



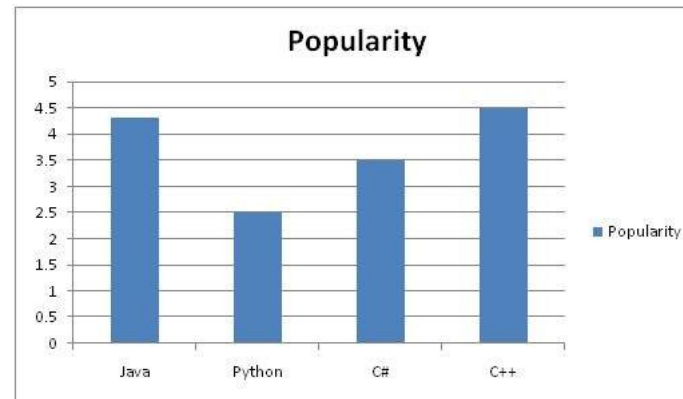
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AND TECHNOLOGY**

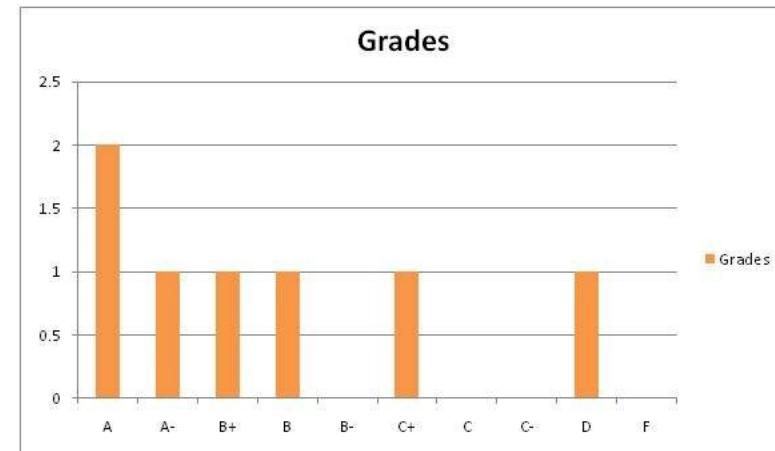
Histogram Stretching Filter

Histogram

A graph that shows frequency of anything



Name	Grade
John	A
Jack	D
Carter	B
Tommy	A
Lisa	C+
Derek	A-
Tom	B+



Histogram of an image

The x axis of the histogram shows the range of pixel values. Since its an 8 bpp image, that means it has 256 levels of gray or shades of gray in it. Thats why the range of x axis starts from 0 and end at 255 with a gap of 50. Whereas on the y axis, is the count of these intensities.

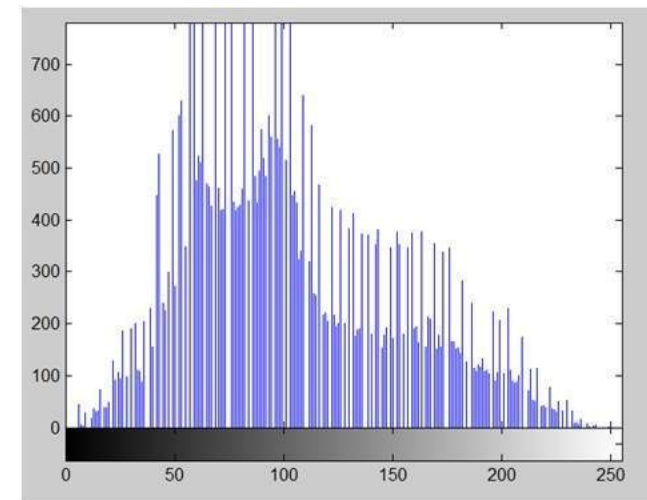
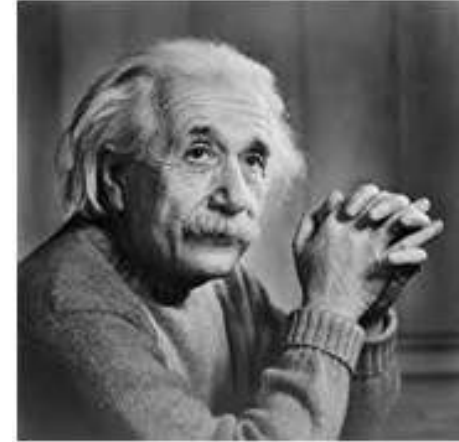


