Title: Hall effect

Experiment No. 13
1. **Goal**
To determine the sample conductivity, Hall coefficient, mobility and concentration of carriers.

2. **What to learn?**

3. **Background**
Consider a semiconductor sample placed in the magnetic field that is in the $z$-direction (Fig. 1).

Suppose we pass a current through that sample perpendicular to the magnetic field, say in the $x$-direction.
The charge carriers are electrons and they drift with drift speed $v_d$ in the opposite direction $-x$.
At the instant shown in Fig. 2a, an external magnetic field $\vec{B}$ pointing out of the plane of the figure has just been turned out. Magnetic force $\vec{F}_B$ acts on each drifting electron, pushing it toward the left edge of the sample.
As time goes on, electrons move to the left piling up on the left edge of the strip, leaving uncompensated positive charges in fixed positions at the right edge. The separation of positive and negative charges produces an electric field \( \vec{E} \) within the sample pointing from right to left (Fig. 2b). This field acts an electric force \( \vec{F}_E \) on each electron tending to push it to the left.

In an equilibrium, the electric force \( \vec{F}_E \) balances the magnetic force \( \vec{F}_B \) so as to keep current flowing along the x-direction.

\[
\vec{F}_B = q(\vec{v} \times \vec{B})
\]

\[
F_B = ev_x B_z
\]

\[
\vec{v} = (v_x, 0, 0), \quad \vec{B} = (0, 0, B_z)
\]

\[
\vec{F}_E = q\vec{E}
\]

\[
F_E = eE_y
\]

\[
\vec{E} = (0, E_y, 0)
\]

Upsurge of the electrical field in the y-direction is called **Hall Effect**. A Hall potential difference \( U_H \) is associated with the electric field across the sample of width \( w \) as follows:

\[
U_H = E_y w
\]

Considering the balance of forces on the charge carriers we see that:

\[
eE_y = ev_x B_z
\]

The total current density is according to the microscopic Ohm’s Law \( j_x = nev_x \) which yields:

\[
eE_y = \frac{eB_z j_x}{ne}
\]

\[
E_y = \frac{1}{ne} j_x B_z = R_H j_x B_z
\]

The Hall coefficient \( R_H \) is defined as:

\[
R_H = \frac{E_y}{j_x B_z} = \frac{1}{ne}
\]

Note that the sign of \( R_H \) indicates if the sample is **n-type** or **p-type**. When the carriers are electrons, the Hall coefficient is negative. When the carriers are holes, the Hall coefficient is
positive. The Hall coefficient enables also establishing the concentration of the carriers in the sample $n$. Carrier concentration is the number or carriers per unit volume.

![Fig. 2. The idea of Hall effect](image)

Electron velocity $v_d$ is taken to vary linearly with the field $E_x$:

$$v_d = \mu E_x$$

which is only true under ohmic condition. The proportionality constant between $v$ and $E$ is $\mu$ – mobility - and it can be expressed as:

$$\mu = \frac{v_d}{E_x} = \frac{j_x}{neE_x} = \frac{1}{E_x} B_z \frac{E_y}{j_z B_z} = \frac{1}{B_z} \frac{E_y}{w} \frac{U_{H_y}}{l} = \frac{1}{B_z} \frac{U_{H_y}}{U_c w}$$

Voltage $U_c = E_x l$ is called the conductivity voltage i.e. voltage between the contacts in the direction of the current and $U_H = E_z w$ is called the Hall voltage and it is the resultant voltage between the contacts that are perpendicular to the direction of the current.

The mobility $\mu$ is used to express conductivity $\sigma = ne\mu$.

Taking into account the dimensions of the sample, the total current is related to the density of charge carriers $n$ and their drift velocity $v_d$ as:

$$I_x = j_x S = j_x wd = (env_d)(wd)$$
i.e. as the product of the current density $j_x$ and the cross-sectional area of the sample $S$.

Finally, we obtain the following expressions for the:

$$\mu = \frac{1}{R_H} \frac{U_H}{l} = \frac{1}{B_z} \frac{l}{U_c} = \frac{1}{B_z} \frac{l}{B_z},$$
$$R_H = \frac{E_y}{j_x B_z} = \frac{U_H}{l} \frac{w}{B_z} = \frac{U_H}{l} \frac{d}{B_z},$$
$$\sigma = \frac{1}{R_H} \mu = \frac{1}{\mu} \frac{1}{B_z} \frac{l}{U_c} \frac{1}{U_H} \frac{d}{B_z} \frac{1}{U_c} \frac{w}{d} \frac{l}{1}.$$

4. Equipment

Electromagnet. Hall probe. Measurement panel containing DC power supply to furnish current to Hall probe, voltmeter of high input impedance to measure Hall voltage $U_H$, voltmeters to measure conductivity voltage $U_c$ and voltmeter to measure voltage $U_R$ across the resistor $R=1000 \, \Omega \pm 1 \, \Omega$ connected in series with the Hall probe.

