# FLUIDIZATION

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## Chapter 1

# Introduction to fluidization

### 1.1 Goal of this course

The aim of this course is to provide basic knowledge about fluidization and aerodynamics of gas-solid systems as well as mathematical tools that enable to simulate basic fluidized systems.

## 1.2 The phenomenon of fluidization

To simply decribe the idea behind fludization process one can say that it is the operation that can change a system a solid particles into fluidlike suspension in gas or liquid. This method of contacting of this two phase mixture have some unusual characteristics that are widely used in many fields of chemical industry. Simplified diagram showing the idea of fluidization is presented in Fig. 1.1. Gas is delivered from the bottom of the reactor, goes through a gas distributor to provide inform distribution through whole profile of bed and flows through packed bed of solids.

At low gas velocities the drag force is to small to lift the bed, which remains fixed. Increasing gas velocity causes solids to move upward and create fluid bed. Depending on the velocity of gas we can distinguish different modes of fluidization (Fig. 1.2) from bubbling fluidization, through turbulent and fast fluidization modes up to pneumatic transport of solids.

Another important issue concerning the fluidization proces is pressure drop through a fixed bed. Fig. 1.3 presents changes in pressure drop with changing gas velocity. At first one can observe increasing pressure drop, up to some level where it becomes constant, despite increasing gas velocity. This change in pressure drop trend can be connected



FIGURE 1.1: Schematic diagram of fluidization

with the creation of dense phase of fluidized bed and that is the moment when the fluidization occurs. The velocity at which the pressure is stabilized is called minimum fludization velocity. Pressure drop is stable in a certain range of velocities, then a slight increase can be observed which precede a drastic decrease in pressure drop. This is due to the entrainment of smaller particles which are suspended in the section over the dense fluidized bed. Further increase in gas velocity will cause more fractions to be carried over which leads to disappearance of dense phase and start of pneumatic transport. Although, as it will be shown later in some cases gas velocities exceeding the terminal velocity can be applied for the so called fast fluidization.



FIGURE 1.2: Fluidization type depending on gas velocity

FIGURE 1.3: Pressure drop vs. gas velocity.



## 1.3 Discussion points

- 1. Definition of fluidization phenomenon.
- 2. Various types of beds with gas flowing through a bed of fine particles.
- 3. Circulating fluid bed characteristics.
- 4. Liquidlike features of fluid beds
- 5. Advantages and disadvantages of fluidized beds.
- 6. Heat transport phenomenon in fluidized bed.

- 7. Drying process in fluid bed basic process configuration.
- 8. Catalytic cracking in fluid bed.
- 9. Fluid bed boilers for energy generation.
- 10. Fluid bed pyrolysis and gasification.

## Chapter 2

# Fluidization velocities

### 2.1 Characterization of particles

Usually the bed contains particles with a wide range of size and shapes, which causes the necessity to provide a proper and uniform description of size of material forming a bed. If the particles are spherical the bed can be described by means of their diameter distribution, but in real application most particles are nonspherical which yields a question about the way to decribe this kind of beds. There exists a wide range of nonsphericity measures [?]. However, the most wildly used is the one called sphericity ( $\phi_s$ ) defined as the ratio of the surface of sphere to the surface of particle with the same volume. For spherical particles can be found in [?]. Other important parameter describing nonspherical particles is their specific surface, given as the ratio between surface and volume of the particle:

$$a' = \frac{6}{\phi_s d_{sph}} \tag{2.1}$$

where  $d_{sph}$  is a diameter of the sphere having the same volume as the considered particle. The same concept can be applied to the whole bed of particles:

$$a = \frac{6(1 - \epsilon_m)}{\phi_s d_{sph}} \tag{2.2}$$

where  $\epsilon_m$  is the fractional voidage, which usually can be found experimentally for each specific system.

