AGH University of Science and Technology

Essential Thinking.
Introduction to Constraint Programming
Example Problems IV

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AGH'2011
Kraków, Poland

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3. What is CSP?
4. Backtracking Search and Heuristics
5. Exploring Problem Structure
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References

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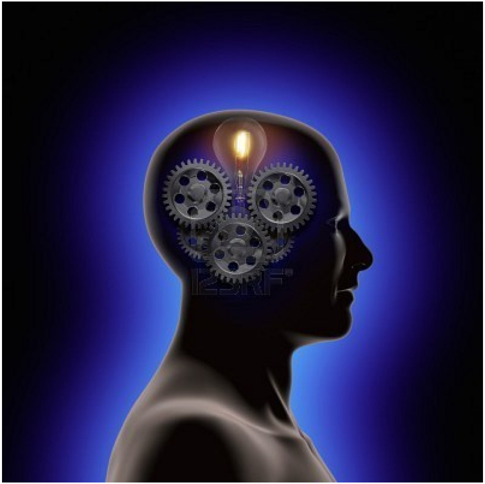
What is worth learning?

A bit provocative position statement

- Languages — enable communication and knowledge representation; Wieviel Sprachen du sprichst, sooftmal bist du Mensch; Goethe
- Problem Solving — analytical thinking; cross-curricular competencies,
- Learning — persistent learning, quick learning, focused learning, learning on-demand, ...
- Tools — finding and using proper tools for specific problems.
  - conceptual tools,
  - software tools.

Thinking — What is the Essence of it?



An Inspiration Example

A cryptoarithemtic problem

```
   SEND
+  MORE
------
  MONEY
```

An Intuitive Introduction

```
?- [library(clpfd)].

?- X #> 3.
X in 4..sup.

?- X #\= 20.
X in inf..19\/21..sup.

?- 2*X #= 10.
X = 5.

?- X*X #= 144.
X in -12\/12.

?- 4*X + 2*Y #= 24, X + Y #= 9, [X,Y] ins 0..sup.
X = 3, Y = 6.

?- Vs = [X,Y,Z], Vs ins 1..3, all_different(Vs), X = 1, Y #\= 2.
Vs = [1, 3, 2], X = 1, Y = 3, Z = 2.
```

Basic Constraints

clp(fd) — intro

```
?- [library(clpfd)].

Expr1 #>= Expr2 % Expr1 is larger than or equal to Expr2

Expr1 #=< Expr2 % Expr1 is smaller than or equal to Expr2

Expr1 #= Expr2 % Expr1 equals Expr2

Expr1 #\= Expr2 % Expr1 is not equal to Expr2

Expr1 #> Expr2 % Expr1 is strictly larger than Expr2

Expr1 #< Expr2 % Expr1 is strictly smaller than Expr2
```

Basic Constraints

clp(fd) — intro

```
?- [library(clpfd)].

#\ Q       % True iff Q is false

P #\/ Q      % True iff either P or Q

P #/\ Q      % True iff both P and Q

P #<==> Q    % True iff P and Q are equivalent

P #==> Q     % True iff P implies Q

P #<== Q     % True iff Q implies P
```

Basic Constraints

clp(fd) — intro

```
?Var in Domain

?Vars ins Domain

 Var is an element of Domain. Domain is one of:

   Integer
        Singleton set consisting only of Integer.
   Lower .. Upper
        All integers I such that Lower =< I =< Upper.
        Lower must be an integer or the atom inf, which den
        Upper must be an integer or the atom sup, which den
   Domain1 \/ Domain2
        The union of Domain1 and Domain2.
```

Basic Constraints

clp(fd) — intro

`label(+Vars)`

`labeling(+Options, +Vars)`

```
Labeling means systematically trying out values for
the finite domain variables Vars until all of them are grou
The domain of each variable in Vars must be finite.
Options is a list of options that let you exhibit
some control over the search process.
```

clp(fd) — intro

`indomain(+Var)`

```
Bind Var to all feasible values of its domain on backtracki
The domain of Var must be finite.
```

Basic Constraints

clp(fd) — intro

```
all_different(+Vars)

all_distinc(+Vars)

Vars are pairwise distinct.

The second command has stronger propagation
(can detect inconsistency).
```

