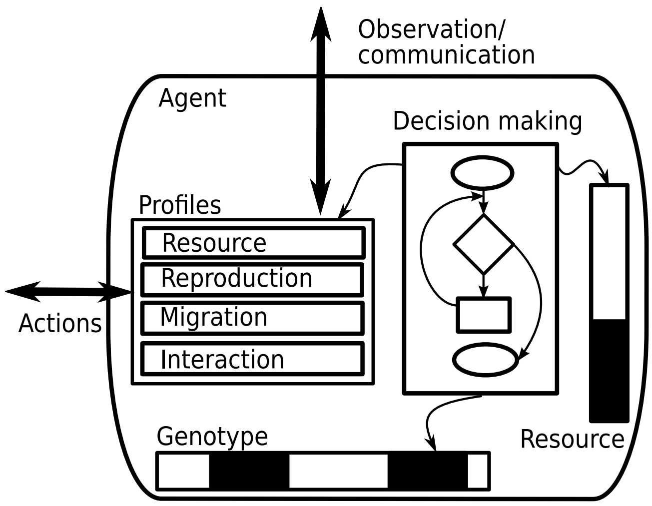


Fig. 1.2: Agent in the $CoEMAS$

Agent a (see Fig. 1.2) of sex sx and species s (in order to simplify the notation we assume that $a \equiv a^{sx,s}$) is defined as follows:

$$a = \langle gn^a, Z^a, \varGamma^a, \varOmega^a, PR^a \rangle \tag{1.15}$$

where:

- gn^a is the genotype of agent a, which may be composed of any number of chromosomes (for example: $gn^a = \langle (x_1, x_2, \ldots, x_k) \rangle$, where $x_i \in \mathbb{R}$, $gn^a \in \mathbb{R}^k$);
- Z^a is the set of actions, which agent a can perform;
- $\varGamma^a$ is the set of resource types, which are used by agent a ($\varGamma^a \subseteq \varGamma$);
- $\varOmega^a$ is the set of information, which agent a can possess and use ($\varOmega^a \subseteq \varOmega$);
- PR^a is partially ordered set of profiles of agent a ($PR^a \equiv \langle PR^a, \trianglelefteq \rangle$) with defined partial order relation $\trianglelefteq$.

Algorithm 1. Basic activities of agent a in $CoEMAS$

$r^{\gamma} \leftarrow r^{\gamma}_{init}$; /* r^{γ}_{init} is the initial amount of resource given to the agent */
while $r^{\gamma} > 0$ **do**
  activate the profile $pr_i \in PR^a$ with the highest priority and with the active goal $gl^*_j \in GL^{pr_i}$;
  if *pr_i is the resource profile* **then**
    if $0 < r^{\gamma} < r^{\gamma}_{min}$ **then** ; /* r^{γ}_{min} is the minimal amount of resource needed by the agent to realize its activities */
      choose the strategy $st_k \in ST^{pr_i}$ with the highest priority that can be used to take some resources from the environment or other agent;
      perform actions contained within the st_k;
    else if $r^{\gamma} = 0$ **then**
      execute $\langle die \rangle$ strategy;
    end
  else if *pr_i is the reproduction profile* **then**
    if $r^{\gamma} > r^{rep,\gamma}_{min}$ **then** ; /* $r^{rep,\gamma}_{min}$ is the minimal amount of resource needed for reproduction */
      choose the strategy $st_k \in ST^{pr_i}$ with the highest priority that can be used to reproduce;
      perform actions contained within the st_k;
    end
  else if *pr_i is the migration profile* **then**
    if $r^{\gamma} > r^{mig,\gamma}_{min}$ **then** ; /* $r^{mig,\gamma}_{min}$ is the minimal amount of resource needed for migration */
      choose the strategy $st_k \in ST^{pr_i}$ with the highest priority that can be used to migrate;
      perform actions contained within the st_k;
      give $r^{mig,\gamma}_{min}$ amount of resource to the environment;
    end
  end
end

Relation $\trianglelefteq$ is defined in the following way:

$$\trianglelefteq = \Big\{ \langle pr_i, pr_j \rangle \in PR^a \times PR^a : \text{realization of active goals of profile } pr_i \text{ has equal or higher priority than the realization of active goals of profile } pr_j \Big\} \tag{1.16}$$

The active goal (which is denoted as gl^*) is the goal gl, which should be realized in the given time. The relation $\trianglelefteq$ is reflexive, transitive and antisymmetric and partially orders the set PR^a:

$$pr \trianglelefteq pr \quad \text{for every } pr \in PR^a \tag{1.17a}$$
$$(pr_i \trianglelefteq pr_j \wedge pr_j \trianglelefteq pr_k) \Rightarrow pr_i \trianglelefteq pr_k \quad \text{for every } pr_i, pr_j, pr_k \in PR^a \tag{1.17b}$$
$$(pr_i \trianglelefteq pr_j \wedge pr_j \trianglelefteq pr_i) \Rightarrow pr_i = pr_k \quad \text{for every } pr_i, pr_j \in PR^a \tag{1.17c}$$

The set of profiles PR^a is defined in the following way:

$$PR^a = \{pr_1, pr_2, \ldots, pr_n\} \tag{1.18a}$$
$$pr_1 \trianglelefteq pr_2 \trianglelefteq \cdots \trianglelefteq pr_n \tag{1.18b}$$

Profile pr_1 is the basic profile—it means that the realization of its goals has the highest priority and they will be realized before the goals of other profiles.

Profile pr of agent a ($pr \in PR^a$) can be the profile in which only resources are used:

$$pr = \langle \Gamma^{pr}, ST^{pr}, RST^{pr}, GL^{pr} \rangle \tag{1.19}$$

in which only information are used:

$$pr = \langle \Omega^{pr}, M^{pr}, ST^{pr}, RST^{pr}, GL^{pr} \rangle \tag{1.20}$$

or resources and information are used:

$$pr = \langle \Gamma^{pr}, \Omega^{pr}, M^{pr}, ST^{pr}, RST^{pr}, GL^{pr} \rangle \tag{1.21}$$

where:

- Γ^{pr} is the set of resource types, which are used within the profile pr ($\Gamma^{pr} \subseteq \Gamma^a$);
- Ω^{pr} is the set of information types, which are used within the profile pr ($\Omega^{pr} \subseteq \Omega^a$);
- M^{pr} is the set of information representing the agent's knowledge about the environment and other agents (it is the model of the environment of agent a);
- ST^{pr} is the partially ordered set of strategies ($ST^{pr} \equiv \langle ST^{pr}, \preccurlyeq \rangle$), which can be used by agent within the profile pr in order to realize an active goal of this profile;
- RST^{pr} is the set of strategies that are realized within the profile pr—generally, not all of the strategies from the set ST^{pr} have to be realized within the profile pr, some of them may be realized within other profiles;
- GL^{pr} is partially ordered set of goals ($GL^{pr} \equiv \langle GL^{pr}, \preccurlyeq \rangle$), which agent has to realize within the profile pr.