### 2.2 Particle size distribution

Variety in shapes of particles is not an only problem in describing the bed, because in most cases one also has a bed of particles with different sizes. For this description we can define two functions of size distribution  $\mathbf{p}$  and  $\mathbf{P}$ . Assuming that we have a bed of solids with diameters  $dp_i$ , for  $i \in (1, 2, ..., N)$  then  $\mathbf{p}$  gives the fraction (mass, volume, number) of particles that are of the diameter  $d \in (dp_1, dp_{i+1})$ . The function  $\mathbf{P}$  gives the so called cumulative distribution, which means the fraction of solids that are smaller than the given value dp. Examples of such distributions are shown in Fig.2.1



FIGURE 2.1: Difference between  $\mathbf{P}$  (left) and  $\mathbf{p}$  (right)

Next issue is to provide an average size than can best describe properties of the system and can be used in further calculations. This is done by harmonic diameter:

$$\overline{dp} = \frac{1}{\sum_{i=1}^{N} \frac{x_i}{dp_i}}$$
(2.3)

where  $x_i$  is a fraction of solids with diameter  $(dp_i, dp_{i+1} \text{ and } dp_i = \frac{dp_i + dp_{i+1}}{2}$ . Then mean specific surface can be obtained using equation (2.1):

$$\overline{a'} = \frac{6}{\phi_s \overline{dp}} \tag{2.4}$$

#### 2.3 Fluidization velocities

First step in the process of description of fluidized bed is to calculate the velocity of gas needed in the system. We can distinguish two basic velocities describing fluidization: minimum and terminal. In this section we present the procedure used to calculate both of them. All calculations can be performed for a bed with single or multi-size particles (in case of multi-size bed it is necessary to calculate its mean diameter).

#### 2.3.1 Pressure drop

Pressure drop through fixed bed of solids of uniform size (dp) of the length L is given by Ergun [XX] correlation:

$$\frac{\Delta P}{L} = 150 \frac{(1 - \epsilon_m)^2}{\epsilon_m^3} \frac{\mu u_0}{(\phi_s d_p)^2} + 1.75 \frac{1 - \epsilon_m}{\epsilon_m^3} \frac{\rho_g u_o^2}{\phi_s dp}$$
(2.5)

where  $\mu$  is gas viscosity, dp is solid diameter,  $\rho_g$  is gas density,  $u_0$  is superficial gas velocity.

#### 2.3.2 Minimum fluidizing velocity

At the beginning of this section one has to revise definitions of two dimensionless numbers: Reynolds(2.6) and Archimedes (2.7)

$$Re = \frac{d_p u_{mf} \rho_g}{\mu} \tag{2.6}$$

$$Ar = \frac{d_p^3 \rho_g (\rho_s - \rho_g)g}{\mu^2} \tag{2.7}$$

Now, remembering that the phenomenon of fludization occurs when drag force created by the upward flow of gas is a least equal to the weight of particles in the bed. Mathematically it can be presented with the following equation:

$$\Delta P_{bed}A_t = A_t L_{mf} (1 - \epsilon_{mf}) [(\rho_s - \rho_g)g]$$
(2.8)

Rearranging and combining with equation (2.5) gives a quadratic in  $u_m f$  which can be presented in dimensionless form of the equation (2.9) (for the details see "Problem solving", ex. 3)

$$\frac{1.75}{\epsilon_{mf}^3 \phi_s} R e_{p,mf}^2 + \frac{150(1 - \epsilon_{mf})}{\epsilon_{mf}^3 \phi_s^2} R e_{p,mf} = Ar$$
(2.9)

Solving equation (2.9) can be laborious but gives reliable estimation of  $u_{mf}$  if sphericity and voidage are known. For rough estimation without knowledge of voidage and sphericity of the system some simplifications can be used. For fine particles expression (2.10) proposed by Wen and Yu [xx] can be used to obtain reynolds number in minimum fludization conditions:

$$Re_{p,mf} = (33.7^2 + 0.0494Ar)^{1/2} - 33.7$$
(2.10)

#### 2.3.3 Terminal fluidization velocity

Terminal fluidization velocity can be calculated from the equation given in the dimentionless form (2.11)

$$C_D R e_t^2 = \frac{4}{3} A r \tag{2.11}$$

where  $C_D$  is drag coefficient, which can be obtained experimentally or calculated from one of many empirical relationships,  $C_D$  is a function of Reynolds number. One of the correlation that enables to calculate drag coefficient for a wide range of Reynolds numbers  $(10^{-1} \div 10^6)$  was proposed by Kaskas [xx] (2.12

$$C_D(Re) = \frac{24}{Re} + \frac{4}{\sqrt{Re}} + 0.4 \tag{2.12}$$

Substituting (2.12) to (2.11) we obtain equation (2.13). Solving numerically for  $Re_t$  we can obtain terminal velocity for the system.

