Agent-Based Simulation Model of Sexual Selection Mechanism

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Abstract. Agent-based approach is especially applicable and useful in modeling and simulation of social and biological systems and mechanisms. In this paper a formal agent-based model of sexual selection mechanism is presented. The paper includes results of simulation experiments aimed at showing whether sexual selection can trigger speciation processes and maintain genetic population diversity. We show that in certain conditions sexual selection mechanism and co-evolution of sexes resulting from it, can lead to formation of new species.

Keywords: agent-based modeling and simulation, speciation, sexual selection

1 Introduction

In recent years agent-based approach to modeling and simulation becomes more and more popular, especially when social and biological simulations are taken into consideration. Agent-based modeling and simulation (ABMS) approach has been applied in many different areas, such as research on complex biological, social, and economical systems [11, 12, 15, 16] crowd behavior and traffic simulation [19].

ABMS allows for elegant and explicit representation of environment, objects, resources, individuals (agents) and relations between them [15]. It is relatively easy to introduce learning mechanisms, to model dynamic formation of organizations and teams, also with learning capabilities at the organization/team level. Spatial relations are also easy to model because agents and objects are located within the environment with some topography. Usually scientists can develop agent-based model, implement it with the use of some existing tools (review of some of such tools can be found in [15, 19]) and experiment with the simulation model in order to observe some emergent phenomena that was not explicitly represented within it.

In this paper we will mainly focus on processes of species formation resulting from sexual selection and on agent-based modeling and simulation of such phenomena. Biological models of speciation include [14]:

- *allopatric models* that require geographical separation of sub-populations,

- *parapatric models* that require that zones of organisms only partially overlap and
- *sympatric models* where speciation takes place within one population without physical barriers as a result of, for example, co-evolutionary interactions or sexual selection.

Sexual selection results from the fact that cost of the reproduction for one of the sexes (usually females) is much higher than for the second one and the proportions of both sexes within the population are almost equal [14, 17]. These facts cause that females choose males on the basis of some features. These male's features and female's preferences are inherited by children and reinforced. Such phenomena leads to co-evolution of sexes where one of the sexes evolves in direction of keeping the reproduction rate on optimal level and the second one evolves in a direction that leads to increasing of the reproduction rate.

Model of multi-agent system with biological and social mechanisms (BSMAS) that we use in this paper to describe formally the simulation model of sexual selection mechanism was presented in [6]. BSMAS is reformulated and enhanced version of the model of co-evolutonary multi-agent system (CoEMAS) proposed in [4]. CoEMAS model was based on the idea of combining together multi-agent systems and evolutionary algorithms presented in [3]. CoEMAS approach introduced the possibility of existing of multiple species and sexes within a population and possibility of defining relations between them. BSMAS approach additionally introduced possibilities of defining social relations and structures. CoEMAS approach was used in many different areas, such as multi-modal optimization [5], multi-objective optimization [10, 8] and generating investment strategies [9, 7].

In this paper we apply BSMAS approach to modeling and simulation of sexual selection. We then investigate whether sexual selection can cause speciation and whether it promotes genetic diversity.

2 Agent-Based Model of Sexual Selection Mechanism

The notions *agent* and *multi-agent system* are used in literature, in rather not very formal and strict manner, for naming different kinds of systems and approaches. In this paper we follow the definitions proposed by J. Ferber [13]. In his approach *agent* is considered to be a physical or virtual entity capable of acting within an environment and capable of communicating with other agents. Its activities are driven by individual goals, it possesses some resources, it may observe the environment (but only local part of it), it possesses only partial knowledge about the environment. It has some abilities and may offer some services, it may also be able to reproduce.

Multi-agent system is a system composed of environment, objects (passive elements of the system) and active elements of the system (agents). There can be different kinds of relations between elements of the system. Multi-agent system also includes set of operations that allow agents to observe and interact with other elements of the system, and set of operators, which aim is to represent agent's actions and reactions of the other elements of the system [13].

2.1 Agent-Based Model of Sexual Selection Mechanism

In the model presented in this section sympatric speciation takes place as a result of sexual selection. There are two sexes within the population: female and male (Fig. 1). Each agent needs resources for living so it tries to get them from the environment. During movement agents lose resources and when an agent runs out of resources it dies.

