TOPOLOGICAL INSULATORS

A romance with many dimensions

Electric charges on the boundaries of certain insulators are programmed by topology to keep moving forward when they encounter an obstacle, rather than scattering backwards and increasing the resistance of the system. This is just one reason why topological insulators are one of the hottest topics in physics right now.

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One of the best role models for 'thinking outside the box' has to be A. Square, the hero of Edwin A. Abbott's novella *Flatland*. Despite being confined to two dimensions, the hapless Square nevertheless imagines and then experiences worlds truly beyond his own, venturing into both lower and higher dimensions. *Flatland* is a timeless story embedded in the mathematics of geometry and, at the same time, it is a clever, edgy satire on life in the Victorian era. But often forgotten is the fact that *Flatland* is, at its core, a romance — the book's full title is *Flatland: A Romance of Many Dimensions* — between a tireless explorer and forbidden knowledge. In recent years physicists and engineers have plunged head over heels into their own interdimensional love affair. The target of their affections is a new class of materials known as topological insulators. To understand this relatively recent rapture, we need to introduce Square to two new concepts that he did not know about at the time — topology and quantum mechanics.

Flatland has a rigid social hierarchy in which social class depends on the number of sides a person has. Square is in the middle class, above the triangular serfs but below the pentagons and hexagons — with the high-priest circles being at the top of the social hierarchy. Ironically, if topology had trumped basic geometry in *Flatland*, then everyone would have belonged to the same social class because topology is concerned with classifying properties — so-called topological invariants — that do not change when objects are subjected to continuous deformations. In other words, triangles, squares, pentagons, hexagons and circles all share the same topology.

In one episode, Square visits one-dimensional (1D) Lineland and 0D Pointland. As a higher-dimensional being, he has the power to see into these worlds in ways that the inhabitants could not fathom. He sees among other things that the residents of these lower-dimensional landscapes are hobbled by a lack of mobility: Lineland's citizens know only their neighbours on the left and right, and Pointland's single inhabitant, the King, is so full of himself that he cannot even comprehend the existence of another being. Things would have been different if quantum mechanics had applied in *Flatland*: for example, two or more quantum objects can simultaneously occupy the same space with little or no interactions (such as electrons in a 0D quantum dot), and quantum mechanics allows more information to be stored by a system or embedded in a region of space than is possible with classical mechanics.

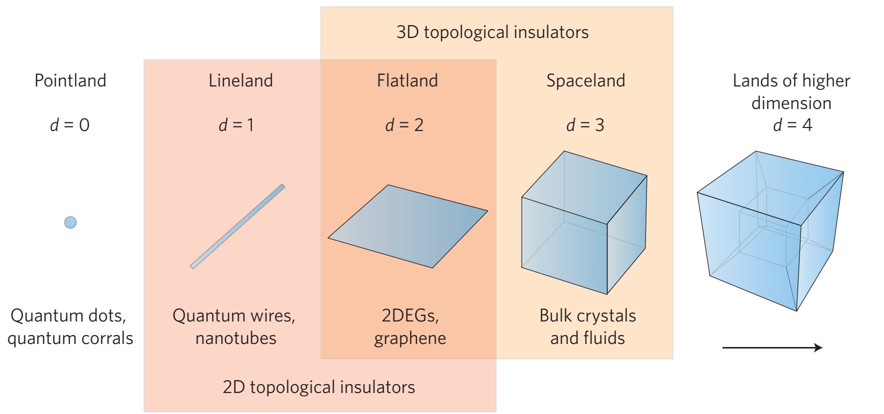

Figure 1 | Different dimensions. A topological insulator is an insulating material that allows electric charge to flow along its boundary in spin-polarized channels that are topologically protected from impurity scattering. The physics of topological insulators involves interactions between hosts of dimension *d* and boundaries of dimension *d*–1. Topological states have been observed on the edges of 2D systems and the surfaces of the 3D materials. The four-dimensional shape on the right is called a tesseract.