$$Re^{2}\left(\frac{24}{Re} + \frac{4}{\sqrt{Re}} + 0.4\right) = \frac{4}{3}Ar$$
(2.13)

To avoid numerical calculations of equation (2.13) some simplification can be used, depending on the type of low in the reactor. For laminar flow we obtain the only analytical solution to the equation (2.13)

$$C_D = \frac{24}{Re} \text{ for } Re < 0.4 \tag{2.14}$$

For larger Reynolds numbers we can use one of the following approximations:

$$Cd = \frac{10}{\sqrt{Re}} \text{ for } 0.4 < Re < 500$$
 (2.15)

$$Cd = 0.43 \text{ for } 500 < Re < 2 * 10^5$$
 (2.16)

An example of minimum fluidization velocity and terminal velocity as a function of particle diameter is presented in Fig. 2.2



FIGURE 2.2: Minimum (red) and terminal (blue) velocity as a function of particle diameter.

## 2.4 Gas distributor design

The design of fluid bed gas distributors may have a marked influence on the performance of a fluid bed reactor. The primary physical reason for this influence is that the distributor design influences the hydrodynamics and thus the gas/solid contacting pattern in the fluidized bed. Particle and gas properties play a key role in successful design together with the critical pressure drop ratio, and hole size, geometry and spacing; these strongly influence jet penetration, dead zones, particle sifting, attrition and mixing. [Geldart, 1985; Bauer, 1981].

This section deals with the simple algorithm that enables to design a perforated plate distributor using just an orifice theory.

1. Calculate pressure drop across the distributor (??):

$$\Delta p_d = (0.2 \div 0.4) \Delta p_b \tag{2.17}$$

where  $\Delta p_b$  can be calculated form (2.5) or (2.8).

2. Calculate the vessel Reynolds number  $(Re_t = \frac{d_t u_0 \rho_g}{\mu})$ , where  $d_t$  is tube (reactor) diameter) and select the corresponding drag coefficient from the table below

$Re_t$	100	300	500	1000	2000	> 3000
$C_{D,or}$	0.68	0.70	0.68	0.64	0.61	0.60

3. Calculate gas velocity through the orifice

$$u_{or} = C_{D,or} * \left(\frac{2\Delta p_d}{\rho_g}\right)^{1/2}$$
(2.18)

Check the ratio  $\frac{u_0}{u_{or}}$  which gives the fraction of open area in the distributor and should be less than 10%.

4. Assume orifice diameter ( $d_{or}$  and calculate the number of orifices per unit area of distributor using equation (2.19)

$$u_o = -\frac{\pi}{4} d_{or}^2 u_{or} N_{or}.$$
 (2.19)

## 2.5 Discussion and problem solving

- 1. Dimensionless numbers define Archimedes and Reynolds number.
- 2. Describe the procedure of calculating minimum and terminal velocity for a polydispersed system.
- 3. Prove, that starting from combined equations (2.5) and (2.8) and using following assumptions one can obtain equation (2.10).

$$\frac{1}{\phi_s\epsilon_{mf}^3}=14~\frac{1-\epsilon_{mf}}{\phi_s^2\epsilon_{mf}^3}=11$$

4. Prove that in steady state condition equation

$$U_s \frac{dU_s}{dz} = \frac{3}{4} C_D \frac{\rho_g}{\rho_s d_p} (U_g - U_s)^2 - g \frac{\rho_s - \rho_g}{\rho_s} \leftrightarrow (2.11).$$
(2.20)

- 5. Calculate mean diameter for the system of particles presented below
  - (a)

(b)

- 6. Calculate minimum fluidization velocity for presented system. Perform the calculation on mean diameter. Check if applying calculated velocity wile cause any fraction to be carried over?
- 7. Design gas distributor for the given system.
- 8. Describe Geldart's powder classification.

