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WALKING DROPLETS

A FORM OF WAVE-PARTICLE DUALITY AT MACROSCOPIC SCALE?

A droplet bouncing on the surface of an oscillating liquid may couple with the surface waves it generates and thus start to propagate. The resulting "walker" is a coherent macroscopic object that associates the droplet and its waves. Due to their waves, the mutual interactions of these walkers, as well as their reactions to their environment have surprising assets.

Masses and waves have long been the constitutive elements of classical physics. The wave-particle duality, which characterizes the behaviour of elementary physical objects on a microscopic scale, appeared only with quantum mechanics [1]. Until now, this duality had no equivalent at the macroscopic scale, for which masses and waves are different objects. We have recently introduced, in classical physics, a system that couples a material particle and a wave. It exhibits surprising properties and some of its observed features can be compared to those attributed to the wave-particle duality in quantum mechanics. The simple fact that such a comparison is possible (in spite of large differences) is in itself a surprise.

Bouncing droplets and surface waves

Our working tool is a droplet bouncing on a bath of the same liquid, and which becomes dynamically associated with the surface wave it emits.

Before presenting its properties, we shall first describe the conditions for the existence of this new system. Normally a liquid drop falling on the surface of the same liquid disappears rapidly (a few tenths of a second). In a first step [2], we have shown that this coalescence may be inhibited when the bath is made to vibrate vertically with a characteristic acceleration exceeding g, the acceleration of gravity. This causes the drop to bounce on the surface periodically. As the droplet collides with the interface, it remains separated from it by a continuous air film. Before this air film has had the

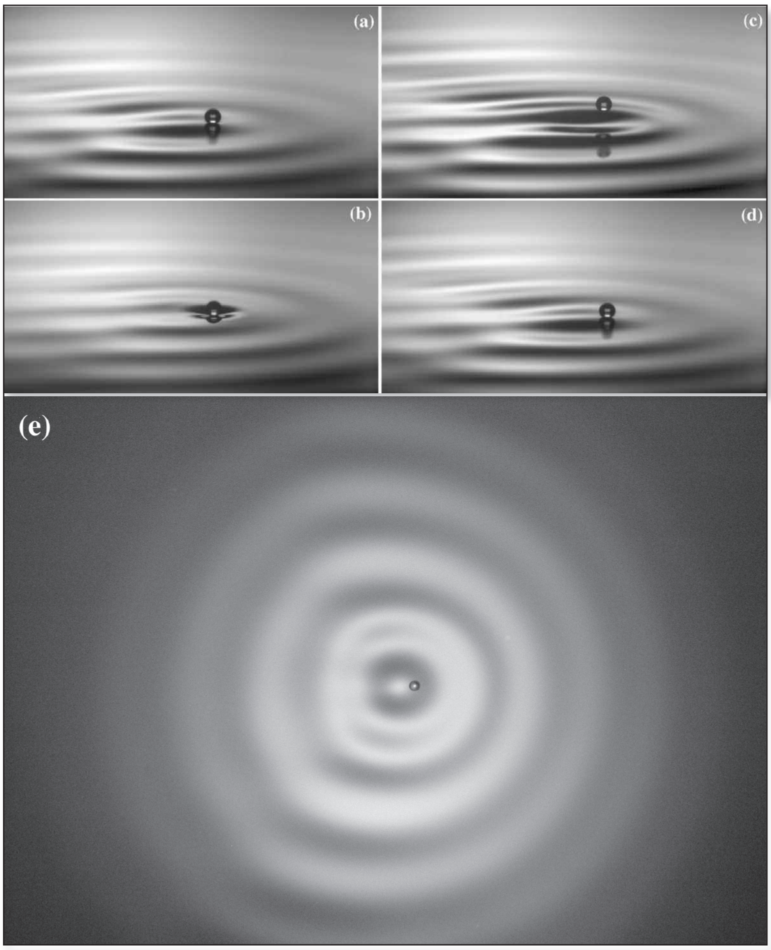
▲ Falling drops of water. © iStockPhoto

time to break, the drop lifts off again. A silicone oil droplet of millimetre size can thus be maintained for an unlimited time in a kind of "oscillating levitation" on the liquid surface.

