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Interactions of surface and ground waters

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ABSTRACT

An extension of the Green-Ampt model for the case of infiltration with varying water contents above the wetting front has been presented in this paper. Both increase and decrease of soil water contents caused by changes in supply of water at the surface were considered. The presented model uses a balance approach and in this way evades the problem of soil moisture hysteresis. The theoretical considerations are illustrated by two examples. One of them enables comparison of results obtained using the piston model with a numerical solution of the diffusion equation which takes the hysteresis into account. The second example shows typical solution cases.

INTRODUCTION

Interaction between surface and ground water has usually been modeled using a discrete model of flow in aeration zone. If even the applied model is one-dimensional (vertical cylinder) it is seriously complicated because of the spatial distribution of parameters. A large number of detailed data on infiltrating stream is however needed by the model. In case when we are interested in general groundwater movement rather than in infiltration itself it becomes uneconomical to use the full model. In this case a model which would supply only a couple of most important data but which would be simpler in use, is much more needed. It has however be able to describe a fully unsteady process. The above conditions are met by the piston model of infiltration.

ASSUMPTIONS FOR THE PISTON MODEL

Varying intensity of rainfall may cause increase or decrease of soil water content behind the wetting front. This process could be expressed by a simplified piston model with only a few variables describing the moisture content. A relative stable moisture profile connected with a wetting front that moves without shape changes – like a piston, is adopted in the model. Only two variables are necessary for description of a single front motion – the position of a front z_f and water content behind the front θ_z. In an unsteady model several wetting fronts may appear and each one is characterized by a pair of variables (z_i, θ_i). Before a head of each wetting front water content θ_{i+1} corresponds to the previous wetting front of z_{i+1} position. Just before the lowest water front there is only the initial water content θ_0. Water content above each water front remains stable while a front position changes continuously. Moisture changes may occur behind the highest front only, hence a successive wetting front is described by j index.

To simulate changes of water content the classical Green-Ampt model for a sharp front is applied together with the piston model of unsaturated infiltration and the Morel-Seytoux model of moisture redistribution.

Front velocity

While fronts are balanced one can determine velocity of their movements and then their present positions. This contributes to a general spatial picture of a soil water content distribution (Figure 1). Regardless of the kind of a motion front velocity v_f is calculated from the following water balance:

$$v_f = \frac{v_{i,j} - k(\theta_{i+1})}{\theta_{i,j} - \theta_{i+1}} \tag{1}$$

In result of redistribution water content behind the first front θ1j may decrease. In case of its disappearance its function and features are overtaken by the preceding front. This means in particular that since then the corresponding water content may change again.

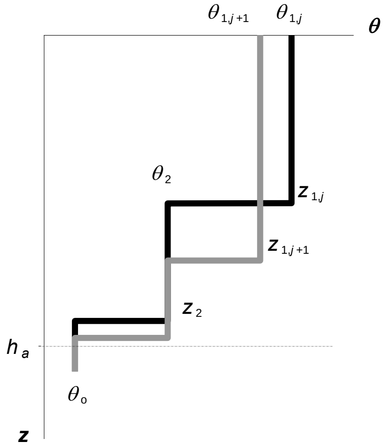

Figure 1. Water budget for two wetting fronts.

Velocity of infiltration v_{ij} for the first front is determined on the basis of pluviometric data and only after the surface is sunk it is limited to the soil infiltration capacity. For the following fronts this velocity is equal to unsaturated hydraulic conductivity behind a wetting front $v_{ij} = k_i$, which is constant for constant water content after a wetting front. It can be derived from equation (1) that velocities of all wetting fronts are different and they allow to calculate positions of fronts after Δt time interval:

$$z_{i,j+1} = z_{i,j} + v_f \, \Delta t = z_i^{\,j} + \frac{v_{i,j} - k(\theta_{i+1})}{\theta_{i,j} - \theta_{i+1}} \Delta t \tag{2}$$