## Chapter 3

## Bubbling fluidized bed

## 3.1 Bubbles in fluidized bed

Knowledge of the general behavior of a fluidized bed is insufficient for some purposes, for example reaction kinetics and heat transfer depend on details of the gas-solids interaction in the bed. Hence, a satisfactory treatment of these phenomena requires a reasonable model representing the gas flow through the bed and its interaction with bed material. As a consequence, the bubble size, rise velocity, shape, distribution, frequency and flow patterns are of key interest. As it was presented in chapter one, increasing the velocity of gas flowing through a bed o solids causes changes fluidization mode (see Fig. 1.2) At relatively low gas velocities we can observe a so called dense bubbling fluidized bed, which is characterized by the presence of regions with low solid concentration which are called bubbles. The dense phase, with higher solid concentration is called emulsion.

#### 3.1.1 Bubble formation

#### The following calculations are presented in CGS unit system!

Initial diameter of a bubble formed directly above the gas distributor can be calculated form equation (3.1). Note that this equation is true for a gas flowing with higher velocities, causing the bubbles to overlap when formed  $(d_{b0} < l_{or})$ .

$$d_{b0} = \frac{2.78}{g} (U_0 - U_{mf})^2 \tag{3.1}$$

Bubbles moving upward change their size (grow with height over the distributor). To describe the size of bubbles on the given height of bed we can use to different correlations proposed by Mori and Wen (3.3) or Werther (3.4). Using Mori-Wen model also requires

calculating of the bubble's maximum diameter (3.2) which occurs at the end of dense part of fluid bed.

Mori-Wen model:

$$d_{bmax} = 0.65 \left[\frac{\pi}{4} D_t^2 (U_0 - U_{mf})\right]^{0.4}$$
(3.2)

$$d_b(h) = d_{bmax} - (d_{bmax} - d_{b0})exp(-0.3\frac{h}{D_t})$$
(3.3)

Werther model

$$d_b(h) = 0.853[1 + 0.272(U_0 - U_{mf})]^{0.333}(1 + 0.0684h)^{1.21}$$
(3.4)

FIGURE 3.1: Changes of bubble's diameter[cm] with height of bed [cm] according to Mori-Wen (blue) and Werther (red)



Finally we can calculate the velocity of a single bubble flowing upward:

$$U_{br}(h) = 0.711 [g * d_b(h)]^{0.5}$$
(3.5)

where  $d_b(h)$  is bubble velocity calculated according to Werther.

### 3.2 Bubbling fluidization

As it was previously mentioned bubbling bed must be treated as a two phase system, with solids in dense phase and gas bubbles in lean phase. From previous paragraph we already know how to asses change of bubbles size in bed and the following part deals with the problem of two phase approach to a bubbling fluid bed. One should remember that bubbles contain very small amounts of solids and are not necessarily spherical. The schematic figure showing elements of such system can be seen in Fig. 3.2.

FIGURE 3.2: Schematic bubble in bubbling bed



As can be seen the bubbles are approximately hemispherical, with pushed-in bottom. The part directly under the bubble is called a wake, containing significant amount of solids. Moreover every bubble is surrounded by cloud - a part of the emulsion that was penetrated by gas from a rising bubble. Concentration of solid in the cloud is higher than that inside the bubble, but lower than the one in emulsion.

#### 3.2.1 Kuni-Levenspiel model

Kuni-Levenspiel model (later simply called K-L) is based on following assumptions: [http://www.umich.edu/ elements/12chap/html/FluidizedBed.pdf, page 9]

- 1. All bubbles are of the same size.
- 2. The solids forming emulsion phase flow downward.
- 3. Emulsion phase exists at minimum fludizing velocity. The gas occupies the same void fraction in this phase as it had in the entire bed at the minimum fluidization

point. Minimum fluidizing velocity refers to the gas velocity relative to moving solids.

- 4. In the wake, concentration of solid is said to be the same as in the emulsion phase. However, the wake is turbulent and the average velocities of solids and gas are equal to the upward velocity of a rising bubble.
- Fig. 3.3 shows the KL model with its assumptions.