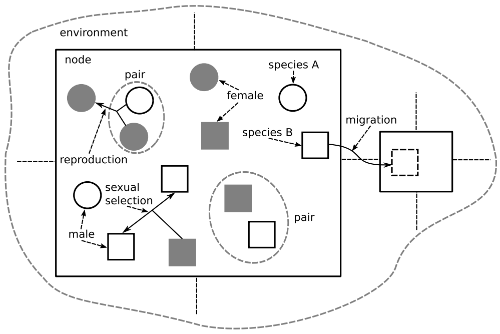


Fig. 1: Multi-Agent System with Sexual Selection

When the level of the resource is above some minimal value, an agent is ready for reproduction and searches for a partner. Female agent chooses a partner from the set of male agents that are ready for reproduction in the given period of time and that are located within the same node of the environment—the decision is made on the basis of genotype similarity (the more the agents are similar the more probable is the decision of choosing the given male agent). Then a pair is formed and agents move together within the environment for some time (Fig. 1)—such behavior helps during reproduction because agents do not have to search for a partner every time when they are ready for reproduction. The offspring is generated with the use of mutation and recombination operators (intermediate recombination [2], and mutation with self-adaptation [1]). The offspring receives some of the resources from parents—more from female agent, so female agents' cost of reproduction is much higher than male agents' cost of reproduction.

The multi-agent system with sexual selection is defined as follows:

$$\begin{aligned} sBSMAS(t) = \big\langle & EnvT(t) = \{et\}, Env(t) = \{env\}, ElT(t) = \\ & VertT(t) \cup ObjT(t) \cup AgT, ResT(t) = \{rt\}, \\ & InfT(t) = \emptyset, Rel(t), Attr(t) = \{genotype\}, \\ & Act(t)\big\rangle \end{aligned} \tag{1}$$

where:

- $EnvT(t)$ is the set of environment types in the time t;
- $Env(t)$ is the set of environments of the $sBSMAS$ in the time t;
- $ElT(t)$ is the set of types of elements that can exist within the system in time t;
- $VertT(t) = \{vt\}$ is the set of vertice types that can exist within the system in time t;
- $ObjT(t) = \emptyset$ is the set of object (not an object in the sense of object-oriented programming but object as an element of the simulation model) types that may exist within the system in time t;
- $AgT(t) = \{female, male\}$ is the set of agent types that may exist within the system in time t;
- $ResT(t)$ is the set of resource types that exist in the system in time t, the amount of resource of type $rest(t) \in ResT(t)$ will be denoted by $res^{rest}(t)$;
- $InfT(t)$ is the set of information types that exist in the system, the information of type $inft(t) \in InfT(t)$ will be denoted by $inf^{inft}(t)$;
- $Rel(t)$ is the set of relations between sets of agents, objects, and vertices;
- $Attr(t)$ is the of attributes of agents, objects, and vertices;
- $Act(t)$ is the set of actions that can be performed by agents, objects, and vertices.

The set of actions is defined as follows:

$$Act = \{die, reproduce, get\_resource, give\_resource, migrate, choose\} \tag{2}$$

Environment type et is defined in the following way:

$$et = \langle EnvT^{et} = \emptyset, VertT^{et} = VertT, ResT^{et} = ResT, InfT^{et} = \emptyset \rangle \tag{3}$$

$EnvT^{et} \subseteq EnvT$ is the set of environment types that may be connected with the et environment. $VertT^{et} \subseteq VerT$ is the set of vertice types that may exist within the environment of type et. $ResT^{et} \subseteq ResT$ is the set of resource types that may exist within the environment of type et. $InfT^{et} \subseteq InfT$ is the set of information types that may exist within the environment of type et.

Environment env of type et is defined as follows:

$$env = \langle gr^{env}, Env^{env} = \emptyset \rangle \tag{4}$$

where gr^{env} is directed graph $gr^{env} = \langle Vert, Arch, cost \rangle$, $Vert$ with the $cost$ function defined is the set of vertices, $Arch$ is the set of arches. The distance between two nodes is defined as the length of the shortest path between them in graph gr^{env}. $Env^{env} \subseteq Env$ is the set of environments of types from $EnvT$ connected with the environment env.