Image transformation

$$G(x,y) = T\{ f(x,y) \}$$

$F(x,y)$ = input image on which transformation function has to be applied.

$G(x,y)$ = the output image or processed image.

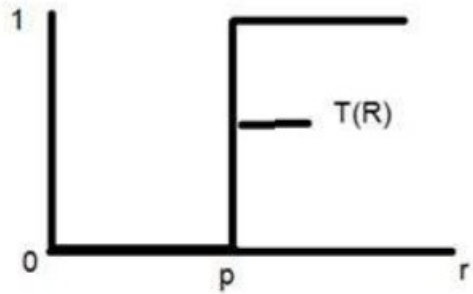
T is the transformation function.

This relation between input image and the processed output image can also be represented as.

$$s = T (r)$$

where r is actually the pixel value or gray level intensity of $f(x,y)$ at any point. And s is the pixel value or gray level intensity of $g(x,y)$ at any point.

Image transformation – example - thresholding



$$g(x,y) = \begin{cases} 0 & f(x,y) < p \\ 1 & f(x,y) > p \end{cases}$$

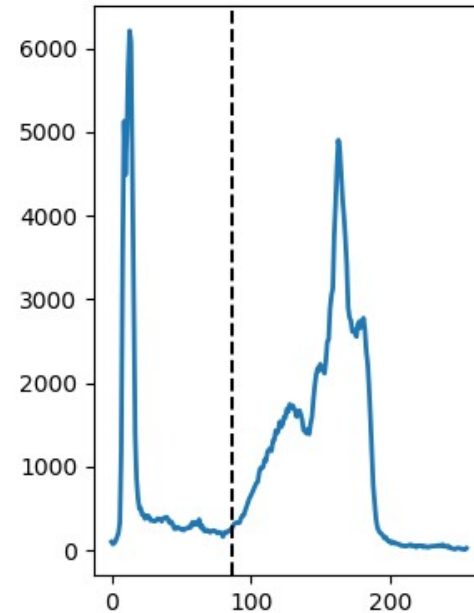


Image transformation – example – thresholding Otsu's method

In computer vision and image processing, Otsu's method, named after Nobuyuki Otsu (大津展之 , Ōtsu Nobuyuki), is used to perform automatic image thresholding. In the simplest form, the algorithm returns a single intensity threshold that separate pixels into two classes, foreground and background. This threshold is determined by minimizing intra-class intensity variance, or equivalently, by maximizing inter-class variance.

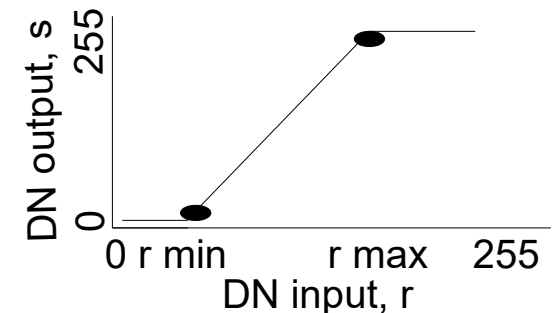


Image transformation – example - stretching

Contrast stretching is only possible if minimum intensity value and maximum intensity value are not equal to the possible minimum and maximum intensity values. Otherwise, the image generated after contrast stretching will be the same as input image.