The relation $\preccurlyeq$ is defined in the following way:

$$\begin{aligned}\preccurlyeq = \big\{ \langle st_i, st_j \rangle \in ST^{pr} \times ST^{pr} :\ & \text{strategy } st_i \text{ has equal or higher} \\ & \text{priority than strategy } st_j \big\}\end{aligned} \tag{1.22}$$

This relation is reflexive, transitive and antisymmetric and partially orders the set ST^{pr}. Every single strategy $st \in ST^{pr}$ is consisted of actions, which ordered performance leads to the realization of some active goal of the profile pr:

$$st = \langle z_1, z_2, \dots, z_k \rangle, \quad st \in ST^{pr}, \quad z_i \in Z^a \tag{1.23}$$

The relation $\preccurlyeq$ is defined in the following way:

$$\begin{aligned} \preccurlyeq = \big\{ \langle gl_i, gl_j \rangle \in GL^{pr} \times GL^{pr} : \ & \text{goal } gl_i \text{ has equal or higher} \\ & \text{priority than the goal } gl_j \big\} \end{aligned} \tag{1.24}$$

This relation is reflexive, transitive and antisymmetric and partially orders the set GL^{pr}.

The partially ordered sets of profiles PR^a, goals GL^{pr} and strategies ST^{pr} are used by the agent in order to make decisions about the realized goal and to choose the appropriate strategy in order to realize that goal. The basic activities of the agent a are shown in Algorithm 1.

In *CoEMAS* systems the set of profiles is usually composed of resource profile (pr_1), reproduction profile (pr_2), and migration profile (pr_3):

$$PR^a = \{pr_1, pr_2, pr_3\} \tag{1.25a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \tag{1.25b}$$

The highest priority has the resource profile, then there is reproduction profile, and finally migration profile.

1.3 Co-Evolutionary Multi-Agent Systems for Multi-Objective Optimization

In this section we will describe two co-evolutionary multi-agent systems used in the experiments. Each of these systems uses different co-evolutionary mechanism: co-operation and predator-prey interactions. All of the systems are based on general model of co-evolution in multi-agent system described in Section 1.2—in this section only such elements of the systems will be described that are specific for these instantiations of the general model. In all the systems presented below, real-valued vectors are used as agents' genotypes. Mutation with self-adaptation and intermediate recombination are used as evolutionary operators [1].

1.3.1 Co-Evolutionary Multi-Agent System with Co-Operation Mechanism (CCoEMAS)

The co-evolutionary multi-agent system with co-operation mechanism is defined as follows (see Eq. (1.1)):

$$CCoEMAS = \langle E, S, \Gamma, \Omega \rangle \tag{1.26}$$

The number of species corresponds with the number of criteria (n) of the multi-objective problem being solved $S = \{s_1, \ldots, s_n\}$. Three information types ($\Omega = \{\omega_1, \omega_2, \omega_3\}$) and one resource type ($\Gamma = \{\gamma\}$) are used. Information of type ω_1 denotes nodes to which agent can migrate. Information of type ω_2 denotes (for the agent of given species) all agents from other species that are located within the same node in time t. Information of type ω_3 denotes (for the given agent) all agents from the same species located within the same node.

1.3.1.1 Species

The species s is defined as follows:

$$s = \langle A^s, SX^s = \{sx\}, Z^s, C^s \rangle \tag{1.27}$$

where SX^s is the set of sexes which exist within the s species, Z^s is the set of actions that agents of species s can perform, and C^s is the set of relations of s species with other species that exist in the $CCoEMAS$.

Actions

The set of actions Z^s is defined as follows:

$$Z^s = \{die, seek, get, give, accept, seekPartner, clone, rec, mut, migr\} \tag{1.28}$$

where:

- *die* is the action of death (agent dies when it is out of resources);
- *seek* is the action of finding a dominated agent from the same species in order to take some resources from it;
- *get* action gets some resource from another agent located within the same node, which is dominated by the agent that performs *get* action;
- *give* action gives some resources to the agent that performs *get* action;
- *accept* action accepts partner for reproduction when the amount of resource possessed by the agent is above the given level;

- *seekPartner* action seeks for partner for reproduction, such that it comes from another species and has the amount of resource above the minimal level needed for reproduction;
- *clone* is the action of producing offspring (parents give some of their resources to the offspring during this action);
- *rec* is the recombination operator (intermediate recombination is used [1]);
- *mut* is the mutation operator (mutation with self-adaptation is used [1]);
- *migr* is the action of migrating from one node to another. During this action agent loses some of its resource.

Relations

The set of relations of s_i species with other species that exist within the system is defined as follows:

$$C^{s_i} = \left\{ \xrightarrow{s_i, get-}, \xrightarrow{s_i, accept+} \right\} \tag{1.29}$$

The first relation models intra species competition for limited resources:

$$\xrightarrow{s_i, get-} = \{\langle s_i, s_i \rangle\} \tag{1.30}$$

The second one models co-operation between species:

$$\xrightarrow{s_i, accept+} = \{\langle s_i, s_j \rangle\} \tag{1.31}$$

1.3.1.2 Agent

Agent a of species s ($a \equiv a^s$) is defined as follows:

$$a = \langle gn^a, Z^a = Z^s, \Gamma^a = \Gamma, \Omega^a = \Omega, PR^a \rangle \tag{1.32}$$

Genotype of agent a is consisted of two vectors (chromosomes): $\mathbf{x}$ of real-coded decision parameters' values and $\boldsymbol{\sigma}$ of standard deviations' values, which are used during mutation with self-adaptation. Agents of the given species are evaluated according to only one criteria associated with this species. $Z^a = Z^s$ (see Eq. (1.28)) is the set of actions which agent a can perform. Γ^a is the set of resource types used by the agent, and Ω^a is the set of information types. Basic activities of agent a in $CCoEMAS$ with the use of profiles are presented in Alg. 2.

Algorithm 2. Basic activities of agent a in $CCoEMAS$

$r^{\gamma} \leftarrow r^{\gamma}_{init}$;
while $r^{\gamma} > 0$ **do**
  activate the profile $pr_i \in PR^a$ with the highest priority and with the active goal $gl^*_j \in GL^{pr_i}$;
  if pr_1 *is activated* **then**
    if $0 < r^{\gamma} < r^{\gamma}_{min}$ **then**
      $\langle seek, get\rangle$;
      $r^{\gamma} \leftarrow \left(r^{\gamma} + r^{\gamma}_{get}\right)$;
    else if $r^{\gamma} = 0$ **then**
      $\langle die\rangle$;
    end
  else if pr_2 *is activated* **then**
    if $r^{\gamma} > r^{rep,\gamma}_{min}$ **then**
      $\langle seekPartner, clone, rec, mut\rangle$;
      $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{rep,\gamma}_{give}\right)$;
    end
  else if pr_3 *is activated* **then**
    if $\langle accept\rangle$ *is activated* **then**
      $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{rep,\gamma}_{give}\right)$;
    else if $\langle give\rangle$ *is activated* **then**
      $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{\gamma}_{get}\right)$;
    end
  else if pr_4 *is activated* **then**
    if $r^{\gamma} > r^{mig,\gamma}_{min}$ **then**
      $\langle migr\rangle$;
      $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{mig,\gamma}_{min}\right)$;
    end
  end
end

Profiles

The partially ordered set of profiles includes resource profile (pr_1), reproduction profile (pr_2), interaction profile (pr_3), and migration profile (pr_4):

$$PR^a = \{pr_1, pr_2, pr_3, pr_4\} \tag{1.33a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \trianglelefteq pr_4 \tag{1.33b}$$

The resource profile is defined in the following way:

$$\begin{aligned} pr_1 = \big\langle & \mathit{\Gamma}^{pr_1} = \mathit{\Gamma}, \mathit{\Omega}^{pr_1} = \{\omega_3\}, M^{pr_1} = \{i^{\omega_3}\}, \\ & ST^{pr_1}, RST^{pr_1} = ST^{pr_1}, GL^{pr_1}\big\rangle \end{aligned} \tag{1.34}$$

The set of strategies include two strategies:

$$ST^{pr_1} = \{\langle die\rangle, \langle seek, get\rangle\} \tag{1.35}$$

The goal of the pr_1 profile is to keep the amount of resources above the minimal level or to die when the amount of resources falls to zero. This profile uses the model $M^{pr_1} = \{i^{\omega_3}\}$.