FIGURE 3.3: K-L bed model



Following algorithm of calculation **K-L model** will use Werther model to obtain size of of bubbles.

1. Calculate bubbles velocity based on single bubble velocity (3.5:

$$U_b(h) = U_0 - U_{mf} + U_{br}(h)$$
(3.6)

2. Calculate average diameter of bubbles (3.7), using mean value theorem for integration on function  $d_b(h)$  (3.4) and average velocity of bubbles (3.8). In this point we have to assume some value of  $L_f$  (height of bed).

$$d_{bs}(h) = \frac{1}{L_f} \int_0^{L_f} d_b(h) dh$$
 (3.7)

$$U_{bs}(h) = U_0 - U_{mf} + 0.711 [g * d_{bs}(h)]^{0.5}$$
(3.8)



FIGURE 3.4: Wake volume to bubble volume (Kuni, Levenspiel; 1991

3. The downflow velocity of solids, can be calculated based on material balance of solid particles present in the system.

Total solids = Solids flowing downward in emulsion + solids flowing upward in wakes

$$u_s == \frac{f_w \delta U_b}{1 - \delta - f_w \delta} \tag{3.9}$$

where  $f_w$  is the ratio of wake volume to bubble volume and can be found from the Fig. 3.4

4. Velocity of gas in the emulsion phase comes from the material balance of gas:
Total gas = Gas in bubbles + gas in wakes + gas in emulsion

$$U_e = \frac{U_{mf}}{\varepsilon_{mf}} - U_s \tag{3.10}$$

5. Volume fraction of bubbles in bed

$$\delta = \frac{U_0 - U_{mf}}{U_{bs} - Umf} \tag{3.11}$$

6. Calculate porosities:

- (a) in emulsion phase is assumed to be constant  $\varepsilon_e = \varepsilon_{mf}$
- (b) average bed porosity:

$$\varepsilon_f = \delta + (1 - \delta)\varepsilon_e \tag{3.12}$$

7. Calculate height of bed (checkpoint if the assumption in (3.7)) was correct.

$$L_f = L_{mf} \frac{1 - \varepsilon_{mf}}{1 - \varepsilon_f} \tag{3.13}$$

#### 3.2.2 Extended K-L model

1. Volume fraction of clouds in bed

$$f_c = \frac{3}{U_{brs}\frac{\varepsilon_{mf}}{U_{mf}} - 1} \tag{3.14}$$

where  $U_{brs}$  is a velocity of single bubble (3.5) calculated for average bubble diameter (3.7)

- 2. Volume fraction of wake is assumed to be constant  $f_w = 0.33$ .
- 3. Volume fraction of emulsion

$$f_e = 1 - \delta - f_w \delta - f_c \tag{3.15}$$

- 4. Fraction of solids in bubbles was specified experimentally  $\gamma_b = 0.005$
- 5. Fraction of solids in clouds and wakes

$$\gamma_c = (1 - \varepsilon_{mf})(f_c + f_w) \tag{3.16}$$

6. Fraction of solids in emulsion

$$\gamma_e = \frac{1 - \varepsilon_{mf}(1 - \delta)}{\delta} - \gamma_b - \gamma_c \tag{3.17}$$

- 7. Wake velocity is constant and equal to the velocity of bubbles (3.8  $U_w = U_{bs}$ .
- 8. Emulsion downflow velocity

$$U_e = \frac{f_w \delta U_{bs}}{1 - \delta - f_w \delta} \tag{3.18}$$

9. Relative gas velocity in emulsion

$$U_{ge} = \frac{U_{mf}}{\varepsilon_{mf}} - U_e \tag{3.19}$$

#### 3.3 Entrainment and elutriation

Fluidized reactor can be divided into two parts, the bottom one called the dense phase which was described in previous sections and dispersed phase, where the concentration of solid decreases. We showed that by using equation (3.13) we can find the height of fluidized bed or to be more specific its dense part. That's were more or less distinct border between the two phases occurs and the bubbles present in the dense phase disappear. The "disappearance" is a reason for the presence of the lean phase in the reactor. This is shown in Fig. 3.5. Spraying of solids into lean phase can have three different mechanisms (Kuni, Levenspiel):

- bubbles have higher pressure than the surface of bed, so by reaching the top of dense phase they spray solids form its roof into lean phase;
- reaching the surface, bubbles can explode, and then the arising forces cause the solids present in the wake to be sprayed to lean phase;
- two bubbles can coalesce at the surface and create energetic ejection of solids from under the bottom bubble.