General Formula for Contrast Stretching:

$$s = (r - r_{\min}) \frac{(I_{\max} - I_{\min})}{(r_{\max} - r_{\min})} + I_{\min}$$



For $I_{\min} = 0$ and $I_{\max} = 255$ (for standard 8-bit grayscale image)

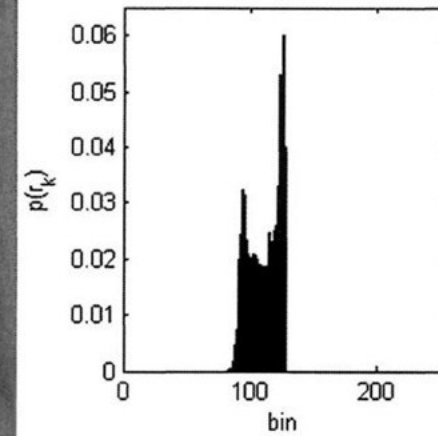
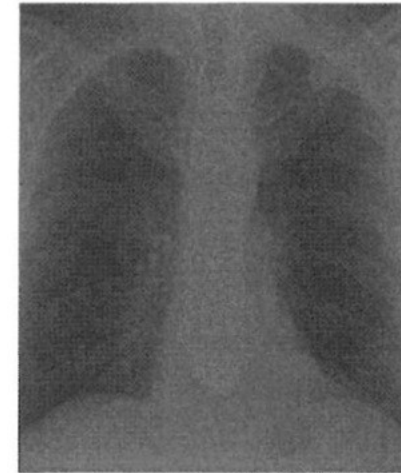
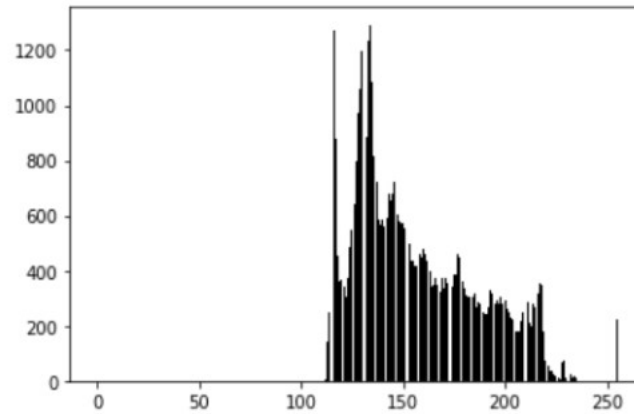
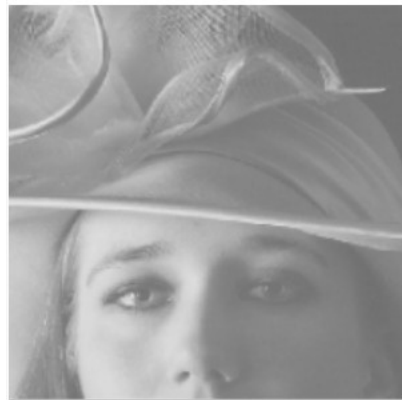
$$s = 255 \times \frac{(r - r_{\min})}{(r_{\max} - r_{\min})}$$