The reproduction profile is defined as follows:

$$\begin{aligned} pr_2 = \big\langle \varGamma^{pr_2} = \varGamma, \varOmega^{pr_2} = \{\omega_2\}, M^{pr_2} = \{i^{\omega_2}\}, \\ ST^{pr_2}, RST^{pr_2} = ST^{pr_2}, GL^{pr_2} \big\rangle \end{aligned} \tag{1.36}$$

The set of strategies include one strategy:

$$ST^{pr_2} = \{\langle seekPartner, clone, rec, mut \rangle\} \tag{1.37}$$

The only goal of the pr_2 profile is to reproduce. In order to realize this goal agent can use strategy of reproduction: $\langle seekPartner, clone, rec, mut \rangle$. During the reproduction agent transfers the amount of $r_{give}^{rep,\gamma}$ resources to the offspring.

The interaction profile is defined as follows:

$$\begin{aligned} pr_3 = \big\langle \varGamma^{pr_3} = \varGamma, \varOmega^{pr_3} = \{\omega_2, \omega_3\}, M^{pr_3} = \{i^{\omega_2}, i^{\omega_3}\}, \\ ST^{pr_3} = \{\langle accept \rangle, \langle give \rangle\}, RST^{pr_3} = ST^{pr_3}, GL^{pr_3} \big\rangle \end{aligned} \tag{1.38}$$

The goal of the pr_3 profile is to interact with agents from another species with the use of $\langle accept \rangle$ and $\langle give \rangle$ strategies.

The migration profile is defined as follows:

$$\begin{aligned} pr_4 = \big\langle \varGamma^{pr_4} = \varGamma, \varOmega^{pr_4} = \{\omega_1\}, M^{pr_4} = \{i^{\omega_1}\}, \\ ST^{pr_4} = \big\{\big\langle migr \big\rangle\big\}, RST^{pr_4} = ST^{pr_4}, GL^{pr_4} \big\rangle \end{aligned} \tag{1.39}$$

The goal of the pr_4 profile is to migrate within the environment. In order to realize such a goal the migration strategy $\big\langle migr \big\rangle$ is used, which firstly chooses the node on the basis of information $\{i^{\omega_1}\}$ and then realizes the migration. As a result of migrating agent loses some of its resources.

1.3.2 Co-Evolutionary Multi-Agent System with Predator-Prey Interactions (PPCoEMAS)

The co-evolutionary multi-agent system with predator-prey interactions (*PP-CoEMAS*) is defined as follows (see Eq. (1.1)):

$$PPCoEMAS = \langle E, S, \varGamma, \varOmega \rangle \tag{1.40}$$

The set of species includes two species, preys and predators $S = \{prey, pred\}$. Two information types ($\varOmega = \{\omega_1, \omega_2\}$) and one resource type ($\varGamma = \{\gamma\}$) are used. Information of type ω_1 denote nodes to which agent can migrate.

Information of type ω_2 denote such prey that are located within the particular node in time t.

1.3.2.1 Prey Species

The prey species ($prey$) is defined as follows:

$$prey = \langle A^{prey}, SX^{prey} = \{sx\}, Z^{prey}, C^{prey} \rangle \tag{1.41}$$

where SX^{prey} is the set of sexes which exist within the $prey$ species, Z^{prey} is the set of actions that agents of species $prey$ can perform, and C^{prey} is the set of relations of $prey$ species with other species that exist in the $PPCoEMAS$.

Actions

The set of actions Z^{prey} is defined as follows:

$$\begin{aligned} Z^{prey} = \big\{&die, seek, get, give, accept, seekPartner, \\ &clone, rec, mut, migr\big\} \end{aligned} \tag{1.42}$$

where:

- die is the action of death (prey dies when it is out of resources);
- $seek$ action seeks for another prey agent that is dominated by the prey performing this action or is too close to it in criteria space.
- get action gets some resource from another a^{prey} agent located within the same node, which is dominated by the agent that performs get action or is too close to it in the criteria space;
- $give$ action gives some resource to another agent (which performs get action);
- $accept$ action accepts partner for reproduction when the amount of resource possessed by the prey agent is above the given level;
- $seekPartner$ action is used in order to find the partner for reproduction when the amount of resource is above the given level and agent can reproduce;
- $clone$ is the action of producing offspring (parents give some of their resources to the offspring during this action);
- rec is the recombination operator (intermediate recombination is used [1]);
- mut is the mutation operator (mutation with self-adaptation is used [1]);
- $migr$ is the action of migrating from one node to another. During this action agent loses some of its resource.

Relations

The set of relations of *prey* species with other species that exist within the system is defined as follows:

$$C^{prey} = \left\{ \xrightarrow{prey,get-}, \xrightarrow{prey,give+} \right\} \tag{1.43}$$

The first relation models intra species competition for limited resources:

$$\xrightarrow{prey,get-} = \{\langle prey, prey \rangle\} \tag{1.44}$$

The second one models predator-prey interactions:

$$\xrightarrow{prey,give+} = \{\langle prey, pred \rangle\} \tag{1.45}$$

1.3.2.2 Predator Species

The predator species (*pred*) is defined as follows:

$$pred = \left\langle A^{pred}, SX^{pred} = \{sx\}, Z^{pred}, C^{pred} \right\rangle \tag{1.46}$$

Actions

The set of actions Z^{pred} is defined as follows:

$$Z^{pred} = \{seek, getFromPrey, migr\} \tag{1.47}$$

where:

- The *seek* action allows finding the "worst" (according to the criteria associated with the given predator) prey located within the same node as the predator;
- $getFromPrey$ action gets all resources from the chosen prey,
- $migr$ action allows predator to migrate between nodes of the graph H— this results in losing some of the resources.

Relations

The set of relations of *pred* species with other species that exist within the system are defined as follows:

$$C^{pred} = \left\{ \xrightarrow{pred,getFromPrey-} \right\} \tag{1.48}$$

This relation models predator-prey interactions:

$$\xrightarrow{pred,getFromPrey-}= \{\langle pred, prey\rangle\} \tag{1.49}$$

As a result of performing $getFromPrey$ action and taking all resources from selected prey, it dies.