Developments in areas such as materials growth, device fabrication and imaging mean that we can now explore similar 'outside-the-box' ideas from topology and quantum mechanics in the real world. In the past, these pursuits often concentrated on physics in one particular dimension (for example, the 1D physics of quantum wires or the 2D physics of graphene). In particular, as physicists explored the quantum Hall effect in 2D electron gases (2DEGs) in greater detail, it became clear that 1D edge states had an important role in the effect, prompting more theoretical and experimental work on the interactions between different dimensions. However, although the 2D surface states of bulk 3D crystals have been investigated for decades, the 3D 'host' material was usually ignored. In contrast, the physics of topological insulators is fundamentally linked to structures in *d* dimensions, their boundaries in *d*–1 dimensions, and the information overlap between these two worlds (Fig. 1). This work could lead to breakthroughs in areas as diverse as spintronics, quantum information and even particle physics — theorists have predicted that an exotic type of particle that is its own antiparticle (a Majorana particle) might be observed in topological insulators. One can see why the physics community is smitten.

Despite all this potential, the physics that underpins topological insulators will be familiar to anyone who has studied crystals. The periodic potential experienced by the charge carriers in a solid material

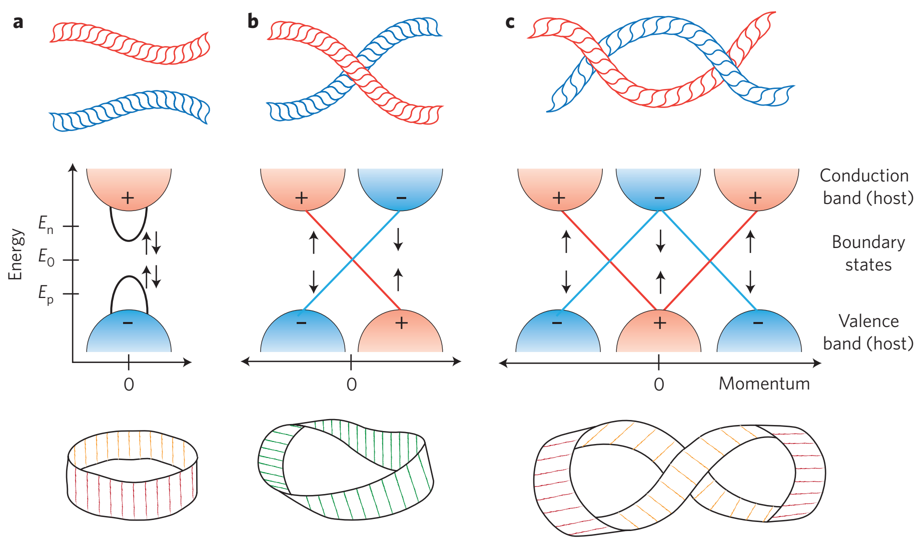

Figure 2 | Topological insulators and band structures. **a**, The conduction and valence bands of a typical 3D solid (middle section). The shaded regions are the bands in the bulk of the solid, and the thick black lines are the bands at the surface. (Similar behaviour is observed in a 2D system with a 1D boundary.) In general, the conduction band is symmetric (red), the valence band is antisymmetric (blue), and spin-up and spin-down electrons (black arrows) have the same energy. E_n, E_0 and E_p are the Fermi energies of a negatively doped, neutral and positively doped solid, respectively. If the Fermi energy lies in the energy gap between the conduction and valence bands, the solid is an insulator; if it intersects either band, the material will conduct electric charge. The top image shows the conduction and valence bands as strings, and the closed strip in the bottom section represents the Fermi surface. **b**, Spin–orbit coupling lifts the degeneracy of the electron spins and leads to other changes: in the bulk, for example, the conduction band becomes antisymmetric (–) and the valence band becomes symmetric (+) for positive momenta. At the boundary the bands (the red and blue lines) actually cross over each other, and the Fermi energy is forced to intersect both bands, which results in the conduction of electric charge along the boundary. The flow of charge is not impeded by obstacles in this example because, as explained in the text, it is not possible for electrons to be backscattered. The electron current in a particular direction is spin-polarized and robust against perturbations such as disorder and interactions. **c**, Increasing the spin–orbit coupling further leads to more changes. Electrons can be backscattered in this system.