r = current pixel intensity value

r_{\min} = minimum intensity value present in the whole image

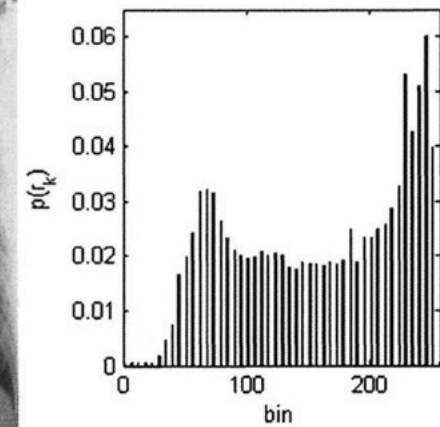
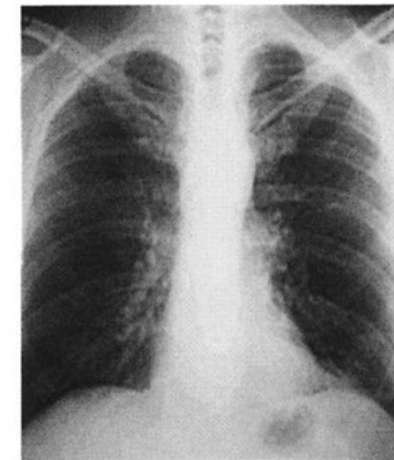
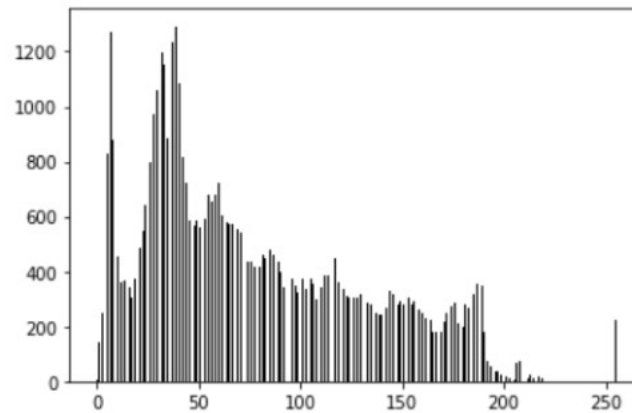
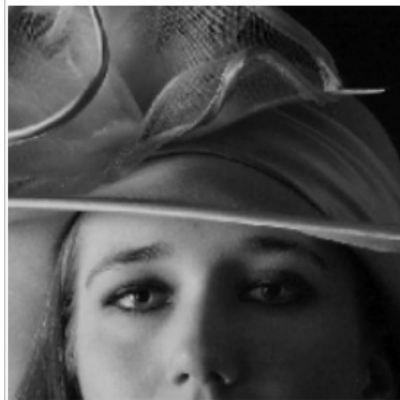
r_{\max} = maximum intensity value present in the whole image

Image transformation – example - stretching



(a)

(b)

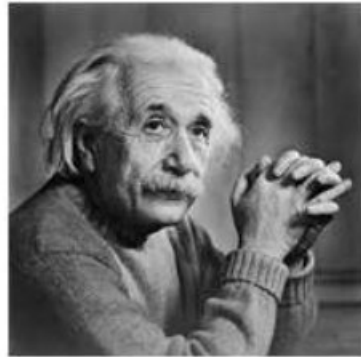


(a)

(b)

Negative transformation

Input Image



Output Image

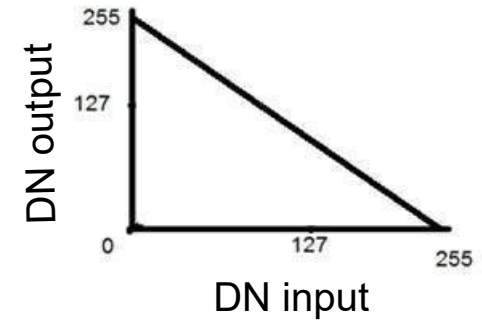


Image transformation – stretching

Contrast stretching

- Linear
- Linear with saturation i.e. 5% of histogram is cut-off
-

Image enhancement

Enhancing an image provides better contrast and a more detailed image as compare to non enhanced image. Image enhancement has very applications. It is used to enhance medical images, images captured in remote sensing, images from satellite e.t.c