1.3.2.3 Prey Agent

Algorithm 3. Basic activities of agent $a \equiv a^{prey}$ in $PPCoEMAS$

$r^{\gamma} \leftarrow r^{\gamma}_{init}$;
while $r^{\gamma} > 0$ **do**
  activate the profile $pr_i \in PR^a$ with the highest priority and with the active goal $gl^*_j \in GL^{pr_i}$;
  if pr_1 *is activated* **then**
    if $0 < r^{\gamma} < r^{\gamma}_{min}$ **then**
      $\langle seek, get\rangle$;
      $r^{\gamma} \leftarrow \left(r^{\gamma} + r^{\gamma}_{get}\right)$;
    else if $r^{\gamma} = 0$ **then**
      $\langle die\rangle$;
    end
  else if pr_2 *is activated* **then**
    if $r^{\gamma} > r^{rep,\gamma}_{min}$ **then**
      if $\langle seekPartner, clone, rec, mut\rangle$ *is performed* **then**
        $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{clone,\gamma}_{give}\right)$;
      else if $\langle accept\rangle$ *is performed* **then**
        $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{accept,\gamma}_{give}\right)$;
      end
    end
  else if pr_3 *is activated* **then**
    if $\langle get\rangle$ *is performed by prey agent* **then**
      $\langle give\rangle$;
      $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{\gamma}_{give}\right)$;
    else if $\langle get\rangle$ *is performed by predator agent* **then**
      $\langle give\rangle$;
      $r^{\gamma} \leftarrow 0$;
    end
  else if pr_4 *is activated* **then**
    if $r^{\gamma} > r^{mig,\gamma}_{min}$ **then**
      $\langle migr\rangle$;
      $r^{\gamma} \leftarrow \left(r^{\gamma} - r^{mig,\gamma}_{min}\right)$;
    end
  end
end

Agent a of species $prey$ ($a \equiv a^{prey}$) is defined as follows:

$$a = \langle gn^a, Z^a = Z^{prey}, \Gamma^a = \Gamma, \Omega^a = \Omega, PR^a \rangle \tag{1.50}$$

Genotype of agent a is consisted of two vectors (chromosomes): $\mathbf{x}$ of real-coded decision parameters' values and $\boldsymbol{\sigma}$ of standard deviations' values, which are used during mutation with self-adaptation. $Z^a = Z^{prey}$ (see Eq. (1.42)) is the set of actions which agent a can perform. Γ^a is the set of resource types used by the agent, and Ω^a is the set of information types. Basic activities of agent a are presented in Alg. 3.

Profiles

The partially ordered set of profiles includes resource profile (pr_1), reproduction profile (pr_2), interaction profile (pr_3), and migration profile (pr_4):

$$PR^a = \{pr_1, pr_2, pr_3, pr_4\} \tag{1.51a}$$

$$pr_1 \trianglelefteq pr_2 \trianglelefteq pr_3 \trianglelefteq pr_4 \tag{1.51b}$$

The resource profile is defined in the following way:

$$\begin{aligned} pr_1 = \big\langle & \Gamma^{pr_1} = \Gamma, \Omega^{pr_1} = \{\omega_2\}, M^{pr_1} = \{i^{\omega_2}\}, \\ & ST^{pr_1}, RST^{pr_1} = ST^{pr_1}, GL^{pr_1} \big\rangle \end{aligned} \tag{1.52}$$

The set of strategies include two strategies:

$$ST^{pr_1} = \{\langle die \rangle, \langle seek, get \rangle\} \tag{1.53}$$

The goal of the pr_1 profile is to keep the amount of resources above the minimal level or to die when the amount of resources falls to zero. This profile uses the model $M^{pr_1} = \{i^{\omega_2}\}$.

The reproduction profile is defined as follows:

$$\begin{aligned} pr_2 = \big\langle & \Gamma^{pr_2} = \Gamma, \Omega^{pr_2} = \{\omega_2\}, M^{pr_2} = \{i^{\omega_2}\}, \\ & ST^{pr_2}, RST^{pr_2} = ST^{pr_2}, GL^{pr_2} \big\rangle \end{aligned} \tag{1.54}$$

The set of strategies include two strategies:

$$ST^{pr_2} = \{\langle seekPartner, clone, rec, mut \rangle, \langle accept \rangle\} \tag{1.55}$$

The only goal of the pr_2 profile is to reproduce. In order to realize this goal agent can use strategy of reproduction $\langle seekPartner, clone, rec, mut \rangle$ or can accept partners for reproduction ($\langle accept \rangle$).

The interaction profile is defined as follows:

$$pr_3 = \left\langle \Gamma^{pr_3} = \Gamma, \Omega^{pr_3} = \emptyset, M^{pr_3} = \emptyset, ST^{pr_3} = \{\langle give \rangle\}, \right. \\ \left. RST^{pr_3} = ST^{pr_3}, GL^{pr_3} \right\rangle \tag{1.56}$$

The goal of the pr_3 profile is to interact with predators and preys with the use of strategy $\langle give \rangle$.

The migration profile is defined as follows:

$$pr_4 = \left\langle \Gamma^{pr_4} = \Gamma, \Omega^{pr_4} = \{\omega_1\}, M^{pr_4} = \{i^{\omega_1}\}, \right. \\ \left. ST^{pr_4} = \left\{\langle migr \rangle\right\}, RST^{pr_4} = ST^{pr_4}, GL^{pr_4} \right\rangle \tag{1.57}$$

The goal of the pr_4 profile is to migrate within the environment. In order to realize such a goal the migration strategy is used, which firstly chooses the node and then realizes the migration. As a result of migrating prey loses some amount of resource.

1.3.2.4 Predator Agent

Algorithm 4. Basic activities of agent $a \equiv a^{pred}$ in $PPCoEMAS$

$r^\gamma \leftarrow r^\gamma_{init}$;
while $r^\gamma > 0$ **do**
    activate the profile $pr_i \in PR^a$ with the highest priority and with the active goal $gl^*_j \in GL^{pr_i}$;
    if pr_1 *is activated* **then**
        if $0 < r^\gamma < r^\gamma_{min}$ **then**
            $\langle seek, getFromPrey \rangle$;
            $r^\gamma \leftarrow \left(r^\gamma + r^{prey,\gamma}_{get}\right)$; `/*` $r^{prey,\gamma}_{get}$ `are all resources of the prey agent that was chosen by` a `*/`
        end
    else if pr_2 *is activated* **then**
        if $r^\gamma > r^{mig,\gamma}_{min}$ **then**
            $\langle migr \rangle$;
            $r^\gamma \leftarrow \left(r^\gamma - r^{mig,\gamma}_{min}\right)$;
        end
    end
end

Agent a of species *pred* is defined analogically to *prey* agent (see eq. (1.50)). There exist two main differences. Genotype of predator agent is consisted only of the information about the criterion associated with the given agent. The set of profiles is consisted only of two profiles, resource profile (pr_1), and migration profile (pr_2): $PR^a = \{pr_1, pr_2\}$, where $pr_1 \trianglelefteq pr_2$. Basic activities of agent a are presented in Alg. 4.

Profiles

The resource profile is defined in the following way:

$$
\begin{aligned}
pr_1 = \Big\langle & \varGamma^{pr_1} = \varGamma, \varOmega^{pr_1} = \{\omega_2\}, M^{pr_1} = \{i^{\omega_2}\}, \\
& ST^{pr_1} = \{\langle seek, getFromPrey\rangle\}, RST^{pr_1} = ST^{pr_1}, GL^{pr_1} \Big\rangle
\end{aligned}
\tag{1.58}
$$

The goal of the pr_1 profile is to keep the amount of resource above the minimal level with the use of strategy $\langle seek, getFromPrey\rangle$.

The migration profile is defined as follows:

$$
\begin{aligned}
pr_2 = \Big\langle & \varGamma^{pr_2} = \varGamma, \varOmega^{pr_2} = \{\omega_1\}, M^{pr_2} = \{i^{\omega_1}\}, \\
& ST^{pr_2} = \Big\{\Big\langle migr\Big\rangle\Big\}, RST^{pr_2} = ST^{pr_2}, GL^{pr_2} \Big\rangle
\end{aligned}
\tag{1.59}
$$

The goal of pr_2 profile is to migrate within the environment. In order to realize this goal the migration strategy $\Big\langle migr\Big\rangle$ is used. The realization of the migration strategy results in losing some of the resource possessed by the agent.

