Book Erratum
Understanding The Analytic Hierarchy Process

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Mistakes are proof that you are trying
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Errors found in the book Understanding The Analytic Hierarchy Process [1].

Page 40, line 3 from the top

is

$$n_{ij} = \begin{cases} 0 & \text{if } i = j \text{ or } \{a_i, a_j\} \in E \\ 1 & \text{if } i \neq j \text{ and } \{a_i, a_j\} \notin E \end{cases}$$

should be

$$n_{ij} = \begin{cases} 0 & \text{if } i = j \text{ or } \{a_i, a_j\} \notin E \\ 1 & \text{if } i \neq j \text{ and } \{a_i, a_j\} \in E \end{cases}$$

Page 75, the numerical example is faulty. The correct version is:

and the constant term vector r is given as

$$r = \begin{pmatrix} \ln 6 \\ \ln \frac{21}{8} \\ -\ln 3 \\ \ln \frac{36}{7} \\ -\ln 27 \end{pmatrix}.$$

Solving $G\widehat{w} = r$ leads to the following logarithmized ranking vector

$$\widehat{w} = \begin{pmatrix} \frac{1}{100}\left(22\ln 3 + 43\ln 6 + 2\ln 27 - 7\ln\frac{36}{7} + 8\ln\frac{21}{8}\right) \\ \frac{1}{25}\left(8\ln 3 + 2\ln 6 + 3\ln 27 + 2\ln\frac{36}{7} + 12\ln\frac{21}{8}\right) \\ \frac{1}{100}\left(22\ln 3 - 7\ln 6 + 2\ln 27 + 43\ln\frac{36}{7} + 8\ln\frac{21}{8}\right) \\ \frac{1}{100}\left(22\ln 3 - 7\ln 6 + 2\ln 27 + 43\ln\frac{36}{7} + 8\ln\frac{21}{8}\right) \\ \frac{1}{50}\left(-4\ln 3 - \ln 6 - 14\ln 27 - \ln\frac{36}{7} - 6\ln\frac{21}{8}\right) \end{pmatrix} = \begin{pmatrix} 1.04 \\ 1.484 \\ -2.29 \\ 0.963 \\ -1.19 \end{pmatrix}.$$

Hence, the (unscaled) ranking vector is

$$w = \begin{pmatrix} e^{1.04064} \\ e^{1.48464} \\ e^{-2.2937} \\ e^{0.96356} \\ e^{-1.19512} \end{pmatrix} = \begin{pmatrix} 2.83103 \\ 4.4134 \\ 0.100889 \\ 2.62103 \\ 0.302668 \end{pmatrix}.$$

The last step to receive the ranking in the usual form is scaling so that the entries of the ranking vector sum up to 1. The final form of the ranking vector is as follows:

$$w_{gm} = \begin{pmatrix} 0.275 \\ 0.429 \\ 0.0098 \\ 0.255 \\ 0.0294 \end{pmatrix}.$$

According to the computed ranking, the most preferred alternative is a_1 with the ranking value $w(a_2) = 0.429$. The second place is taken by a_4 with $w(a_1) = 0.275$, then a_4, a_5 and a_3.

Of course, one may verify that GMM applied to the following matrix

$$\begin{pmatrix} 1 & \frac{2}{3} & \frac{0.275}{0.0098} & \frac{0.275}{0.255} & 9 \\ \frac{3}{2} & 1 & \frac{0.429}{0.0098} & \frac{7}{4} & \frac{0.429}{0.0294} \\ \frac{0.0098}{0.275} & \frac{0.0098}{0.429} & 1 & \frac{0.0098}{0.255} & \frac{1}{3} \\ \frac{0.255}{0.275} & \frac{4}{7} & \frac{0.255}{0.0098} & 1 & 9 \\ \frac{1}{9} & \frac{0.0294}{0.429} & 3 & \frac{1}{9} & 1 \end{pmatrix}$$

results in w_{gm}.

Page 80, line 1 from the top

is

where p_i is the number of existing comparisons in the i-th row of C,

should be

where p_i is the number of existing comparisons in the i-th row of C except the diagonal

Page 81, line 12 from the bottom

is

$$q_3 = \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} \left(2 - \frac{a_{ik}}{a_{ij}a_{jk}} - \frac{a_{ij}a_{jk}}{a_{ik}} \right)$$

should be

$$q_3 = \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} \left(2 - \frac{c_{ik}}{c_{ij}c_{jk}} - \frac{c_{ij}c_{jk}}{c_{ik}} \right)$$

Page 113, line 14 from the bottom

is

6.3.8.1 Effectiveness of the Koczkodaj index

should be

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Page 123, line 5 from the top

is

$$T_{ijk} = \begin{pmatrix} 1 & c_{ij} & c_{ik} \\ 1/c_{ij} & 1 & c_{kj} \\ c_{ik} & 1/c_{kj} & 1 \end{pmatrix}$$

should be

$$T_{ijk} = \begin{pmatrix} 1 & c_{ij} & c_{ik} \\ 1/c_{ij} & 1 & c_{jk} \\ c_{ik} & 1/c_{jk} & 1 \end{pmatrix}$$

Page 150, line 11 from the bottom

is

$$C = \begin{pmatrix} 1 & \left(\prod_{q=1}^{r} c_{1,2,q}^{\eta_q}\right)^{1/r} & \cdots & \left(\prod_{q=1}^{r} c_{1,n,q}^{\eta_q}\right)^{1/r} \\ \left(\prod_{q=1}^{r} c_{2,1,q}^{\eta_q}\right)^{1/r} & 1 & \cdots & \vdots \\ \vdots & \cdots & \ddots & \left(\prod_{q=1}^{r} c_{n-1.n,q}^{\eta_q}\right)^{1/r} \\ \left(\prod_{q=1}^{r} c_{n,1,q}^{\eta_q}\right)^{1/r} & \cdots & \cdots & 1 \end{pmatrix},$$

should be

$$C = \begin{pmatrix} 1 & \prod_{q=1}^{r} c_{1,2,q}^{\eta_q} & \cdots & \prod_{q=1}^{r} c_{1,n,q}^{\eta_q} \\ \prod_{q=1}^{r} c_{2,1,q}^{\eta_q} & 1 & \cdots & \vdots \\ \vdots & \cdots & \ddots & \prod_{q=1}^{r} c_{n-1.n,q}^{\eta_q} \\ \prod_{q=1}^{r} c_{n,1,q}^{\eta_q} & \cdots & \cdots & 1 \end{pmatrix},$$

Page 150, line 7 from the bottom

is

$$w(a_i) = \left(\prod_{k=1}^{n} \left(\prod_{q=1}^{r} c_{i,k,q}^{\eta_q} \right)^{1/r} \right)^{1/n}$$

should be

$$w(a_i) = \left(\prod_{k=1}^{n} \prod_{q=1}^{r} c_{i,k,q}^{\eta_q} \right)^{1/n}.$$

References

[1] K. Kułakowski. *Understanding the Analytic Hierarchy Process.* Chapman and Hall / CRC Press, 2020.