leads to a band structure that describes the relationship between the energy and momentum of the electrons (or holes). This band structure typically consists of a conduction band composed of bonding orbitals (which have symmetric wave functions), and a valence band composed of antibonding orbitals (antisymmetric wave functions). In an insulator the Fermi energy lies in the bandgap between the minimum of the conduction band and the maximum of the valence band. The situation changes at the surface of a 3D host (or the edge of a 2D host) and new electron states appear at these boundaries. Depending on doping and crystal structure, the Fermi level may intersect either the conduction or valence band at the boundary, which will lead to conducting behaviour. However, if the Fermi level does not intersect either band, the boundary will remain insulating (Fig. 2a).

The situation becomes very interesting if spin–orbit coupling (Fig. 3) is added to the picture. In a semiconductor, spin–orbit coupling typically leads to various effects, such as warping of the valence band and the splitting of spin degeneracies. (The spin-up and spin-down electrons in a conventional semiconductor tend to have the same energy.) However, if the spin–orbit coupling is sufficiently large, it can actually lead to antisymmetric states having higher energies than symmetric states in certain regions of momentum space (whereas antisymmetric states normally have lower energies). This inversion leads to topological 'twists' in the band structure (Fig. 2b,c).

The changes caused by spin–orbit coupling can be even more dramatic at the boundary, with the conduction and valence bands actually crossing over. If the host is 3D and the valence and conduction bands cross over twice (or an even number of times), the 2D surface states form a pair of Dirac cones (Fig. 2c) — this is similar in some ways to what is found in graphene. However, if the host is 3D and the valence and conduction bands cross over once (or an odd number of times), the 2D surface states are completely different: indeed, theorists have shown that these even and odd boundary states are topologically distinct.

A distinguishing characteristic of the odd states (which are known as strong topological insulators) is that backscattering is forbidden: this means that electrons can, in principle, propagate with little or no resistance along the edge or surface of the system — even if the host is an insulator. This is a property that could prove to be very useful for applications. To see why backscattering is forbidden, consider Fig. 2b: if an electron is backscattered so that its momentum (**k**) is changed from +**k** to –**k**, then its spin must also be flipped from up to down, or vice versa. However, something is needed to flip the spin, such as a magnetic impurity or a magnetic field. If nothing is available to flip the spin, the electrons cannot be backscattered, so they can travel along the boundary unimpeded. If we look at Fig. 2c, we can see that it is possible to backscatter an electron without flipping its spin in a system where there is an even number of twists.

The first topological insulators were 2D hosts with a spin structure on a 1D edge (known as the quantum spin Hall effect). They were first elaborated theoretically[1–3], then predicted in a specific HgTe heterostructure system[4], and experimentally verified in a carefully tuned nanostructure[5]. The speed of the theoretical and experimental cycle was remarkable and a testament to the skill of the investigators involved, as well as to the mature state of nanofabrication technologies such as molecular beam epitaxy. Then, the 3D version of topological insulators was proposed[6], predicted in a BiSb alloy[7], and experimentally detected by angle-resolved photoemission spectroscopy (ARPES)[8]. Again it was a stunning sequence of developments that launched a new field.

As the 3D topological insulators have surface states with chiral patterns of spins in momentum space (Fig. 3), surface-sensitive techniques such as ARPES and scanning tunnelling microscopy (STM) have been unleashed in full force on these materials. The Fermi 'surface' of the 2D boundary state is a circle and can be represented by a closed strip (Fig. 2, bottom panels). As the spin–orbit coupling increases, and the band structure changes shape, twists are introduced into the strip that represents the Fermi surface. Again the properties of the system depend on

