abstract id: 231

Data processing in hydrogeology

5.2
Groundwater flow and solute transport modelling

title: A semi-analytical model for estimating groundwater recharge through fractured till

author(s): Mark O. Cuthbert
University of Birmingham, United Kingdom, m.cuthbert@bham.ac.uk

Kate E. Thatcher
University of Birmingham, United Kingdom, k.e.thatcher@bham.ac.uk

John H. Tellam
University of Birmingham, United Kingdom, jhtellam@bham.ac.uk

Rae Mackay
University of Birmingham, United Kingdom, r.mackay@bham.ac.uk

keywords: groundwater recharge, till, fractures, semi-analytical modelling
The geology, material properties and hydraulics of a c. 6 m thick till deposit overlying a regional sandstone aquifer have been investigated at a field site in Shropshire, UK. The data have enabled a robust conceptual model of the processes contributing to groundwater recharge through till to be derived for the site. The hydraulic data indicate that water table responses to rainfall during the summer occur while large soil moisture tensions (suctions) are present higher up the profile. Given the very low permeability of the till at the core scale, these data are highly suggestive of the occurrence of preferential flow. Furthermore, near-vertical, hydraulically active fractures have been observed in a test pit extending to depths of greater than 2 m. The fracture intensity decreases with depth and they are commonly infilled with sediment (often fine grained calcareous material or sand) derived from weathered clasts within the till.

During the summer period, plants will draw moisture from both the fractures and the matrix in the soil zone leading to highly complex hydraulic interactions. However, for the winter period, we show how a simple semi-analytical model can adequately simulate the dynamics of the till water table, and suggest that this may be used to derive information to allow reasonable estimates of recharge to be made for the whole year. The model assumes that, during the winter, flow predominantly occurs through the fracture network, that fracture apertures are constant but that the fracture spacing doubles with depth, consistent with our field observations. Since the fractures are infilled with sediment it is also assumed that flow is Darcian. Thus:

\[ K(z) = \frac{aK_s}{z} \]  \hspace{1cm} (1)

\[ S(z) = \frac{aS_s}{z} \]  \hspace{1cm} (2)

where \( z \) = depth below ground surface, and \( K_s \) and \( S_s \) are the hydraulic conductivity and specific yield respectively at a representative depth, \( a \), at which values of \( K \) and \( S \) are defined.

Based on the assumptions above and the conceptual simplifications embedded in these assumptions, a model was constructed to simulate the position of the water table during the winter months (December 2004 to March 2005). If the mean vertical \( K \) in fractures is estimated as a harmonic mean between the water table and the base of the till, using Darcy’s Law:

\[ Q_t = \frac{(WT_t - GWL_t)}{(WT_t - D)} \cdot \frac{(2aK_s)}{(WT_t + D)} \]  \hspace{1cm} (3)

where \( Q_t \) = vertical flow (L/T) through the base of the till (i.e. recharge to the underlying sandstone) at time \( t \), \( WT_t \) = water table elevation (bgl) at time \( t \), \( GWL_t \) = groundwater level in the sandstone aquifer (bgl) at time \( t \), \( D \) = total thickness of till.

Letting the unit time interval for model calculations be equal to \( \Delta t \), the inflow-outflow imbalance for a given time step, \( \Delta Q \), is given by:

\[ \Delta Q = P_t - AE_t - Q_t \]  \hspace{1cm} (4)

where \( P_t \) and \( AE_t \) are the precipitation and evapotranspiration at time \( t \).

If the mean vertical \( S \) in fractures is estimated as an arithmetic mean over the interval of head changes then it can be shown that:
\[ WT_{t+\Delta t} = \exp\left( \ln(WT_t) - \frac{\Delta Q}{aS_a} \right) \]  

(5)

where \( WT_{t+\Delta t} \) = water table elevation (bgl) at time \( t+\Delta t \).

Combining equations (3), (4) and (5) gives:

\[ WT_{t+\Delta t} = \exp\left( \ln(WT_t) + \frac{(WT_t - GWL_t)(2K_a)}{(WT_t - D)(WT_t + D)S_a} \frac{(P_t - AE_t)}{aS_a} \right) \]  

(6)

Equation (6) was implemented in a spreadsheet to generate a simulated water table for given time series inputs of \( P \) and \( AE \) using a daily timestep. A starting value for \( WT_t \) was assigned and then values of \( K_a, S_a \) and \( a \) were varied to refine the model. Three different scenarios were run based on rainfall values factored at 70, 80 and 90% of the observed values at Bowling Green. The model outputs are time series of water table elevation and groundwater recharge (Fig. 1).

![Figure 1. Model results.](image)

While refining the models, it became apparent that with good fits to the data for the first three months of the modelled period the simulated water table values fall much too low during the month of March. Furthermore, fitting the March data produces poor fits to the first three months of observations. Warmer weather in March may have led to a change in the mode of evaporation at the near surface, most likely due to evapotranspiration gaining significance at the initiation of plant growth from this time on. Plants will thus draw moisture from both the fractures and the matrix in the soil zone whereas in the preceding months it has been assumed...
that evaporation is predominantly drawing water from the fracture network. Hence in March, even though the PE has increased, the proportion of the energy demand from the fracture system is still likely to be smaller than in the previous month because of the increased proportion of energy that will go towards transpiration. Therefore, for the month of March, a factor was applied to the AE input in order to reduce the evaporation from the fractured system resulting in much better model fits.

Model results indicate that the bulk hydraulic conductivity of the till at the field site is in the range $9 \times 10^{-8}$ to $8 \times 10^{-10}$ m s$^{-1}$, up to three orders of magnitude higher than the till matrix hydraulic conductivity measured in the laboratory. Fracture porosities of the till may be in the range 0.1 to 4% with groundwater velocities of a few cm/d ($3 \times 10^{-7}$ m s$^{-1}$). The recharge to the sandstone underlying the field site is likely to have been in the range $49 \pm 28$ mm/a ($1.5 \pm 0.9 \times 10^{-9}$ m s$^{-1}$) during 2004 to 2005 and up to 50% lower during summer than winter.

The paper furthers understanding of the hydraulic processes contributing to recharge through till and makes the link between the detail of these processes and simplified models for recharge estimation, which may be needed for larger scale water resource studies. The results are relevant also to contaminant migration studies and aquifer vulnerability assessments.