5. Measurements

Fig. 3 shows the measurement set-up.

![Fig. 3. The measurement set-up.](image)

Fig. 4 shows the view of the front of the measurement panel. Fig. 5 shows the view of electromagnet.
1. Turn the measurement panel on (Fig. 4).
2. Push the knob $U_R$ and then set $U_R$ with the lower potentiometer on the front of the measurement panel. The instructor will choose the certain values of $U_R$ for your experiment. $U_R$
is a voltage across the resistor $R = 1000 \, \Omega \pm 1\%$, which is connected in series with the Hall probe and it is used to calculate current $I_x$ flowing through the Hall probe:

$$I_x = \frac{U_R}{R}.$$  

3. Turn the electromagnet on (Fig. 5). Variac to control current in electromagnet coils should be set for zero current.

4. Set the direction of current through the electromagnet coils "to the right" using the knob with a white arrow on the front of electromagnet.

5. Push the knob $U_H$ and using the upper potentiometer on the front of the measurement panel set $U_H$ for zero.

6. Set the current through the electromagnet coils for a certain values given by the instructor. Do not exceed electromagnet current of 6A. Always gradually increase and decrease current in electromagnet coils. Large inductive voltage surges may damage the insulation of coils.

7. For every setting of the current measure $U_{H1}$ and $U_{c1}$ voltage using voltmeters included in the measurement panel. Push the knob $U_H$ and measure the Hall voltage $U_{H1}$. Push the knob $U_c$ and measure the voltage conductivity $U_{c1}$.

8. Decrease the current in electromagnetic coils to zero and change the direction of current in coils "to the left" using the knob with white arrows on the front of electromagnet. After this change adjust Hall voltage $U_H$ to zero using the upper potentiometer on the front of the measurement panel.

9. Set the current through the electromagnet coils for the same values as in the point 7. For every setting of the current measure $U_{H2}$ and $U_{c2}$ voltage.

10. Use the electromagnet characteristic $B_z = f(I_{em})$ to establish the magnetic field between the electromagnet poles on the basis of current $I_{em}$ in electromagnet coils (Fig. 6).

11. Write down the results in the Table 1.
6. Data handling

1. Write down the results in the Table 1
2. Draw the relationships: $U_H=f(U_c)$, $U_H=f(U_R)$, $U_R=f(U_c)$ for each $B_z$ that corresponds to $I_{em} = 1, 1.5, 2, 2.5, 3.0, 3.5, 4.0$ A, respectively.
3. For each relationship find the "best fit" straight line through the data using the linear regression method, find the regression coefficients of the line $a$ and $b$ and the uncertainties of coefficients $\Delta a$ and $\Delta b$.
4. Find the carrier mobility $\mu$, Hall coefficient $R_H$ and conductivity $\sigma$ of the Hall probe using the relationships:

$$U_H = \mu B_z \frac{w}{l} U_c$$
$$U_H = R_H \frac{B_z}{Rd} U_R$$
$$U_R = \sigma \frac{Rwd}{l} U_c$$

and the proper linear regression equations. Be careful: use the one parameter linear regression ($b=0$)

5. Find the concentration of carriers $n$ on the basis of the relationship $R_H = \frac{1}{ne}$.

6. Calculate the uncertainties $\Delta \mu$, $\Delta R_H$, $\Delta \sigma$, $\Delta n$.

Literature:

Table 1. Results

<table>
<thead>
<tr>
<th>$U_R$ [V]</th>
<th>$I_{em}$ [A]</th>
<th>$B_z$ [Wb/m²]</th>
<th>$U_{H1}$ [mV]</th>
<th>$U_c$ [mV]</th>
<th>$U_{H2}$ [mV]</th>
<th>$U_{c2}$ [mV]</th>
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\[ U_H = \frac{|U_{H1}| + |U_{H2}|}{2},\quad U_c = \frac{U_{c1} + U_{c2}}{2} \]

Dimensions of the Hall probe:

d = (3,7 ± 0,1) × 10^{-3} m,

w = (1,7 ± 0,1) × 10^{-3} m,

l = (11,5 ± 0,2) × 10^{-3} m,

l_0 = (22,4 ± 0,2) × 10^{-3} m.
Electromagnet characteristic $B_z = f(I_{em})$

Fig. 6. Electromagnet characteristic
6. Hints to the report.