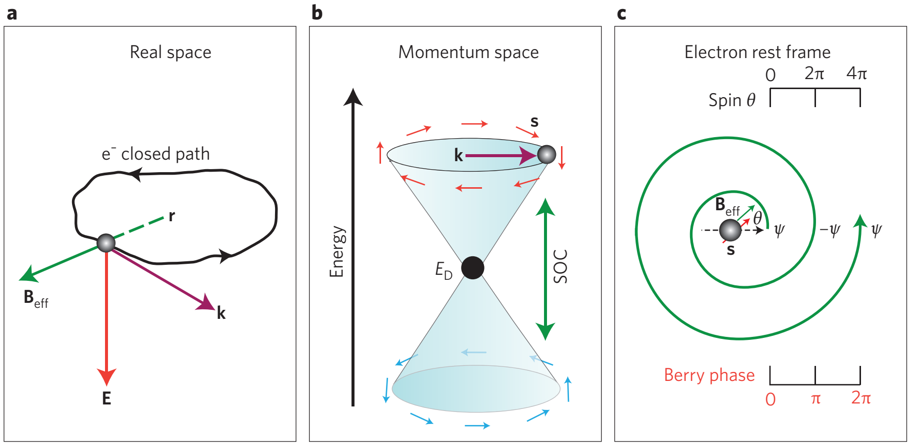

Figure 3 | Inside a topological insulator. **a**, An electron at position **r** executing a closed orbit (as a result of being scattered by impurities) in an electric field **E** experiences an effective magnetic field $\mathbf{B}_{\mathrm{eff}}$ that couples to its spin. **k** is momentum. **b**, This spin–orbit coupling (SOC) splits the degeneracy of spin-up and spin-down electrons and results in a Dirac cone in momentum space, with the spins (**s**) rotating with one chirality above the Dirac point E_D, and the opposite chirality below E_D. **c**, In its rest frame the electron sees a rotating magnetic field that changes both the spin direction θ and the Berry phase of the wavefunction ψ. For each closed orbit completed in real space, θ changes by 2π, but the Berry phase only changes by π, which means that ψ changes sign to become $-\psi$. The electron must therefore complete two complete orbits in real space for ψ to return to its original value. This behaviour corresponds to the Möbius topology of Fig. 2b and underlies the peculiar physics of topological surface states, including the cancellation of backscattering (which results from destructive interference as a result of the Berry phase shift).

whether this strip contains an even or odd number of twists. When there are zero (Fig. 2a) or two (Fig. 2c) twists, the band has two sides and two edges, and an electron moving around the Fermi surface returns to its starting point after one cycle. When there is one twist, on the other hand, a Möbius strip is created with only one side and one edge (Fig. 2b). An electron moving around this Fermi surface does not return to its starting point after one cycle: instead it must complete a second cycle to return to its original quantum state. This property is a key element of topological surface states and results from the electron acquiring a Berry phase[9] (Fig. 3).

As movement around the Fermi surface in momentum space corresponds to scattering in real space, STM can access topological physics through careful scrutiny of quantum interference patterns. These patterns contain an extremely rich and dense set of information that transcends the pure Flatland in which the electrons move, as evidenced by the holographic projection of 3D images from 2D surface states designed by hand[10]. Indeed, one can view topological surface states as the natural holographic projection of the electronic structure that exists in 3D onto a 2D surface[11]. An electron moving in this special topology has a knotted trajectory[12] that can be unravelled by, in effect, reverse engineering the interference patterns. Remarkably, STM experiments in a variety of topological surface states have now glimpsed the tell-tale signs of chiral surface states, Dirac fermions and suppression of backscattering[13–16].

A recent development is the isolation of a single Dirac cone (Fig. 3) in different materials including Bi_2Se_3 (refs 17,18), Bi_2Te_3 (refs 14,15,19) and pure Sb (refs 16,20) (the latter seems to be the simplest 'parent' material exhibiting non-trivial topological order[21]). The marriage of topological insulators and nanotechnology has also led to STM imaging[22] of Bi_2Se_3 nanowires and nanoribbons synthesized by chemical means, and the demonstration of electronic devices in Bi_2Se_3 nanoribbons[23]. Furthermore, the bonding of a thin film of a normal (*s*-wave) superconductor to a topological surface state is predicted to result in physics similar to *p*-wave superconductors (including the elusive Majorana modes)[24,25]. This happens because electrons with opposite momentum and spin pair together in a superconductor (to form Cooper pairs), but such pairs would normally destructively interfere in a topological insulator (Fig. 3). Recent progress in this direction is the demonstration of a bulk superconducting variant of a topological insulator[26] achieved by doping Bi_2Se_3 with Cu. With new STM studies probing the effects of single atomic impurities, similar to previous work on high-temperature superconductors[27], it is clear that the dimensional reach of topological insulators now stretches all the way from Pointland to Spaceland. And although we cannot fabricate a tesseract (the four-dimensional analogue of a cube; see Fig. 1), A. Square's unrelenting curiosity might make him wonder if there are 3D bulk states that inherit topological properties from a higher-dimensional host. Similar ideas[28] were recently exploited in the realization of nanoscale 'quantum drums'[29].