1.4 Experimental Results

Presented formally in section 1.3 agent-based co-evolutionary approaches for multi-objective optimization have been tentatively assessed. Obtained during experiments preliminary results were presented in some of our previous papers and in this section they are shortly summarized.

1.4.1 Test suite, performance metric and state-of-the-art algorithms

As a test problem firstly, slightly modified so-called *Laumanns* multi-objective problem was used, which is defined as follows [15, 18]:

$$
Laumanns = \begin{cases} f_1(x) = x_1^2 + x_2^2 \\ f_2(x) = (x_1 + 2)^2 + x_2^2 \\ -5 \le x_1, x_2 \le 5 \end{cases}
\tag{1.60}
$$

Secondly the so-called *Kursawe* problem was used. Its definition is as follows [18]:

$$Kursawe = \begin{cases} f_1(x) = \sum_{i=0}^{n-1}\left(-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right) \\ f_2(x) = \sum_{i=1}^{n}|x_i|^{0.8} + 5\sin x_i^3 \\ n = 3 \quad -5 \leq x_1, x_2, x_3 \leq 5 \end{cases} \tag{1.61}$$

In one of our experiments discussed shortly in this chapter building effective portfolio problem was used. Assumed definition as well as true Pareto frontier for such a problem can be found in [16].

Obviously during our experiments also well known and commonly used test suites were used. Inter alia such problems as ZDT test suite was used ([19, p. 57–63], [21], [5, p. 356–362], [4, p. 194–199]).

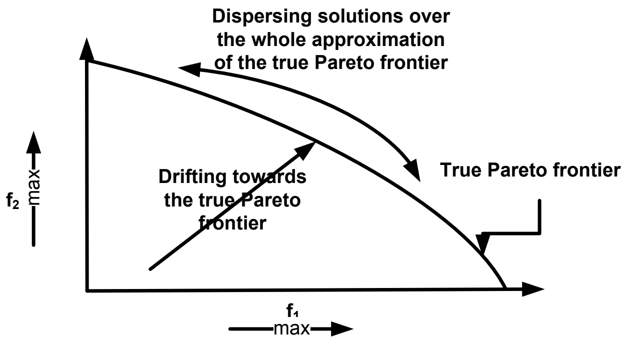


Fig. 1.3: Two goals of multi-objective optimization

Two main distinguishing features of high-quality solution of MOOPs are: closeness to the true Pareto frontier as well as dispersion of found non-dominated solution over the whole (approximation) of the Pareto frontier (see Figure 1.3).

In the consequence, despite that using only one single measure during assessing the effectiveness of (evolutionary) algorithms for multi-objective optimization is not enough [23], since Hypervolume Ratio measure (HVR) [20] allows for estimating both of these aspects—in this chapter discussion and presentation of obtained results is based on this very measure.

Hypervolume or Hypervolume ratio (HVR), describes the area covered by solutions of obtained result set. For each solution, hypercube is evaluated with respect to the fixed reference point. In order to evaluate hypervolume ratio, value of hypervolume for obtained set is normalized with hypervolume value computed for true Pareto frontier. HV and HVR are defined as follows:

$$\mathrm{HV} = v\left(\bigcup_{i=1}^{N} v_i\right) \tag{1.62a}$$

$$\mathrm{HVR} = \frac{\mathrm{HV}(PF^*)}{\mathrm{HV}(PF)} \tag{1.62b}$$

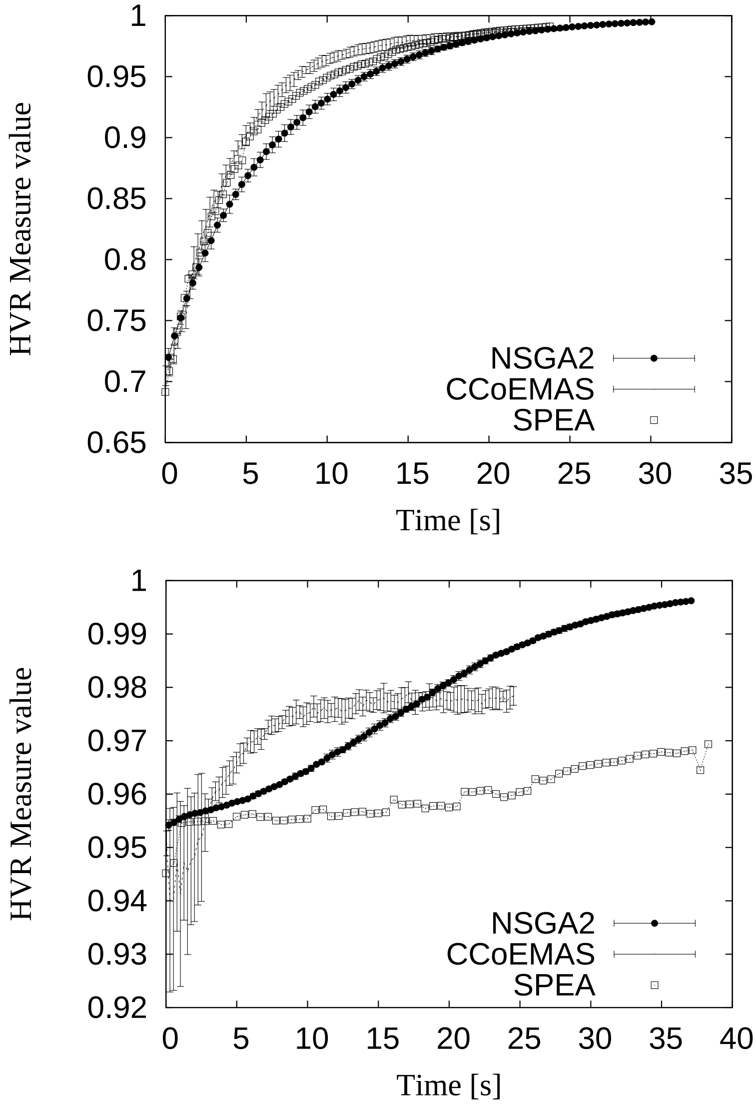
where v_i is hypercube computed for $i - th$ solution, PF^* represents obtained Pareto frontier and PF is the true Pareto frontier.

To assess (in a quantitative way) PPCoEMAS and CCoEMAS the comparison with results obtained with the use of state-of-the-art algorithms has to be made. That is why we are comparing results obtained by discussed in this chapter approaches with results obtained by NSGA-II [6, 7] and SPEA2 [12, 22] algorithms since these very algorithms are the most efficient and most commonly used evolutionary multi-objective optimization algorithms. Additionally, obtained results are compared also with NPGA [13] and PPES [15] algorithms.

1.4.2 A glance at assessing co-operation based approach (CCoEMAS)

Presented in section 1.3.1 co-evolutionary multi-agent system with co-operation mechanism (CCoEMAS) was assessed tentatively using inter alia ZDT test-suite. The size of population of CCoEMAS and the size of benchmarking algorithms (NSGA-II and SPEA2) assumed during presented experiments were as follows: CCoEMAS—200, NSGA-II—300 and SPEA—100. Next, selected parameters and their values assumed during those experiments are as follows: $r^{\gamma}_{init} = 50$ (it represents the level of resources possessed initially by individual just after its creation), $r^{\gamma}_{get} = 30$ (it represents the amount of resources transferred in the case of domination), $r^{rep,\gamma}_{min} = 30$ (it represents the level of resources required for reproduction), $p_{mut} = 0.5$ (mutation probability).