The aim of this section is to provide some insight to what happens over the dense part of the bed. Let us first define number of terms necessary to understand the problem. The flux of solids suspended in gas over the dense phase is called an entrainment  $(G_s)$ . The zone of fluidization vessel above the border between the previously mentioned phases is called a freeboard. The region close to the border between the phases is called the splash zone and that is where the spraying of solids occurs. The entrainment of solids decreases with the increasing height of the freeboard until it reaches some constant level. The height at which it happens is called **TDH** - transport disengaging height. By saturation carrying capacity we understand the largest flux of solids that can entrained by gas above the TDH. Finally elutriation which refers to removal of fine particles from a mixture of solids with different sizes. Larger particles fall back to bed, because they are to heavy to be carried up, but smaller ones are flowing upward with the gas.

Below we present the algorithm that enables to describe the amounts of material in different zones of fluidization vessel.

1. We start with the assumption that the initial velocity of solids  $U_{bf}$  sprayed out of the dense phase of the bed is equal to the velocity of bubbles at this height. We use equation (3.6) with the previously calculated height  $L_f$ . Here one has to remember that all the velocities were calculated in **CGS unit system** and from now on we have to go back to the **SI units!** 



FIGURE 3.5: Mechanism of ejection of solids from dense bed

2. The flux of entrained solids  $\frac{kg}{m^2s}$  is calculated by equation:

$$G_s = 0.1\rho_s (1 - \varepsilon_{mf})U_{bf} - U_{mf} \tag{3.20}$$

3. Now we calculate saturation carrying capacity for gas present in the system.

$$E_{sat} = 0.096 U_0 \rho_g Frt(U_t)^{0.633} Ar^{0.121} (\frac{\rho_s}{\rho_g})^{0.013} (\frac{D_t}{D_0})^{-0.05}$$
(3.21)

where  $Fr_t(U_t)$  is given by (3.22) (Froude number),  $D_t$  is reactor diameter and  $D_0 = 5.9cm$  and it is reference diameter of experimental fludization vessel

$$Fr_t(U_t) = \frac{U_t^2}{g * d_p} \tag{3.22}$$

4. The distribution of solid flux is given by exponential function of height (3.23) and is presented in Fig. 3.6.

$$E(h) = E_{sat} + (G_s - E_{sat})exp(-ah)$$
(3.23)

where a = 4 is an experimentally obtained coefficient. As it can be observed in Fig. 3.6 freeboard zone is about 1m high, because that is where we start to observe constant flux of solids.

5. Porosity in lean phase.

We start from calculating the porosity at the height where the freeboard zone ends (3.24) and then we calculate the distribution of porosity with height, assuming that it is related to the flux of solids (3.25). Example of changes in porosity and solid





conc<br/>ntration is presented in Fig. 3.7

$$\varepsilon_{sat} = 1 - \frac{E_{sat}}{(U_0 - U_t)\rho_s} \tag{3.24}$$

$$\varepsilon(h) = \varepsilon_{sat} + (\varepsilon_f - \varepsilon_{sat})exp(-ah) \tag{3.25}$$

FIGURE 3.7: Porosity( red) and solid concentration (blue) in freeboard zone



6. Average concentration above the dense phase is calculated with mean value theorem for integrals according to equation (3.26)

$$\varepsilon_{es} = 1 - \frac{1}{L} \int_0^L \varepsilon_{sat} + (\varepsilon_f - \varepsilon_{sat}) exp(-ah) dh$$
(3.26)

 Finally we find mass of solids in dense phase (3.27) and mass of solid above the dense phase (3.28)

$$m_d = L_{mf}(1 - \varepsilon_{mf})\rho_s \frac{\pi D_t^2}{4}$$
(3.27)

$$m_d = L_{mf} \varepsilon_{es} \rho_s \frac{\pi D_t^2}{4} \tag{3.28}$$

## 3.4 Fluid bed dimensions

To complete the description of bubbling fluidized bed one needs to be able to calculate its dimensions: height and diameter (Fig. 3.8).