Sadly, Square ended up in prison for his exploits and for having thoughts like this. However, if the field of topological insulators keeps following its meteoric trajectory, this class of exciting materials may well match the longevity of *Flatland* itself. Abbott's novella has been made into a film several times. Here's hoping that *Topological Insulator: The Movie* is not far off. ❐

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References

1. Kane, C. L. & Mele, E. J. *Phys. Rev. Lett.* **95,** 226801 (2005).
2. Kane, C. L. & Mele, E. J. *Phys. Rev. Lett.* **95,** 146802 (2005).
3. Bernevig, B. A. & Zhang, S. C. *Phys. Rev. Lett.* **96,** 106802 (2006).
4. Bernevig, B. A., Hughes, T. L. & Zhang, S. C. *Science* **314,** 1757–1761 (2006).
5. Konig, M. *et al. Science* **318,** 766–770 (2007).
6. Fu, L., Kane, C. L. & Mele, E. J. *Phys. Rev. Lett.* **98,** 106803 (2007).
7. Fu, L. & Kane, C. L. *Phys. Rev. B* **76,** 045302 (2007).
8. Hsieh, D. *et al. Nature* **452,** 970–974 (2008).
9. Berry, M. *Nature Phys.* **6,** 148–150 (2010).
10. Moon, C. R., Mattos, L. S., Foster, B. K., Zeltzer, G. & Manoharan, H. C. *Nature Nanotech.* **4,** 167–172 (2009).
11. Wu, C., Bernevig, B. A. & Zhang, S-C. *Phys. Rev. Lett.* **96,** 106401 (2006).
12. Moore, J. E. *Nature* **464,** 194–198 (2010).
13. Roushan, P. *et al. Nature* **460,** 1106–1109 (2009).
14. Zhang, T. *et al. Phys. Rev. Lett.* **103,** 266803 (2009).
15. Alpichshev, Z. *et al. Phys. Rev. Lett.* **104,** 016401 (2010).
16. Gomes, K. K. *et al.* Preprint at http://arxiv.org/abs/arXiv:0909.0921 (2009).
17. Xia, Y. *et al. Nature Phys.* **5,** 398–402 (2009).
18. Hsieh, D. *et al. Nature* **460,** 1101–1105 (2009).
19. Chen, Y. L. *et al. Science* **325,** 178–181 (2009).
20. Hsieh, D. *et al. Science* **323,** 919–922 (2009).
21. Teo, J. C. Y., Fu, L. & Kane, C. L. *Phys. Rev. B* **78,** 045426 (2008).
22. Kong, D. *et al. Nano Lett.* **10,** 329–333 (2009).
23. Peng, H. *et al. Nature Mater.* **9,** 225–229 (2010).
24. Fu, L. & Kane, C. L. *Phys. Rev. Lett.* **100,** 096407 (2008).
25. Wilczek, F. *Nature Phys.* **5,** 614–618 (2009).
26. Hor, Y. S. *et al. Phys. Rev. Lett.* **104,** 057001 (2010).
27. Balatsky, A. V., Vekhter, I. & Zhu, J. X. *Rev. Mod. Phys.* **78,** 373–433 (2006).
28. Cipra, B. *Science* **255,** 1642–1643 (1992).
29. Moon, C. R. *et al. Science* **319,** 782–787 (2008).