At each successive bounce, the drop generates a surface wave similar to that caused by a stone thrown into water. Generally, this wave is strongly attenuated. However, when the amplitude of the forced oscillation is increased, the amplitude and spatial extension of the waves become larger. A remarkable transition is then observed [3, 4]: the drop starts to move horizontally at a speed V_W along the fluid surface. Henceforth, the set "drop plus associated wave" will be called the "walker". Pictures of such a walker are shown in Fig.1.

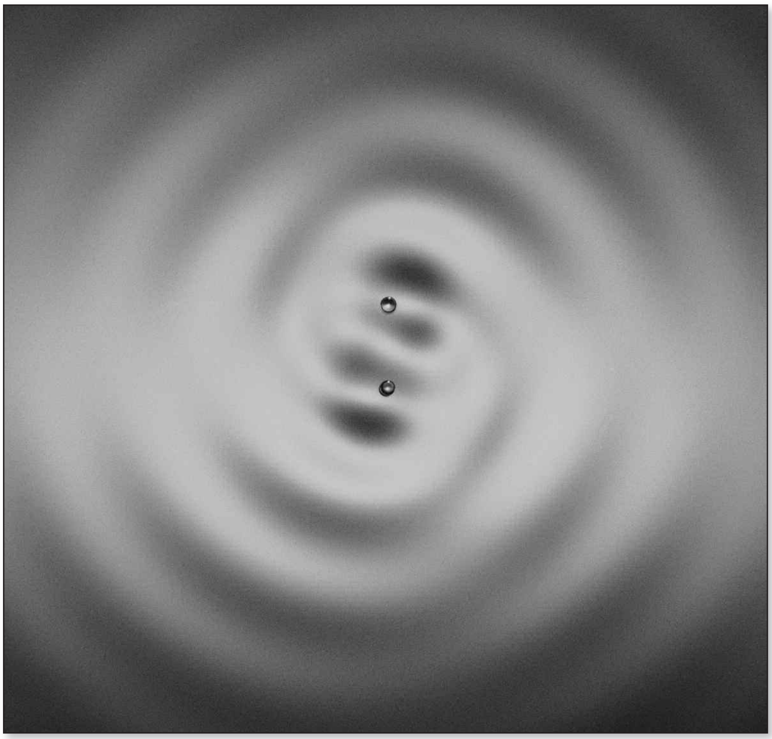
The spontaneous horizontal displacement results from the interaction of the drop with the surface waves. To understand this, one has to remember that a liquid surface is potentially unstable when it is submitted to vertical oscillations. For large oscillating amplitudes, the liquid surface spontaneously forms a system of stationary waves at a frequency $f/2$, the first sub-harmonic of the forced oscillation. This instability [5, 6], discovered by Faraday in 1831 (see box), appears when the acceleration amplitude γ_m of the forced oscillation becomes larger than a threshold value. In our experiment, when the forced oscillation amplitude is below but close to this threshold, the vertical bouncing motion also becomes sub-harmonic so that the droplet collides with the surface only once in two periods of the forced oscillation. The drop becomes an efficient generator of Faraday waves of frequency $f/2$ that are very weakly damped due to the proximity of the instability threshold.