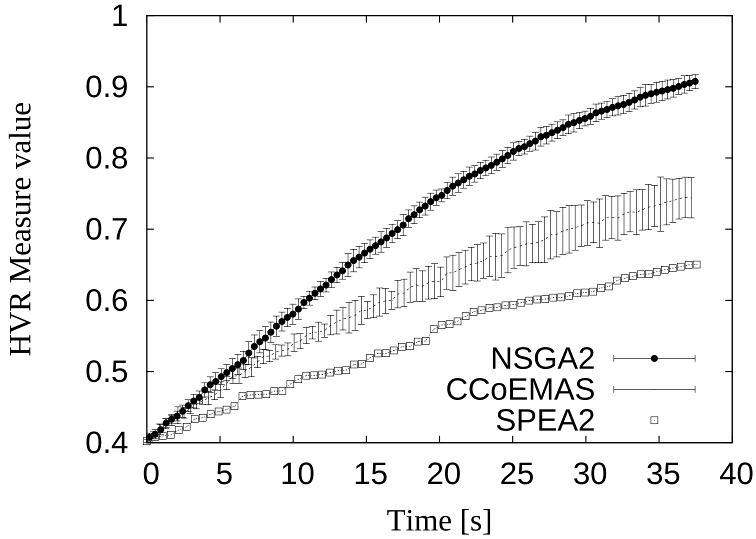
As one may see after the analysis of results presented in figures 1.4 and 1.5—CCoEMAS, as not so complex algorithm as NSGA-II or SPEA2, initially allows for obtaining better solutions, but with time classical algorithms—especially NSGA-II—are the better alternatives. It is however worth to mention that in the case of ZDT4 problem this characteristic seems to be reversed—i.e. initially classical algorithms seem to be better alternatives, but finally CCoEMAS allows for obtaining better solutions (observed as higher values of HVR metrics). Deeper analysis of obtained during presented experiments results can be found in [11].



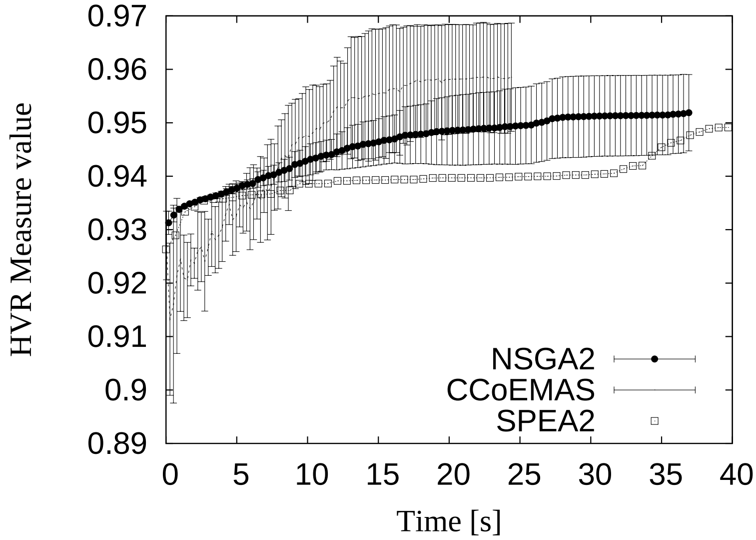
(a)

(b)

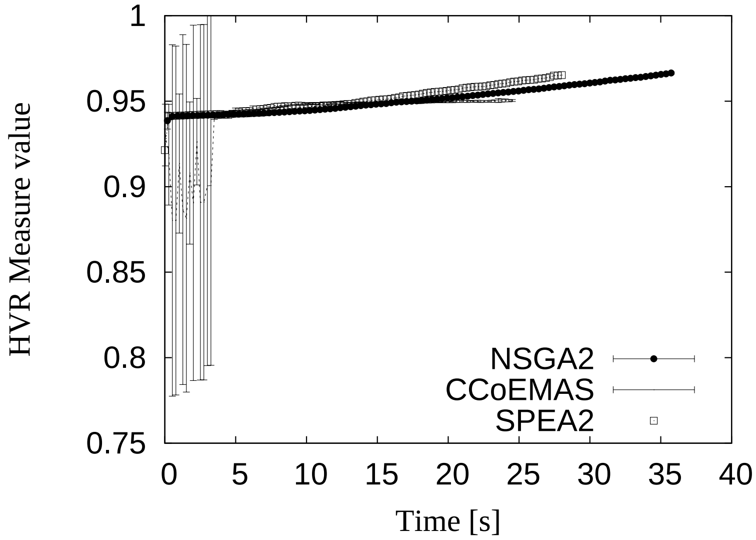
Fig. 1.4: HVR values obtained by CCoEMAS, SPEA2, and NSGA-II run against Zitzler's problems ZDT1 (a) and ZDT2 (b) [11]



(a)



(b)

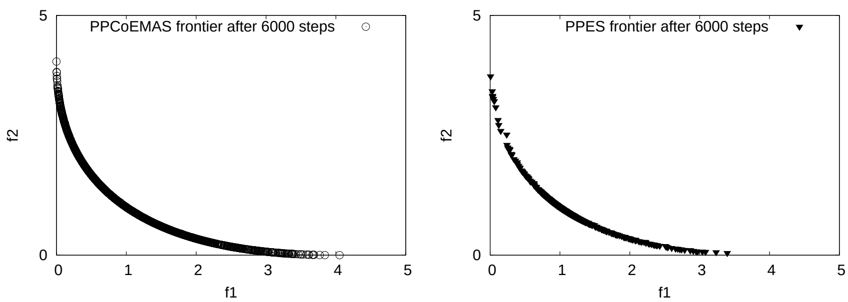


(c)

Fig. 1.5: HVR values obtained by CCoEMAS, SPEA2, and NSGA-II run against Zitzler's problems ZDT3 (a) ZDT4 (b) and ZDT6 (c) [11]

1.4.3 A glance at assessing predator-prey based approach (PPCoEMAS)

In this section some selected results regarding presented in section 1.3.2 co-evolutionary multi-agent system with predator-prey interactions are presented. Among the others, PPCoEMAS was assessed with the use of some presented in section 1.4.1 classical benchmarking problems: firstly Laumanns [15] and Kursawe [14] test problems were used. Also the other than NSGA-II and SPEA2 classical algorithms were used during experiments with predator-prey approach. This time predator-prey evolutionary strategy (PPES) and niched-pareto genetic algorithm (NPGA) were used. In this section only a kind of summary of obtained results is given. More detailed analysis can be found in [9, 16].