Basic Constraints

clp(fd) — intro

```
sum(+Vars, +Rel, +Expr)
```

The sum of elements of the list Vars is in relation Rel to Expr, where Rel is #=, #\=, #<, #>, #=< or #>=. For example:

```
?- [A,B,C] ins 0..sup, sum([A,B,C], #=, 100).
A in 0..100,
A+B+C#=100,
B in 0..100,
C in 0..100.
```

The SEND+MORE=MONEY Example

Code

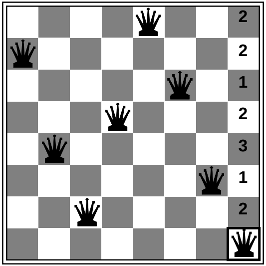
```
:- use_module(library(clpfd)).

puzzle([S,E,N,D] + [M,O,R,E] = [M,O,N,E,Y]) :-
        Vars = [S,E,N,D,M,O,R,Y],
        Vars ins 0..9,
        all_different(Vars),
                  S*1000 + E*100 + N*10 + D +
                  M*1000 + O*100 + R*10 + E #=
        M*10000 + O*1000 + N*100 + E*10 + Y,
        M #\= 0, S #\= 0.

?- puzzle(As+Bs=Cs), label(As).
As = [9, 5, 6, 7],
Bs = [1, 0, 8, 5],
Cs = [1, 0, 6, 5, 2] ;
false.
```

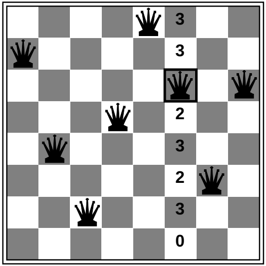
Bidirectional Factorial

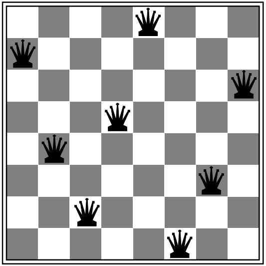
Code

```
:- use_module(library(clpfd)).

n_factorial(0, 1).
n_factorial(N, F) :- N #> 0, N1 #= N - 1, F #= N * F1, n_factorial

?- n_factorial(47, F).
F = 258623241511168180642964355153611979969197632389120000000000 ;
false.

?- n_factorial(N, 1).
N = 0 ;
N = 1 ;
false.

?- n_factorial(N, 3).
false.
```

N-Queens







Examples

A cryptoarithemtic problem

```
   SEND
+  MORE
------
  MONEY
```

Examples

A cryptoarithemtic problem

```
   DONALD
+  GERALD
  ------
   ROBERT
```

Examples

A cryptoarithemtic problem

```
      CROSS
+     ROADS
   ------
     DANGER
```

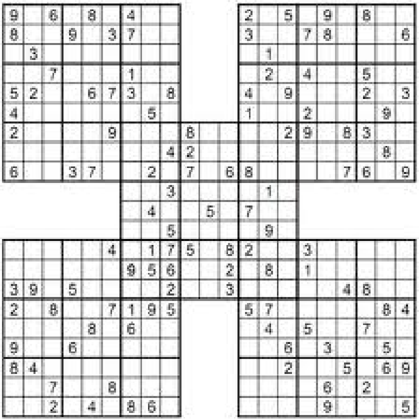
Enoxamples

A cryptoarithemtic problem

```
     TEN
+    TEN
   FORTY
 ------
   SIXTY
```

A Generic Problem Example

A Generic Problem Example



Another Example: The Zebra Puzzle

a) Norweg zamieszkuje pierwszy dom;
b) Anglik mieszka w czerwonym domu;
c) Zielony dom znajduje się po lewej stronie domu białego;
d) Duńczyk pija herbatkę;
e) Palacz Rothmansów mieszka obok hodowcy kotów;
f) Mieszkaniec żółtego domu pali Dunhille;
g) Niemiec pali Marlboro;
h) Mieszkaniec środkowego domu pija mleko;
i) Palacz Rothmansów ma sąsiada, który pija wodę;
j) Palacz Pall Malli hoduje ptaki;
k) Szwed hoduje psy;
l) Norweg mieszka obok niebieskiego domu;
m) Hodowca koni mieszka obok żółtego domu;
n) Palacz Philip Morris pija piwo;
o) W zielonym domu pija się kawę.