Infiltration after increase of rainfall intensity

During intensive rainfalls water almost entirely drives air out of soil and maximal moisture content θ_n is reached. On the other hand velocity of infiltration cannot exceed soil infiltration capacity (Green, Ampt, 1911):

$$v_g = k_n \left(1 + \frac{h_g + h_k}{z_{i,j}} \right) \tag{3}$$

All the rainfall surplus $v_{r\,j}$ above this value remains on the land surface and makes surface run-off. Rainfall capacity smaller than soil infiltration capacity entirely infiltrates. Until recharge drops below hydraulic conductivity $v_{r\,j} \geq k_n$ the full soil saturation is kept while supply velocity changes together with rainfall intensity $v_z = \min(v_{r,j}, v_g) = v_{1j}$ as well as a wetting front velocity v_f.

In case water supply $v_{r\,j}$ does not make soil fully saturated, for a constant moisture (piston model) a hydraulic gradient is equal to 1 behind a wetting front. Hence soil unsaturated conductivity k reaches the value of infiltration velocity and this enables to calculate moisture content after a wetting front (Bouwer, 1976):

$$\theta_{1,j} \Leftarrow k(\theta_{1,j}) = v_{r,j} \tag{4}$$

With the previous partial saturation of the upper soil layer each increase of water supply $v_{r\,j}$ generates appearance of a new wetting front of moisture $\theta_{1j}(v_r)$ calculated from equation (4). However these fronts move with different velocities v_f.

Infiltration after decrease of rainfall intensity

When precipitation exceeds soil infiltration capacity (eq. 3) the decrease of water supply intensity has no direct and considerable impact on infiltration. Small changes of infiltration velocity may be caused by changes of water depth h_g on a ground surface. Only after supply intensity drops below soil infiltration capacity (but for $v_r > k_n$) velocity of infiltration changes together with water supply velocity ($v_{1j} = v_r$) and a wetting front velocity changes respectively. In these both cases moisture after a wetting front remains maximal ($\theta_{1j} = \theta_n$).

For rainfall intensity lower than soil infiltration capacity ($v_r < k_n$) the decrease of water supply v_r causes negative budget of a surface layer up to the nearest wetting front since its velocity does not change at first (Morel-Seytoux, 1984). Front transition that can be then calculated results in decrease of moisture after a wetting front (Figure 1) as well as front velocity but these changes are slow. So, equation (1) can be still approximately used and a front position $z_{1,j+1}$ is calculated on the basis of the value of previous moisture $\theta_{1,j}$ (Charbeneau, 2000):

$$z_{i,j+1} = z_i^{\,j} + \frac{k(\theta_{1,j}) - k(\theta_2)}{\theta_{1,j} - \theta_2}\Delta t \tag{5}$$

At the end of time step Δt humidity is equal to:

$$\theta_{1,j+1} = (v_{1,j} - k_2)\frac{\Delta t}{z_{1,j+1}} + (\theta_{1,j} - \theta_2)\frac{z_{1,j}}{z_{1,j+1}} + \theta_2 \tag{6}$$

In result of humidity decrease velocity of a wetting front gradually drops down until the balance is reached again or a new wetting front appears.

Propagation of a wetting front

Despite of a water supply regime, fronts move down and are exposed to wetting process. It is possible that in a given soil column several fronts exist simultaneously which is caused by changes of water supply intensity. Each higher front is more humid and in consequence it moves with higher velocity. As an exception velocities of fronts may equalize or even an earlier front may move faster. However, most often the difference between velocities leads to superposition of fronts (Książyński, 2007).

Creation of a new front. Each increase of water supply in condition of partial saturation leads to appearance of a new front. This front (no 1) has the initial position $z_{1j}=0$ and it is described by the water content behind a front θ_{1j} calculated from equation (4). Further changes of these parameters undergo principles presented above and they depend on varying conditions of water supply. Numbers of other fronts are then increased by 1 but their water content is stable since then.