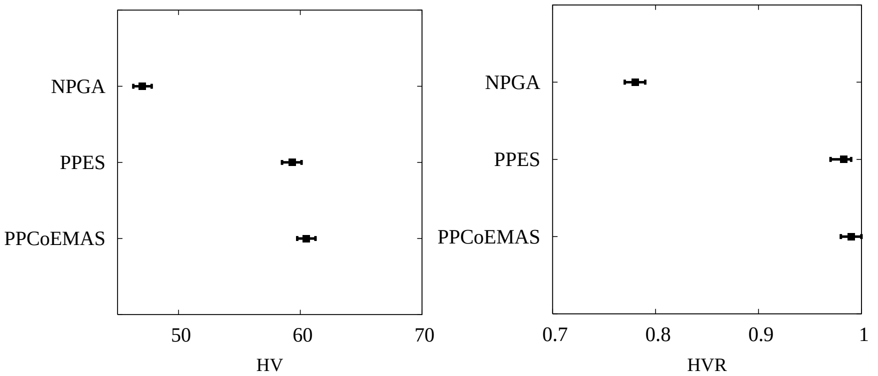


(a) (b)

Fig. 1.6: Pareto frontier approximations obtained by PPCoEMAS (a) and PPES (b) algorithms for Laumanns problem after 6000 steps [9]

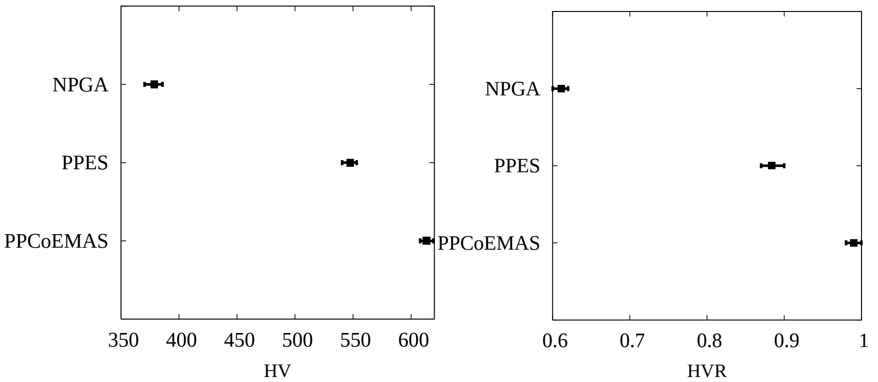
In the very first experiments with PPCoEMAS relatively simple Laumanns test problem was used. In Figure 1.6 there are presented Pareto frontier approximations obtained by PPCoEMAS and PPES algorithms and in Figure 1.7 there are presented values of HV and HVR metrics for all three algorithms being compared (PPCoEMAS, PPES and NPGA). As it can be seen—the differences between algorithms being analyzed are not so distinct, however proposed PPCoEMAS system seems to be the best alternative.

The second problem used was more demanding multi-objective Kursawe problem with disconnected both Pareto set and Pareto frontier. In Figure 1.9 there are presented final approximations of Pareto frontier obtained by PPCoEMAS and by reference algorithms after 6000 time steps. As one may notice, there is no doubt that PPCoEMAS is definitely the best alternative since it is able to obtain Pareto frontier that is located very close to the model



(a) (b)

Fig. 1.7: The value of HV (a) and HVR (b) measure for Laumanns problem obtained by PPCoEMAS, PPES and NPGA after 6000 steps

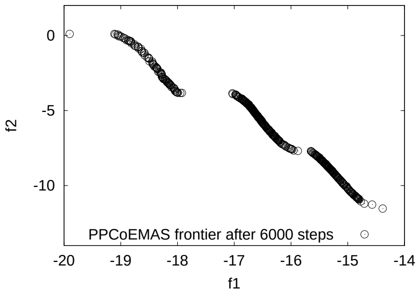


(a) (b)

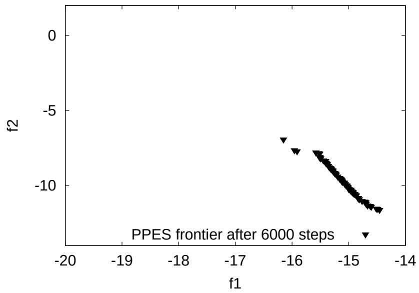
Fig. 1.8: The value of HV (a) and HVR (b) measure for Kursawe problem obtained by PPCoEMAS, PPES and NPGA after 6000 steps

solution, that is very well dispersed and what is also very important—it is more numerous than PPES and NPGA-based solutions. The above observations are fully confirmed by the values of HV and HVR metrics presented in Figure 1.8.

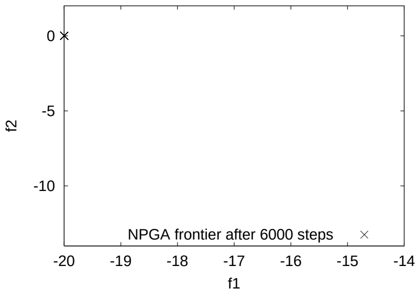
Proposed co-evolutionary multi-agent system with predator-prey interactions was also assessed with the use of building effective portfolio problem. In this case, each individual in the prey population is represented as a p-



(a)



(b)



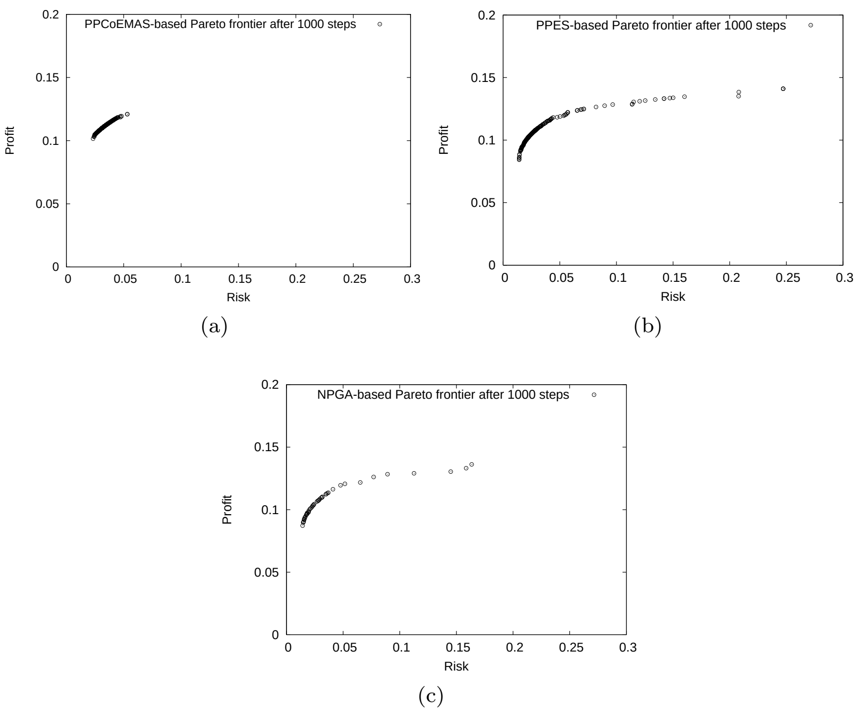
(c)

Fig. 1.9: Pareto frontier approximations for Kursawe problem obtained by PPCoEMAS (a), PPES (b) and NPGA (c) after 6000 steps [9]

dimensional vector. Each dimension represents the percentage participation of i-th ($i \in 1 \ldots p$) share in the whole portfolio.

During presented experiments—Warsaw Stock Exchange quotations from 2003-01-01 until 2005-12-31 were taken into consideration. Simultaneously, the portfolio consists of the following three (experiment I) or seventeen (experiment II) stocks quoted on the Warsaw Stock Exchange: in experiment I: RAFAKO, PONARFEH, PKOBP, in experiment II: KREDYTB, COMPLAND, BETACOM, GRAJEWO, KRUK, COMARCH, ATM, HANDLOWY, BZWBK, HYDROBUD, BORYSZEW, ARKSTEEL, BRE, KGHM, GANT, PROKOM, BPHPBK. As the market index, WIG20 has been taken into consideration.