Heights of the two zones, dense and lean, featured in figure 3.8 are calculated from Kuni-Levenspiel model  $(L_f)$  and entrainment model (TDH) presented in previous paragraph. TDH can be connected with the height at which the outlet to cyclone is mounted. That leaves only diameters of the reactor to be calculated according to the following procedure. To complete this calculation the flow of gas  $(V_{qas})$  MUST be known!

- 1. Determine the maximum amount of fines that can be carried over from the reactor: p %. Knowing size distribution of the particles in the system, determine the maximum diameter of particles that can be carried over and calculate minimum $(u_{mfp})$ and terminal  $(u_{tp})$  fluidization velocities for this diameter. Choose operation velocity  $(u_{op}$  for the bed such that:  $u_{mfp} < u_{op} < u_{utp}$ ).
- 2. Find the minimum fluidization velocity  $(u_{max})$  for the biggest particles present in the system.
- 3. Check if  $u_{max} < u_{op}$ . If the answer is yes, the reactor can have a shape of a simple cylinder and one can calculate tube dimension from eq. (3.29)

$$\frac{V_{gas}}{u_{op}} = \frac{\Pi d_t^2}{4} \tag{3.29}$$

If the answer is no one need to narrow the bottom part of the reactor in order to increase the initial velocity of gas flowing through reactor. In such case the shape of the reactor will be like the one presented in fig. 3.8. In that case one needs to calculate two different diameters  $d_1$  and  $d_2$ .

 $d_2$  is equal to  $d_t$  calculated from eq. (3.29) and  $d_1$  comes from the eq. (3.30).

$$\frac{V_{gas}}{u_{max}} = \frac{\Pi d_1^2}{4} \tag{3.30}$$

4. The height of the narrowing is determined based on the difference  $d_2 - d_1$  and the assumptions that the slope of the walls of the reactor ( $\alpha < 15 \text{ deg}$ ). Than the height h is calculated from the eq. 3.31

$$\frac{(d_2 - d_1)/2}{h} = \tan \alpha \tag{3.31}$$

#### **3.5** Problems and discussions

1. Using extended K-L model describe the bed of solids with a wide size distribution. The conditions of the bed are presented below.

$d_i, mm$	0.2 - 0.4	0.4 - 0.6	0.6 - 1.0	1.0 - 1.5	1.5 - 2.0	2.0 - 3.0
%	32	20	18	15	7	8

 $\rho_s = 1350 kg/m^3$ ;  $\rho_g = 1.5 kg/m^3$ ;  $U_0 = 50 cm/s$ ;  $\nu = 30 * 10^{-6} Pa * s$ ;  $\varepsilon_{mf} = 0.45$ Estimated height of bed h = 1.75m

2. Using extended K-L model describe the bed of solids with a wide size distribution. The conditions of the bed are presented below.

$d_i, mm$	0.1 - 0.2	0.2 - 0.5	0.5 - 0.8	0.8 - 1.0	1.0 - 1.5	1.5 - 2.5
%	15	20	18	32	7	8

 $\rho_s = 1050 kg/m^3$ ;  $\rho_g = 1.1 kg/m^3$ ;  $U_0 = 200 cm/s$ ;  $\nu = 20 * 10^{-6} Pa * s$ ;  $\varepsilon_{mf} = 0.5$ ;  $\varepsilon_f = 0.796$  Estimated height of bed h = 1.95m Drag coefficient for average diameter of particles:  $C_d = 1.53$ .

- 3. Knowing that to obtain the best conversion rate the ration of gas to solid is equal  $1.23m^3/kg$  and the flow of solid material is  $m_s = 1200kg/h$  find the dimensions of the reactor for this process:
  - (a) assuming solid distribution from ex. 2 adn maximum of 15% carryover from dense zone.
  - (b) assuming that the particle size fits in a range between 0.1mm 5mm and only particles smaller than 0.25mm can be carried over from the bubbling zone.