As the drop rebounds on a wave-perturbed surface, there is a spontaneous "breaking of symmetry" by which the droplet starts moving horizontally on the interface. Fast camera recordings show that in this regime, at each bounce, the drop hits the forefront of the central bump of the wave created by the previous shocks (Fig.1). Besides the usual vertical kick, this shock on a locally inclined surface gives the drop a horizontal kick that will be repeated at the next period. On the average this results in a mean force, generating the motion. Because of nonlinear saturation, the droplet reaches a constant limit speed V_W that is of the order of a tenth of the phase velocity V_ϕ of the surface waves (V_W = 20 mm/s and V_ϕ = 189 mm/s for a frequency $f/2$ = 40 Hz). The "walker" is a coherent object, which exists only through the association of the wave and the droplet. If the drop coalesces with the bath, the wave vanishes. Inversely, if the waves are damped (for example in regions where the liquid layer is thin), the drop stops travelling. In the following experiments, the properties given to these walkers by the non-local character of their waves are explored. We will successively describe



▲ **FIG. 1: (a-d) Four successive pictures of a walker, side view; (e) top view. Photographs a and d were taken when the droplet touches the surface on the slope of the central bump of the wave generated by the previous collisions. The droplet diameter is approximately 0.7 mm.**

▼ **FIG. 2: Two orbiting walkers. The picture was taken when the two droplets touch the surface. Droplets diameter: 0.7 mm; orbit diameter: 3.7 mm.**

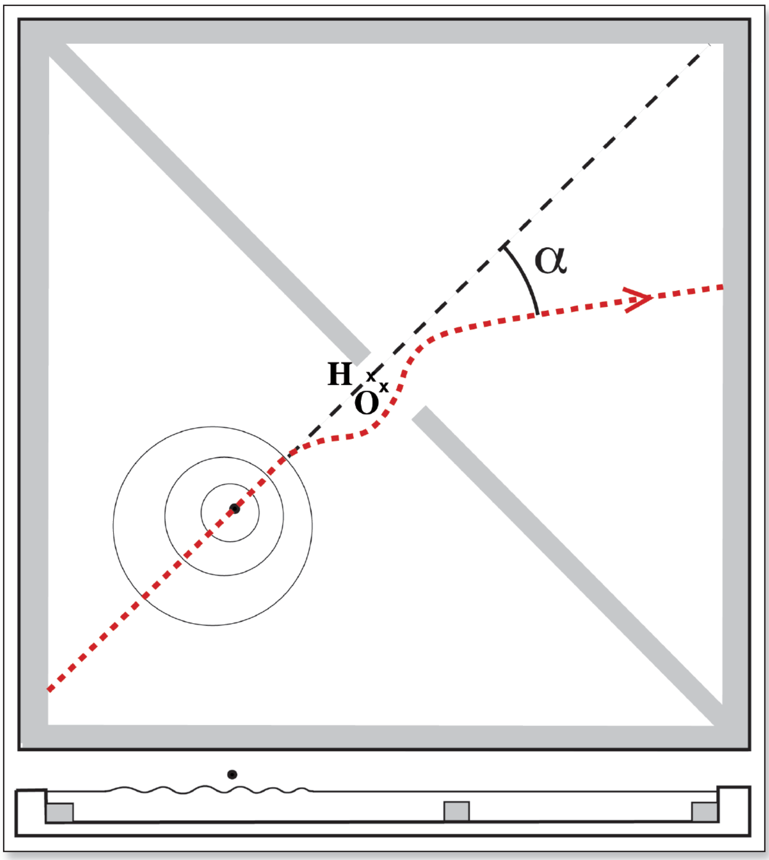


the way a walker reflects from a wall, the interaction between several walkers, and finally the diffraction and interference of single walkers passing through slits.

Reflection from a wall

Walkers are observed to be reflected by the boundaries of the bath somewhat like billiard balls. Here, however, the trajectory has no sharp reflection point; the deflexion is gradual and the droplet follows a curved trajectory, the distance of closest approach being on the order of the Faraday wavelength. Remember that the droplet moves because it bounces off the slanted surface of its own wave. However, close to a boundary, the local slope of the interface results from the superposition of the waves recently emitted by the droplet with earlier waves that have been reflected by the wall. This effect provokes a slight change of direction at each rebound and gradually bends the trajectory of the drop. This is an evidence of an "echolocation" of the walkers. The waves emitted earlier and having propagated faster than the walker itself, come back towards the droplet carrying information on the geometry of the borders. The walker avoids the nearing obstacle as a dolphin or a bat would do, even though it has no brain to process the signal.