Vertice type vt is defined in the following way:

$$\begin{aligned} vt = \langle Attr^{vt} = \emptyset, Act^{vt} = \{give\_resource\}, ResT^{vt} = ResT, \\ InfT^{vt} = \emptyset, VertT^{vt} = VertT, ObjT^{vt} = \emptyset, AgT^{vt} = AgT \rangle \end{aligned} \tag{5}$$

where:

- $Attr^{vt} \subseteq Attr$ is the set of attributes of vt vertice at the beginning of its existence;
- $Act^{vt} \subseteq Act$ is the set of actions, which vt vertice can perform at the beginning of its existence, when asked for it;
- $ResT^{vt} \subseteq ResT$ is the set of resource types, which can exist within vt vertice at the beginning of its existence;
- $InfT^{vt} \subseteq InfT$ is the set of information, which can exist within vt vertice at the beginning of its existence;
- Vt^{vt} is the set of types of vertices that can be connected with the vt vertice at the beginning of its existence;
- $ObjT^{vt} \subseteq ObjT$ is the set of types of objects that can be located within the vt vertice at the beginning of its existence;
- $AgT^{vt} \subseteq AgT$ is the set of types of agents that can be located within the vt vertice at the beginning of its existence;
- $give\_resource$ is the action of giving resource to agents.

Element of the structure of system's environment (vertice) $vert \in Vert$ of type $vt \in VertT^{env}$ is given by:

$$vert = \big\langle Attr^{vert} = \emptyset, Act^{vert} = Act^{vt}, Res^{vert} = \big\{res^{vert}\big\}, Inf^{vert} = \emptyset, \\ Vert^{vert}, Obj^{vert} = \emptyset, Ag^{vert}\big\rangle \tag{6}$$

where:

- $Attr^{vert} \subseteq Attr$ is the set of attributes of vertice $vert$—it can change during its lifetime;
- $Act^{vert} \subseteq Act$ is the set of actions, which vertice $vert$ can perform when asked for it—it can change during its lifetime;
- Res^{vert} is the set of resources of types from $ResT$ that exist within the $vert$;
- Inf^{vert} is the set of information of types from $InfT$ that exist within the $vert$;
- $Vert^{vert}$ is the set of vertices of types from $VertT$ connected with the vertice $vert$;
- Obj^{vert} is the set of objects of types from $ObjT$ that are located in the vertice $vert$;
- Ag^{vert} is the set of agents of types from AgT that are located in the vertice $vert$.

res^{vert} is the amount of resource of type rt that is possessed by the $vert$. $Vert^{vert}$ is the set of four vertices connected with the vertice $vert$ (Fig. 1). Ag^{vert} is the set of agents located within the vertice $vert$.

There are two types of agents in the system: $female$ and $male$. $female$ agent type is defined in the following way:

$$female = \big\langle Gl^{female} = \big\{gl_1, gl_2, gl_3\big\}, Attr^{female} = \big\{genotype\big\}, \\ Act^{female} = \big\{die, reproduce, choose, get\_resource, migrate\big\}, \\ ResT^{female} = ResT, InfT^{female} = \emptyset, ObjT^{female} = \emptyset, \\ AgT^{female} = \emptyset\big\rangle \tag{7}$$

where:

- Gl^{female} is the set of goals of $female$ agent at the beginning of its existence;
- $Attr^{female} \subseteq Attr$ is the set of attributes of $female$ agent at the beginning of its existence;
- $Act^{female} \subseteq Act$ is the set of actions, which $female$ agent can perform at the beginning of its existence;
- $ResT^{female} \subseteq ResT$ is the set of resource types, which can be used by $female$ agent at the beginning of its existence;
- $InfT^{female} \subseteq InfT$ is the set of information, which can be used by $female$ agent at the beginning of its existence;
- $ObjT^{female} \subseteq ObjT$ is the set of types of objects that can be located within the $female$ agent at the beginning of its existence;
- $AgT^{female} \subseteq AgT$ is the set of types of agents that can be located within the $female$ agent at the beginning of its existence.