Superposition of fronts. If two consecutive increases of water supply take place and in consequence moisture changes from θ_{i+1} to θ_i and then to θ_{i-1}, the mentioned difference in velocities of fronts makes the first front reach the second one ($z_{i-1} \rightarrow z_i$) in a given time period. Humidity equalizes at the θ_{i-1} level and because of increase of soil infiltration capacity ($\theta_{ij+1} > \theta_i$) the front velocity drops down. As to ensure balance conditions the calculated positions of fronts z_i and z_{i+1}($<z_i$) should be substituted by positions z_i' ($z_{i+1} < z_i' < z_i$):

$$z'_{i,j+1} = z_{i+1,j+1} + \left(z_{i,j+1} - z_{i+1,j+1}\right)\frac{\theta_i - \theta_{i+1}}{\theta_i - \theta_{i+2}} \tag{7}$$

It can be noticed that the wetting front i+1 disappears and the numbering of others will be decreased by 1. This enables to use this vacant number in further calculations and keep continuity of numbering.

Spread of a front. It may also happen that the first front dissolves and its water content becomes equal to water content of the previous one. Such situation is signaled when moisture θ_{1j} calculated from (eq 6) is smaller than θ_2. This means that the water content of fronts reached the same level and the first front joined up the second one which since then may have negative balance and may diminish its moisture. Since the first front disappeared the numbering of others will be decreased by 1. Moisture θ_{1j+1} is calculated according to following formula:

$$\theta_{1,j+1} = \left(v_{1,j} - k_2\right)\frac{\Delta t}{z_{1,j+1}} + \left(\theta_{1,j} - \theta_2\right)\frac{z_{1,j}}{z_{1,j+1}} + \left(\theta_2 - \theta_3\right)\frac{z_{2,j}}{z_{1,j+1}} + \theta_3 \tag{8}$$

Numbering of fronts on the right side of the equation corresponds to numbering before the fronts are joined.

Changes in groundwater supply

Groundwater recharge is connected to accretion of a wetting front on the upper edge of a capillary fringe $z = h_a$.. Such approach enables to avoid complex calculations of time- and depth-dependant velocity of infiltration. Amount of water reaching an aquifer is then equal to water amount in this part of a moisture profile which passed into a capillary fringe in a given time interval.

Intensity of groundwater recharge. For the calculation of recharge a balance of wetting fronts volume, which at that time step migrated below the border line has been used. After the calculation of every time step the location of fronts is reverted to the border $z_{ij} = h_a$, and so the volume absorbed by groundwater in the next step can be easily specified as:

$$h'_{V,j+1} = \sum\left(z_{i,j} - h_a\right)\left(\theta_i - \theta_{i+1}\right) \tag{9}$$

and positive sum components are included. Basing that, the average supply rate of groundwater can be calculated as:

$$w_{j+1} = \frac{h_{V,j+1}}{\Delta t} = \frac{1}{\Delta t}\sum\left(z_{i,j} - h_a\right)\left(\theta_i - \theta_{i+1}\right) \tag{10}$$

Infiltration velocity through vadose zone. It is easier to assess infiltration velocity at a depth below the surface. If the first (the highest) wetting front did not reach yet a level for which the velocity is calculated, this one corresponds directly to conductivity value $k(\theta_i)$ for the local moisture content but just above the first front moisture changes decide about the velocity. When local moisture θ_{1j} does not vary, infiltration velocity is equal to the the surface supply v_z. In the case of moisture redistribution infiltration velocity increases linearly from the surface to the first wetting front, according to the formula:

$$v = \left(1 - \frac{z}{z_f}\right) v_z + k(\theta_d) \frac{z}{z_f} \tag{11}$$

Exemplary calculation

To illustrate typical phenomena occurring during unsteady infiltration the velocity changes course at different depths were calculated for a specially selected rainfall distribution. The intensity of precipitation in the first hour was 7.2 mm; in the second hour it increased to 14.4, and in the third dropped down to 3.6 mm. Another event occurred after 2 hours break and gave within one hours again 7.2 mm of water, after which rain stopped. The process run in the sandy soil of conductivity for full saturation k_o = 10.8 mm/h, that corresponds to the moisture θ_n=30%, capillary height h_k= 12 cm, initial moisture θ_o= 0.5% and conductivity characteristics described by Irmay-Avierjanov formula (Averjanov, 1950) (power exponent m_k=3.5, residual moisture content θ_r =0).