▼ **FIG. 3: Experimental set-up for the study of walkers diffraction as seen from top and in cross section. The sketch also shows the droplet trajectory (red dotted line). *O* is the centre of the slit and *H* the point where the droplet would have crossed the slit if its trajectory had remained straight (dashed line). The impact parameter *Y* is the distance *OH* and *L* the width of the slit $-L/2 \leq Y \leq L/2$.**



Interaction of two walkers

When several walkers coexist on the same bath, they are observed to have long-distance interactions [3, 4] due to the interference of their waves. In order to characterize this interaction, we organized controlled "collisions" between two identical walkers moving initially towards each other along straight parallel trajectories. The distance d between these lines defines, as usual, the impact parameter of the collision (d is larger than the droplet size so that there is never real contact between the drops).

We find that, depending on the value of d, the interaction is either repulsive or attractive. When repulsive, the drops follow two approximately hyperbolic trajectories. When attractive, there is usually a mutual capture of the two walkers into an orbital motion similar to that of twin stars (fig.2). Allowed orbits have diameters d_n^{orb} quantified by the Faraday wavelength as $d_n^{orb} = (n\text{-}\varepsilon)\lambda_F$ with $n = 1, 2, 3$…if the droplets bounce in phase, and $n = 1/2, 3/2, 5/2$… if they bounce in opposite phases. The shift ε is a constant: $\varepsilon = 0.2$.

In all cases of interaction, the local slope on which one of the droplets bounces results from the superposition of its own wave with the wave generated by the other. The speed of each droplet in its orbit is mostly determined by the interaction with its own wave. Its orbital motion results from the interaction with the other walker and requires an effective attractive force. The observed diameters d_n^{orb} correspond to those distances where, at each collision with the surface, each droplet falls on the internal edge of the circular wave produced by the other (Fig.2). These collisions give each droplet a kick directed towards the other. Repeated collisions thus provide the centripetal force needed for the orbital motion.

Diffraction and interferences of single walkers

The previous experiments have shown that a walker is the association of a localized massive object with a wave that governs its interaction with the external world (cell boundaries, obstacles, other walkers etc…). How can this "particle" and this wave have common dynamics? In particular, what is the trajectory of the droplet when its associated wave is diffracted [7]? As explained above, a linear strip of metal stuck to the bottom of the cell, thus reducing the liquid layer thickness, creates for the walker the equivalent of a wall (Fig.3). An opening of width L in the middle of the strip will be a narrow passage for a walker. As it passes through, its associated wave will be diffracted. Successive images of a walker passing a slit at normal incidence are shown in Fig.4. Is the observed deviation deterministic? In such a case it should depend on the position Y at which the droplet passes the slit. Measuring the deviation angle α far from the slit as a function of Y shows random results. The same walker, crossing the slit at the same place, may deviate toward one side or the other (Fig.5a). Repeating the experiment with the same walker crossing the slit several times, it appears that some values of the deviation angle are more frequent than others. To make this observation quantitative, a histogram of α is shown in Fig.5b for an approximately uniform distribution of the impact parameter Y. Most walkers are in the central lobe of angular width λ_F/L and surrounded by secondary peaks. The envelope of the histogram looks like the amplitude of a plane wave of wavelength λ_F when diffracted by a slit of width L (Fig.5b).

The analogy with Young's double slit experiments can then be investigated by creating two neighbouring slits through the wall. At each crossing the droplet is observed to pass through either one of the slits. However, its associated wave passes through both slits, thus generating interferences that guide the particle. The observed deviation histogram looks like the interference diagram of a plane wave of wavelength λ_F, propagating through two slits of width L at a distance h.