gl_1 is the goal "get resource from environment", gl_2 is the goal "reproduce", and gl_3 is the goal "migrate to other vertice". *die* is the action of death—agent dies when it runs out of resources, *reproduce* is the action of reproducing (with the use of recombination and mutation operators), *choose* is the action of choosing partner for reproduction from the set of *male* agents that are located within the same vertice and are ready for reproduction, *get_resource* is the action of getting resource from environment, and *migrate* is the action of migrating to other vertice.

male agent type is defined in the following way:

$$\begin{aligned} male = \big\langle & Gl^{male} = \{gl_1, gl_2, gl_3\}, Attr^{male} = \{genotype\}, \\ & Act^{male} = \{die, reproduce, get\_resource, migrate, \}, \\ & ResT^{male} = ResT, InfT^{male} = \emptyset, ObjT^{male} = \emptyset, AgT^{male} = \emptyset \big\rangle \end{aligned} \tag{8}$$

where gl_1 is the goal "get resource from environment", gl_2 is the goal "reproduce", and gl_3 is the goal "migrate to other vertice". *die* is the action of death—agent dies when it runs out of resources, *reproduce* is the action of reproducing (with the use of recombination and mutation operators), *get_resource* is the action of getting resource from environment, and *migrate* is the action of migrating to other vertice.

Agent ag^{female} (of type $female$) is defined in the following way:

$$\begin{aligned} ag^{female} = \big\langle & Gl^{ag,female} = Gl^{female}, Attr^{ag,female} = Attr^{female}, \\ & Act^{ag,female} = Act^{female}, Res^{ag,female} = \{r^{ag,female}\}, \\ & Inf^{ag,female} = \emptyset, Obj^{ag,female} = \emptyset, Ag^{ag,female} = \emptyset \big\rangle \end{aligned} \tag{9}$$

where:

- Gl^{female} is the set of goals, which agent $female$ tries to realize—it can change during its lifetime;

- $Attr^{female} \subseteq Attr$ is the set of attributes of agent $female$—it can change during its lifetime;
- $Act^{female} \subseteq Act$ is the set of actions, which agent $female$ can perform in order to realize its goals—it can change during its lifetime;
- Res^{female} is the set of resources of types from $ResT$, which are used by agent $female$;
- Inf^{female} is the set of information of types from $InfT$, which agent $female$ can possess and use;
- Obj^{female} is the set of objects of types from $ObjT$ that are located within the agent $female$;
- Ag^{female} is the set of agents of types from AgT that are located within the agent $female$.

Notation $Gl^{ag,female}$ means "the set of goals of agent ag of type $female$". $r^{ag,female}$ is the amount of resource of type rt that is possessed by the agent ag^{female}.

Agent ag^{male} (of type $male$) is defined in the following way:

$$\begin{aligned} ag^{male} = \Big\langle & Gl^{ag,male} = Gl^{male}, Attr^{ag,male} = Attr^{male}, \\ & Act^{ag,male} = Act^{male}, Res^{ag,male} = \big\{r^{ag,male}\big\}, \\ & Inf^{ag,male} = \emptyset, Obj^{ag,male} = \emptyset, Ag^{ag,male} = \emptyset \Big\rangle \end{aligned} \tag{10}$$

Notation $Gl^{ag,male}$ means "the set of goals of agent ag of type $male$". $r^{ag,male}$ is the amount of resource of type rt that is possessed by the agent ag^{male}.

The set of relations is defined as follows:

$$Rel = \left\{ \xrightarrow[\{get\_resource\}]{\{get\_resource\}}, \xrightarrow[\{reproduce\}]{\{choose,reproduce\}} \right\} \tag{11}$$

The relation $\xrightarrow[\{get\_resource\}]{\{get\_resource\}}$ is defined as follows:

$$\xrightarrow[\{get\_resource\}]{\{get\_resource\}} = \left\{ \left\langle Ag^{\{get\_resource\}}, Ag^{\{get\_resource\}} \right\rangle \right\} \tag{12}$$

$Ag^{\{get\_resource\}}$ is the set of agents capable of performing action $get\_resource$. This relation represents competition for limited resources between agents.