The transformation function has been given below

$$s = T(r)$$

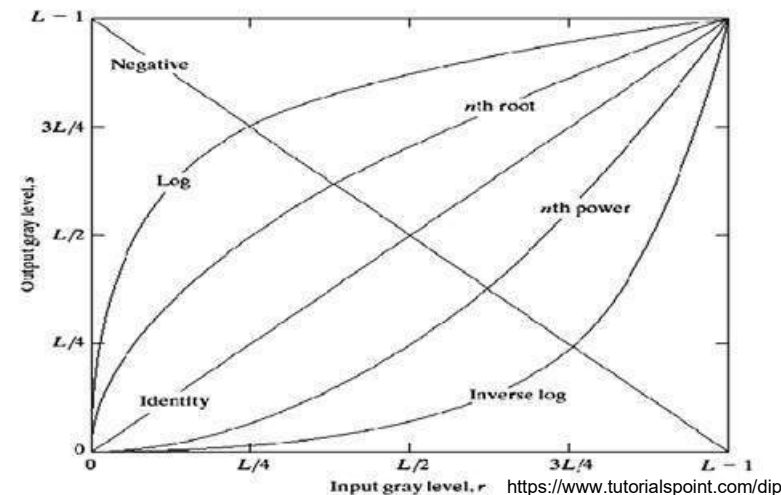
where r is the pixels of the input image and s is the pixels of the output image. T is a transformation function that maps each value of r to each value of s . Image enhancement can be done through gray level transformations which are discussed below.

Gray level transformation

There are three basic gray level transformation.

- Linear
- Logarithmic
- Power – law

The overall graph of these transitions has been shown below.



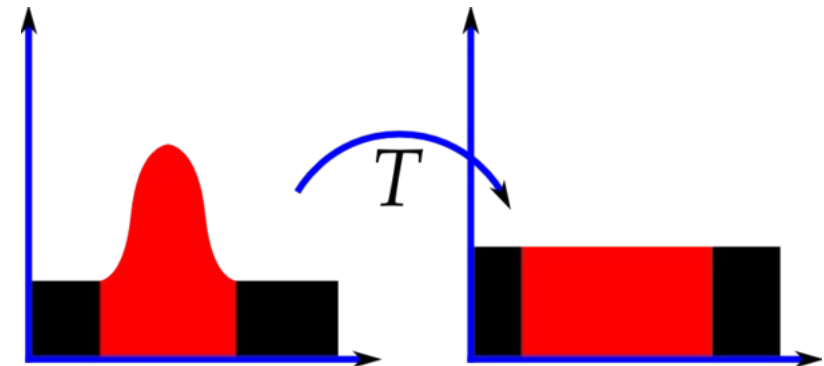
Increasing the contrast of the image – stretching - equalization

Histogram equalization is a method in image processing of contrast adjustment using the image's histogram.

This allows for areas of lower local contrast to gain a higher contrast.

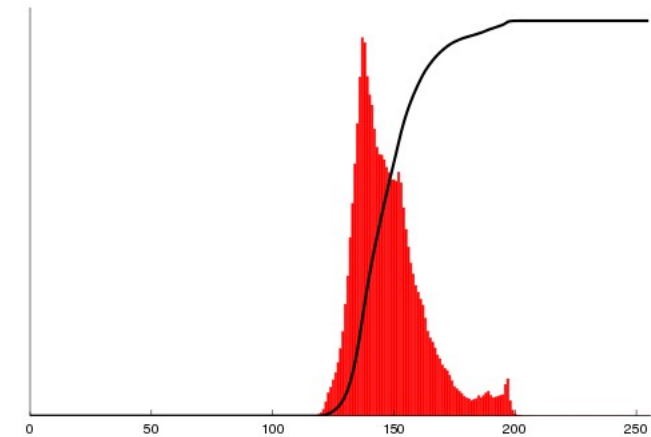
Back projection

The back projection (or "project") of a histogrammed image is the re-application of the modified histogram to the original image, functioning as a look-up table for pixel brightness values.



For each group of pixels taken from the same position from all input single-channel images, the function puts the histogram bin value to the destination image, where the coordinates of the bin are determined by the values of pixels in this input group. In terms of statistics, the value of each output image pixel characterizes the probability that the corresponding input pixel group belongs to the object whose histogram is used.