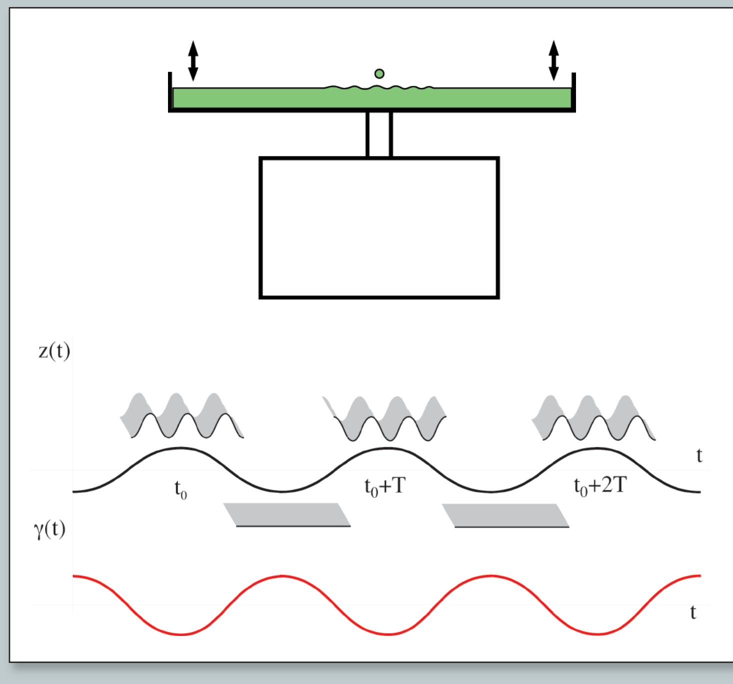
Using a single walker, diffraction and interference patterns are thus obtained here in a cumulative histogram of independent events. Among the experiments that demonstrated the founding paradoxes of quantum mechanics, an essential one concerns the interference with a very weak flux of particles. The first one [8] is due to G.I. Taylor in 1909. It showed that, after a sufficient exposure time, an interference pattern was observed behind Young's slits even when only one single photon was present in the system at a time. The pattern was the result of a statistical behaviour of the photons. The same result was then observed for electrons [9]. The Feynman lectures [1] present a thorough discussion of this effect and of its quantum interpretation in which the probability wave of each particle crosses the two slits at the same time. Surprisingly, we recover a similar behaviour in our system.

To conclude, it is useful as a *caveat* to remember the differences between the present experiments and those on the usual wave-particle duality at the quantum scale.

- The most evident is that, at the macroscopic scale, the Planck constant does not show up.
- The present system is highly dissipative while the quantum situation is non-dissipative.
- The system is two-dimensional.
- The wave is emitted by the particle and propagates at a fixed velocity on a material medium.
- The measurements made on the walkers do not perturb their behaviour. So, the position of the droplet can be observed anytime during interference experiments and the slit it crosses is known. However, this unperturbed observation could be impaired if the measurements had to be done using surface wave detectors only.

Experimental conditions and Faraday instability

In the present experiments a small cell containing a liquid is subjected to a vertical sinusoidal acceleration $\gamma = \gamma_m \cos\omega_0 t$. The liquid is a silicon oil 20 times more viscous than water. The oscillation frequency is 80 Hz and the acceleration amplitude γ_m between g and $5g$. The bouncing of the droplets, with diameters in the mm range ($0.5<D<1.5$ mm), is observed. For acceleration amplitudes $\gamma_m > g$, coalescence of the droplets with the surface no longer occurs. Beyond a threshold $\gamma_m^F = 4.5g$, the Faraday instability (10) is observed and the liquid surface becomes covered with a surface waves network.



It is known since the work of Faraday (10) in 1831 that a liquid surface becomes unstable when subjected to vertical oscillations of sufficient amplitude. In general, a network of stationary waves oscillating at half the forced oscillation frequency is observed. This effect results from what is called parametric forcing, the liquid being submitted to a modulated effective gravity.