The relation $\xrightarrow[\{reproduce\}]{\{choose,reproduce\}}$ is defined as follows:

$$\xrightarrow[\{reproduce\}]{\{choose,reproduce\}} = \left\{ \left\langle Ag^{female,\{choose,reproduce\}}, Ag^{male,\{reproduce\}} \right\rangle \right\} \tag{13}$$

$Ag^{female,\{choose,reproduce\}}$ is the set of agents of type $female$ capable of performing actions $choose$ and $reproduce$. $Ag^{male,\{reproduce\}}$ is the set of agents of type $male$ capable of performing action $reproduce$. This relation represents sexual selection mechanism—$female$ agents choose partners for reproduction form $male$ agents and then reproduction takes place.

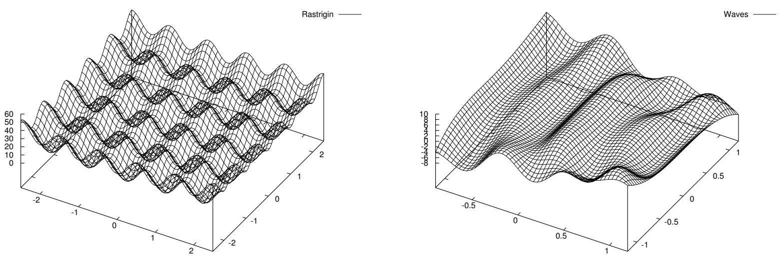
Action *choose* chooses male agent from the set of male agents that are ready for reproduction and are located within the same vertice as the given female agent with probability proportional to genetic similarity of both agents.

3 Results of Simulation Experiments

In this section we will present results of simulation experiments with our agent-based model of sexual selection. The main goal of these experiments was to investigate whether sexual selection mechanism is able to trigger speciation within the population and whether it is able to maintain genetic diversity.

In all experiments we used our own multi-agent simulation system based on BSMAS model and implemented in Java.

During experiments we used two fitness landscapes: Rastrigin and Waves (Fig. 2).



(a) Rastrigin (b) Waves

Fig. 2: Fitness landscapes

Rastrigin multimodal fitness landscape is defined as follows ([18]):

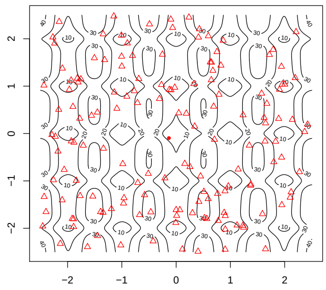
$$
\begin{aligned}
f_2(\boldsymbol{x}) =& 10 * n + \sum_{i=1}^{n} \left(x_i^2 - 10 * \cos(2 * \pi * x_i) \right) \\
& x_i \in [-2.5; 2.5] \text{ for } i = 1, \ldots, n
\end{aligned}
\tag{14}
$$

During experiments $n = 2$ was assumed (Fig. 2a).

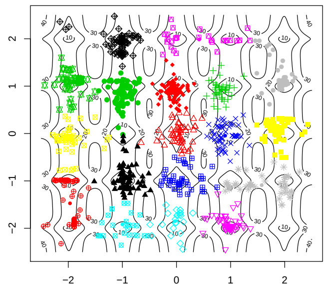
Waves fitness landscape is defined as follows ([20]):

$$
\begin{aligned}
f_4(\boldsymbol{x}) = -\Big(& (0.3 * x_1)^3 - \left(x_2^2 - 4.5 * x_2^2\right) * x_1 * x_2 - \\
& 4.7 * \cos\left(3 * x_1 - x_2^2 * (2 + x_1)\right) * \sin\left(2.5 * \pi * x_1\right)\Big) \\
x_1 \in & [-0.9; 1.2], \; x_2 \in [-1.2; 1.2]
\end{aligned}
\tag{15}
$$

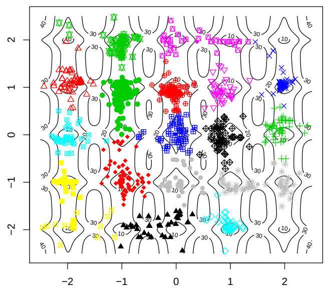
This function has many irregularly placed local minima (Fig. 2b).