In Figure 1.10 there are presented final Pareto frontiers obtained using PPCoEMAS, NPGA and PPES algorithm after 1000 steps in experiment I. As one may notice, in this case frontier obtained by PPCoEMAS is more numerous than NPGA-based and as numerous as PPES-based one. Unfortu-



(a) (b) (c)

Fig. 1.10: Pareto frontier approximations after 1000 steps obtained by PPCoEMAS (a), PPES (b), and NPGA (c) for building effective portfolio consisting of 3 stocks [16]

nately, in this case, diversity of population in PPCoEMAS approach is visibly worse than in the case of NPGA or PPES-based frontiers.

Similar situation can be also observed in Figure 1.11 presenting Pareto frontiers obtained by PPCoEMAS, NPGA and PPES—but this time portfolio that is being optimized consists of 17 shares. Also this time PPCoEMAS-based frontier is quite numerous and quite close to the true Pareto frontier but the tendency for focusing solutions around only selected part(s) of the whole frontier is very distinct. The explanation of observed tendency can be found in [9, 16] and on the very general level it can be said that it is caused by the stagnation of evolution process in PPCoEMAS. Hypothetical, non-dominated average portfolios for experiment I and II are presented in Figure 1.12 and in Figure 1.13 respectively (in Figure 1.13 shares are presented from left to right in the order in which they were mentioned above).

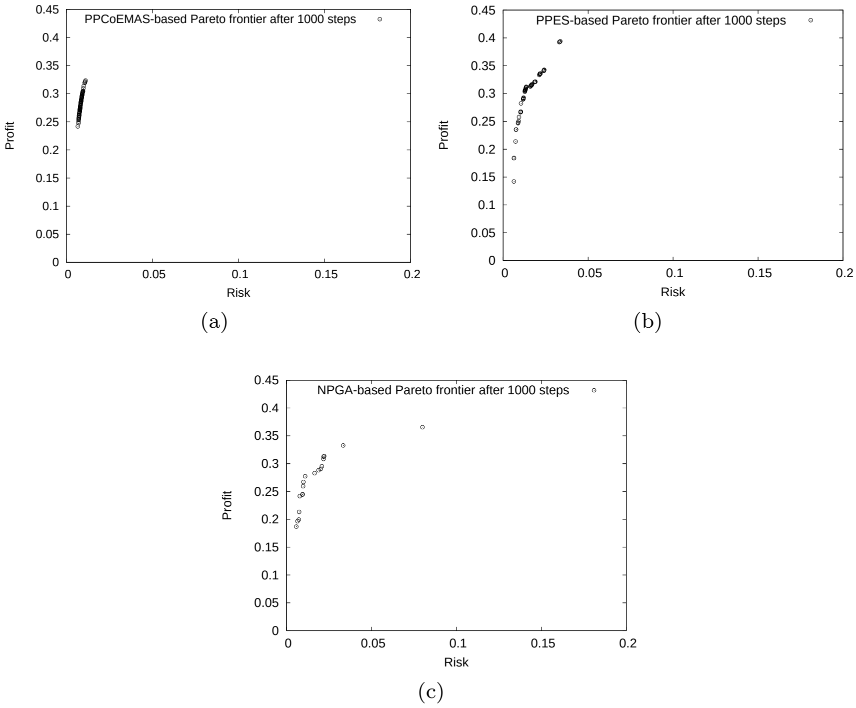


Fig. 1.11: Pareto frontier approximations after 1000 steps obtained by PP-CoEMAS (a), PPES (b), and NPGA (c) for building effective portfolio consisting of 17 stocks [16]

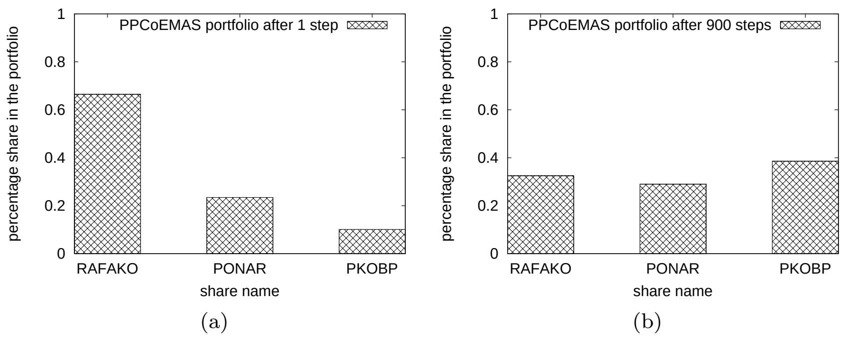
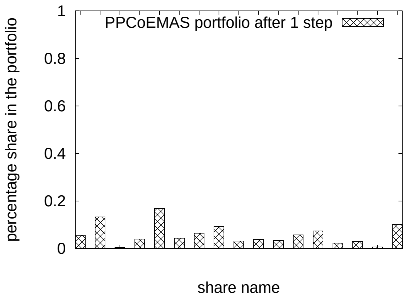
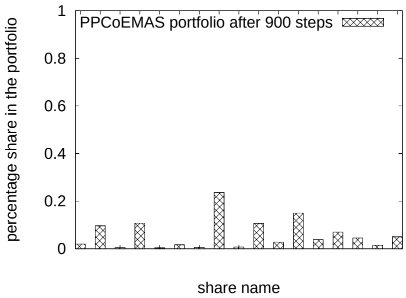


Fig. 1.12: Effective portfolio consisting of three stocks proposed by PP-CoEMAS [16]



(a)



(b)

Fig. 1.13: Effective portfolio consisting of seventeen stocks proposed by PP-CoEMAS [16]

1.5 Summary and Conclusions

Agent-based (co-)evolutionary algorithms have been applied already in many different domains, including multi-modal optimization, multi-objective optimization, and financial problems. Agent-based models of evolutionary algorithms allows for mixing and using simultaneously different bio-inspired techniques and algorithms within one coherent agent model, and adding new biologically and socially inspired operators and mechanisms in a very natural way. Agent-based models of evolutionary algorithm also allow for using parallel and decentralized computations without any additional changes because these models are decentralized and use asynchronous computations.

In this chapter we have presented two selected agent-based co-evolutionary algorithms for multi-objective optimization—one of them used co-operative mechanisms and the other one used predator-prey mechanism. Formal models of these systems as well as results of experiments with standard multi-objective test problems and financial problem of multi-objective portfolio optimization were presented. The results of experiments show that agent-based algorithms may obtain quite satisfactory results, comparable or in the case of some problems even better than state-of-the-art multi-objective evolutionary algorithms, however of course there is still place for improvement and further research. Presented results also lead to conclusion that none of the existing evolutionary algorithms for multi-objective optimization can not alone solve all problems in a best way—there is, and always will be, space for new algorithms and improvements suited for some particular problems.

Future research on the agent-based models will concentrate on improvements to the already proposed algorithms as well as on new algorithms and techniques. Examples of new techniques which may be incorporated into agent-based models of evolutionary algorithms include cultural and immunological mechanisms. Another way of development would be adding social and economical layer to the existing biological one and using such agent-based models for modeling and simulation of complex and emergent phenomena from social and economical life.

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