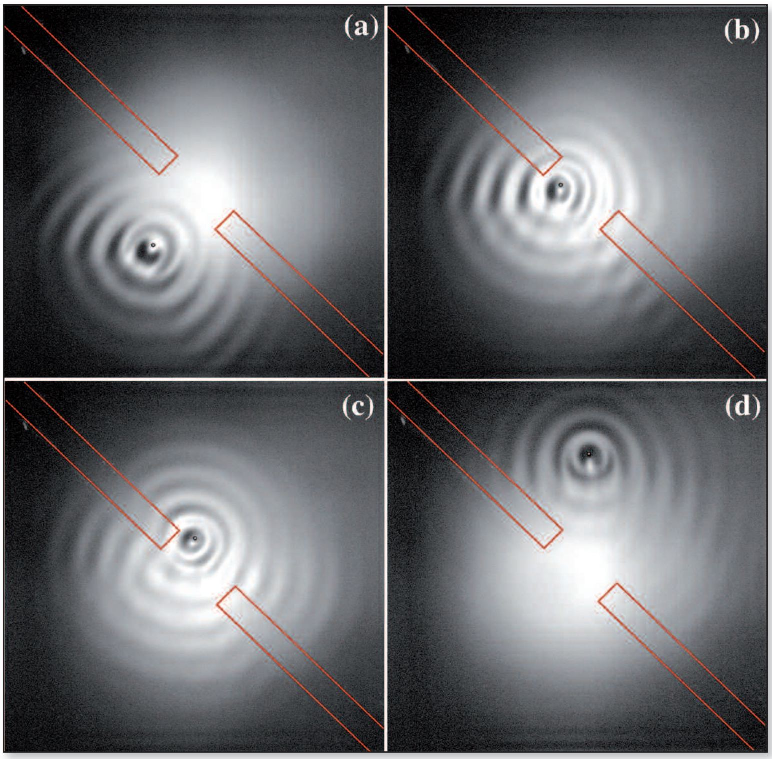
The drawing below shows the oscillation $z(t)$ of the vertical position of the cell and its acceleration $\gamma(t) = \gamma_m \cos 2\pi t/T$ (where T is the forced oscillation period). The sketches in grey show the physical aspect of the liquid surface at different times. At $t = t_0, t_0+T, t_0+2T,\ldots$ the substrate acceleration has the same sign as gravity, but is larger. At these times, some parts of the liquid surface lag behind and periodic crests form spontaneously. At t_0 and t_0+T, maxima and minima are exchanged. Inversely, at $t = t_0+T/2$, $t_0+3T/2$, …, the substrate acceleration opposes gravity and flattens the surface. The interface deformation is therefore periodic and its period is twice the forcing one.

The vertical motion of the droplets is also forced by the substrate acceleration $\gamma(t)$. The droplet leaves the surface at each oscillation when the downward acceleration of the substrate becomes larger than g. For γ_m values slightly larger than g, the droplet has a short free flight at each period. Nearing the Faraday threshold, $\gamma_m < \gamma_m^F$, the droplets have a free flight of such amplitude that they touch the surface only once in two oscillations. Their motion then has the same period $2T$ as the Faraday waves. Since this type of bouncing occurs immediately below the Faraday instability threshold, the droplets become localized emitters of weakly damped Faraday waves and become "walkers".

Keeping these differences in mind, we can now consider the similarities.