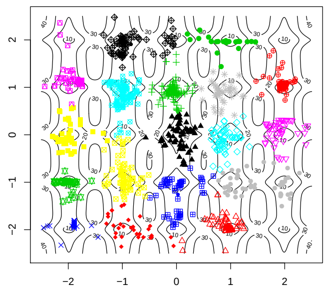
(a) t=0



(b) t=50



(c) t=500



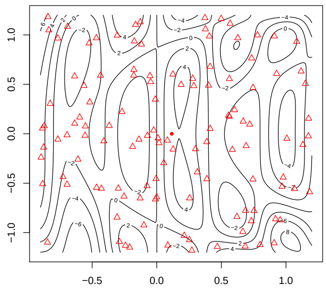
(d) t=5000

Fig. 3: Species formation processes in sBSMAS with Rastrigin fitness landscape

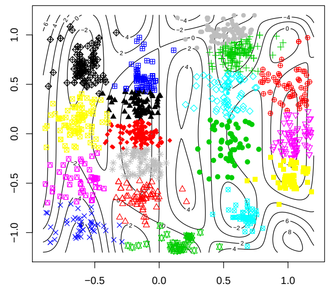
Species formation processes during experiments with different fitness landscapes can be observed in figures 3 and 4.

In all these cases it can be observed that after initial phase, distinct species were formed. In step 0 of each experiment we can see that population is dispersed over the whole genetic space, and later species are formed—distinct species were depicted with the use of different shapes and colors. We used k-means clustering algorithm to find clusters (species) within the population of agents occupying different parts of the genetic space. Sexual selection—selecting males on the basis of genetic similarity—causes that agents tend to form species (isolated clusters within the genetic space).

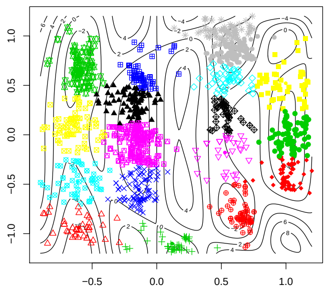
In figures 5a and 5b, results showing population diversity measure are presented. As the measure of genetic population diversity average distance of agents from the centroid of the population was used. It can be observed that sexual se-



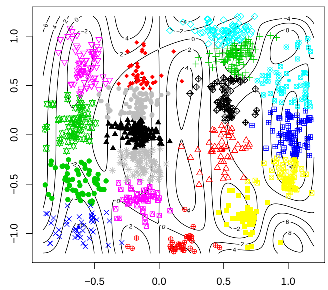
(a) t=0



(b) t=50



(c) t=500



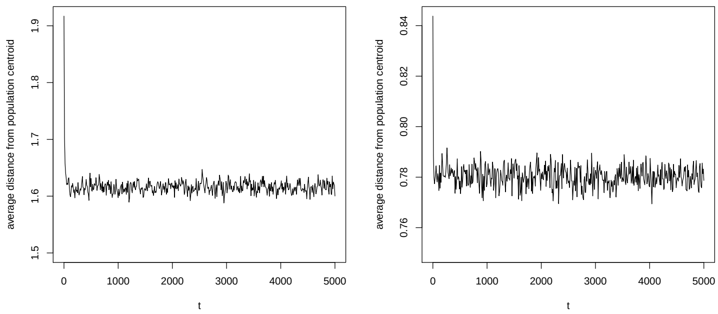
(d) t=5000

Fig. 4: Species formation processes in sBSMAS with Waves fitness landscape

lection maintains genetic population diversity. Diversity of course drops after some time, but it is caused by the fact that distinct species are formed and agents are grouping within some regions of the genetic space. But generally in all the cases genetic diversity is maintained and do not diminish.

4 Conclusions

In this paper we presented agent-based model of species formation processes caused by sexual selection. Presented results show that in particular conditions sexual selection can lead to speciation. Sexual selection also helps maintaining genetic population diversity. It has been demonstrated that the agent-based approach to modeling and simulation in general, and proposed BSMAS model



(a) Rastrigin (b) Waves

Fig. 5: Average distance from population centroid

in particular, are well suited for constructing models of biological and social systems and mechanisms.

In the future we plan to conduct experiments with more realistic environments, without explicit fitness function. Agents would be placed in such environment and would have to deal with changing conditions and different species co-existing within the same environment.

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