Figure 2 presents redistribution of moisture content, computed with the help of the piston model.

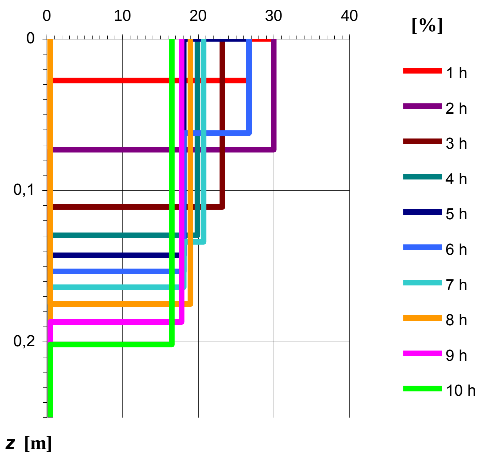

Figure 2. Simulation of moisture distribution during unsteady infiltration according to the piston model.

After two hours a new front appeared resulting from the increase in rainfall intensity which absorbed already the old one and only one fully developed front of water content θ=30% remained. To the fifth hour the moisture redistribution supplemented by too low or zero surface recharge is visible. But in the sixth and seventh hour of the process two wetting fronts occur simultaneously — lower with moisture frozen by the next front at level of 18.1% and the higher with full saturation, above which moisture redistribution takes place. The fronts undergo superposition near the end of eighth hour and further redistribution runs already inside one front zone.

Particular plots in Figure 3 present infiltration process on different depths beneath the surface of the soil computed in 15min. intervals. A wetting front achieves the depth of 3 cm after ca 10 min. from its appearance, which means more than 1 hour's delay. Variety of moisture content causes fluctuation of recharge and disappearance of precipitation only is followed by gradual regression. On the depth of 10 cm infiltration appears after more than three hours, evident flattening of hydrogram is observed. On the depth of 20 cm only one wetting front occurs, gradually declining in the soil.

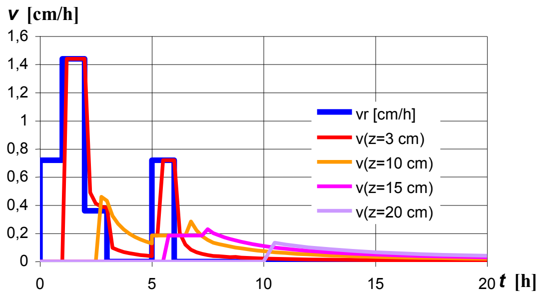

Figure 3. Process of unsteady infiltration according to piston model.

In Figure 4 process of infiltration obtained from the grid model simulation is presented for comparison.

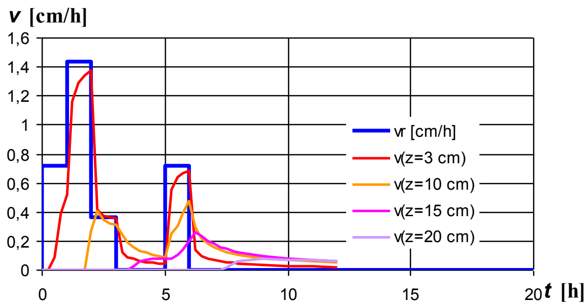

Figure 4. Process of unsteady infiltration according to the discrete model.

CONCLUSIONS

The presented model is a useful tool enabling to determine in an easy way time-dependant groundwater recharge for many soil profiles (Figure 2). Good model results make the model recommendable for simulation of groundwater supply.

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