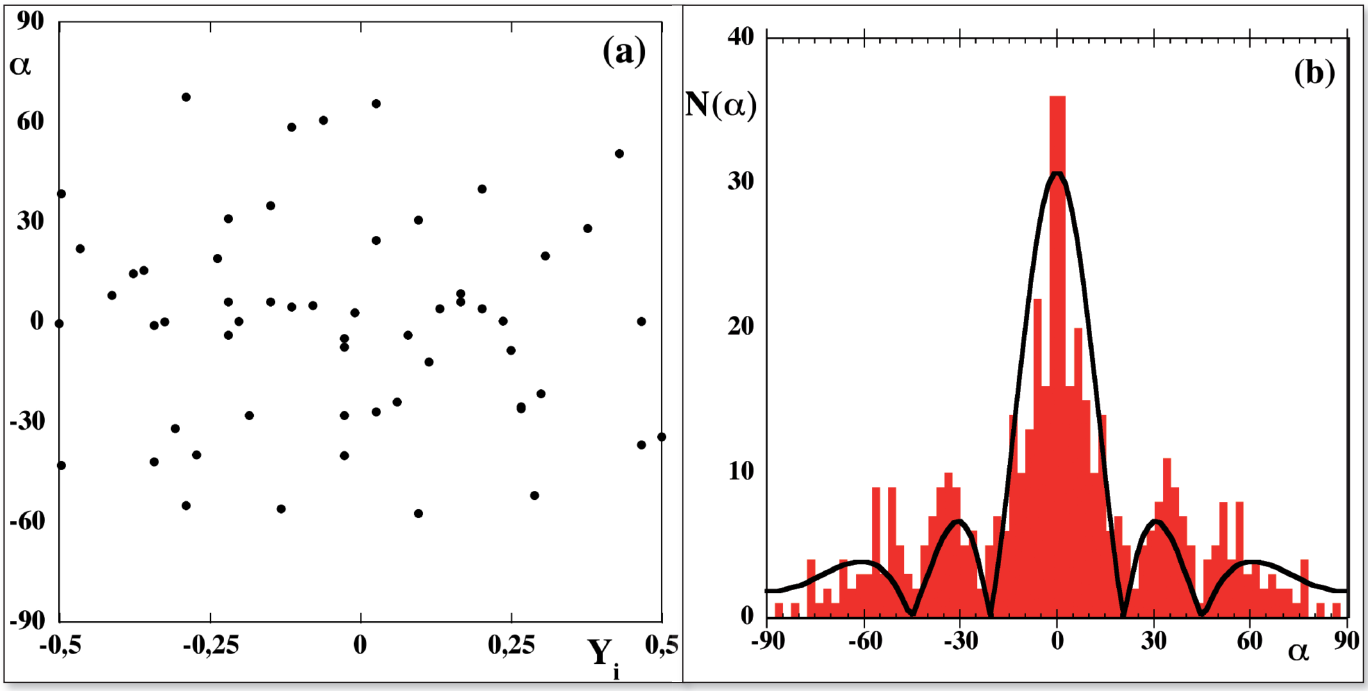
- The wave-induced interactions have been shown to impose quantified diameters to the stationary orbits.
- The diffraction and interference experiments have shown that the transverse momentum of the walker becomes ill-defined when the transverse amplitude of its wave is limited. The diffraction, which is intrinsically bound to the Fourier transform of the wave, is transformed here into an uncertainty of the motion of the droplet.
- Recent experiments (10), not included in this paper, have shown that walkers have a non-zero probability of crossing opaque barriers, a phenomenon reminiscent of the quantum tunnel effect.

All observed phenomena are related to the particle-wave interaction. The droplet moves in a medium modified by previously generated waves. As the droplet moves, the points of the interface it visits keep emitting waves. The wavefield has thus a complex structure that contains a "memory" of the path. More work is needed to fully understand this type of spatial and temporal non-locality. ■



▲ **FIG. 4:** Four successive pictures, a to d, of a walker moving through a slit.

▼ **FIG. 5:** (a) Measured values of individual deviations α as a function of impact parameter *Y*. The Faraday wavelength is here $\lambda_F = 4.75$ mm and the slit width L=14.7 mm. b) Histogram of measured deviations for N=125 successive crossings of single walkers through a slit in the same conditions as in (a). The system being symmetrical with respect to the slit axis, the statistics are improved by adding to each trajectory its symmetric counterpart. The shape of the histogram looks like the diffraction amplitude of a plane wave of the same wavelength crossing that slit (aperture ratio L/λ_F =3.1).



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