Increasing the contrast of the image – stretching - equalization

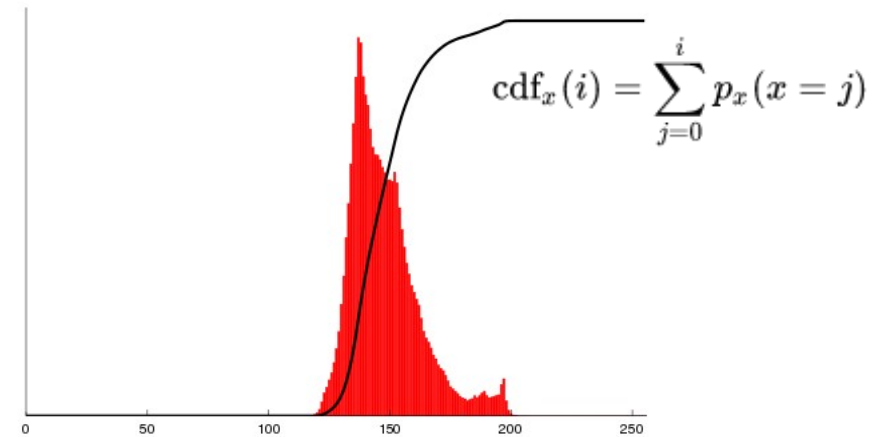


Increasing the contrast of the image – stretching - equalization

$$p_x(i) = p(x = i) = \frac{n_i}{n}, \quad 0 \leq i < L$$

The probability of an occurrence of a pixel of level i in the image

The cumulative distribution function (CDF)



We would like to create a transformation of the form $y = T(x)$ to produce a new image $\{y\}$, with a flat histogram. Such an image would have a linearized cumulative distribution function (CDF) across the value range, i.e.



$$\text{cdf}_y(i) = (i + 1)K \quad \text{for} \quad 0 \leq i < L \quad \text{for some constant } K.$$

Increasing the contrast of the image – stretching - equalization

52	55	61	59	79	61	76	61
62	59	55	104	94	85	59	71
63	65	66	113	144	104	63	72
64	70	70	126	154	109	71	69
67	73	68	106	122	88	68	68
68	79	60	70	77	66	58	75
69	85	64	58	55	61	65	83
70	87	69	68	65	73	78	90

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		



Increasing the contrast of equalization

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

v, Pixel Intensity	cdf(v)	h(v), Equalized v
52	1	0
55	4	12
58	6	20
59	9	32
60	10	36
61	14	53
62	15	57
63	17	65
64	19	73
65	22	85
66	24	93
67	25	97
68	30	117
69	33	130
70	37	146
71	39	154
72	40	158
73	42	166

Increasing the contrast of the image – stretching - equalization

This cdf shows that the minimum value in the subimage is 52 and the maximum value is 154. The cdf of 64 for value 154 coincides with the number of pixels in the image. The cdf must be normalized to [0,255]. The general histogram equalization formula is:

$$h(v) = \text{round} \left(\frac{\text{cdf}(v) - \text{cdf}_{\min}}{(M \times N) - \text{cdf}_{\min}} \times (L - 1) \right)$$

\downarrow
 cdfmax

where cdfmin is the minimum non-zero value of the cumulative distribution function (in this case 1), $M \times N$ gives the image's number of pixels (for the example above 64, where M is width and N the height) and L is the number of grey levels used (in most cases, like this one, 256).

v, Pixel Intensity	cdf(v)	h(v), Equalized v
52	1	0
55	4	12
58	6	20
59	9	32
60	10	36
61	14	53
62	15	57
63	17	65
64	19	73
65	22	85
66	24	93
67	25	97
68	30	117
69	33	130
70	37	146
71	39	154
72	40	158
73	42	166

76	44	174
77	45	178
78	46	182
79	48	190
83	49	194
85	51	202
87	52	206
88	53	210
90	54	215
94	55	219
104	57	227
106	58	231
109	59	235
113	60	239
122	61	243
126	62	247
144	63	251
154	64	255

(Please note that $h(v) = \text{ceil}(\text{cdf}(v)) - 1$ version is not illustrated