1. \( U_H = f(U_c) \) for \( B_z = \text{const.} \)

On the basis of Table 1 create the sub-table:

<table>
<thead>
<tr>
<th>( I_{\text{em}} ) [A] = 1A</th>
<th>( U_c ) [mV]</th>
<th>( U_H ) [mV]</th>
<th>( \mu ) [ ......]</th>
<th>( \Delta \mu / \mu ) [ ......]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_z = \ldots ) Wb/m(^2)</td>
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</table>

Draw the plot \( U_H = f(U_c) \).

Find the regression coefficient \( a \) and the uncertainty \( \sigma_a \) for these data.

Use the one parameter linear regression (\( b = 0 \))

Compare the slopes:

\[
U_H = a \cdot U_c \\
U_H = \mu B_z \frac{d}{l} \cdot U_c \\
a = \mu B_z \frac{d}{l} \\
\mu = \frac{al}{dB_z}
\]

Find the unit of \( \mu \): \( \left[ \mu \right] = \left[ \frac{m}{Wb} \cdot \frac{[m^2]}{Wb} \right] = \left[ \frac{m^2}{Vs} \right] \)

Calculate the uncertainty \( \Delta \mu / \mu \) using the method of logarithmic derivative:

\[
\mu = \frac{al}{dB_z}
\]

Calculate logarithm of both sides of the equation:

\[
\ln \mu = \ln a + \ln l - \ln B_z - \ln d
\]

Derivate both sides of the equation:

\[
\left| \frac{\Delta \mu}{\mu} \right| = \left| \frac{\Delta a}{a} \right| + \left| \frac{\Delta l}{l} \right| + \left| \frac{\Delta B_z}{B_z} \right| + \left| \frac{\Delta d}{d} \right|
\]

where:

\( \Delta a = \sigma_a \) is the uncertainty of the regression coefficient \( a \)

\( \Delta l = 0,2 \cdot 10^{-3} m \), \( \Delta b = 0,1 \cdot 10^{-3} m \) are the geometrical uncertainties of the probe,
$\Delta B_z = ...$ is the uncertainty of the $B_z$ established on the basis of the electromagnet characteristic.

Build-up such sub-tables for each values of $I_{em} = 1, 1.5, 2, 2.5, 3.0, 3.5, 4.0$ A and respective $B_z$. Calculate $\mu$ and $\Delta \mu/\mu$ in each case and collect the results in one final table.

<table>
<thead>
<tr>
<th>$B_z$ [Wb/m²]</th>
<th>$\mu$ [......]</th>
<th>$\Delta \mu/\mu$ [......]</th>
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</table>

2. $U_H=f(U_R)$ for $B_z=const.$

On the basis of Table 1 create the sub-table:

<table>
<thead>
<tr>
<th>$I_{em}$ [A] =1A</th>
<th>$U_R$ [mV]</th>
<th>$U_H$ [mV]</th>
<th>$R_H$ [......]</th>
<th>$\Delta R_H/R_H$ [......]</th>
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</table>

Draw the plot $U_H=f(U_R)$.

Find the regression coefficient $a$ and the uncertainty $\sigma_a$ for these data.

Use the one parameter linear regression (b=0).

Compare the slopes:

$U_H = a \cdot U_R$

$U_H = R_H \frac{B_z}{Rd} U_R$

$a = R_H \frac{B_z}{Rd}$

$R_H = a Rd \frac{B_z}{B_z}$

Find the unit of $R_H$.

Calculate the uncertainty of $\Delta R_H/R_H$ using the method of logarithmic derivative:

Build-up such sub-tables for each values of $I_{em} = 1, 1.5, 2, 2.5, 3.0, 3.5, 4.0$ A and respective $B_z$. Calculate $R_H$ and $\Delta R_H/R_H$ in each case and collect the results in one final table.
3. \( U_R = f(U_{c, \text{average}}) \)

On the basis of Table 1 create the subtable:

<table>
<thead>
<tr>
<th>( B_z [\text{Wb/m}^2] )</th>
<th>( R_H )</th>
<th>( \Delta R_H/R_H )</th>
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<thead>
<tr>
<th>( I_{\text{em}} [A] ) = 1A</th>
<th>( U_c ) [mV]</th>
<th>( U_R ) [mV]</th>
<th>( \sigma )</th>
<th>( \frac{\Delta \sigma}{\sigma} )</th>
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<tr>
<td>( B_z = \ldots ) Wb/m²</td>
<td>4 000</td>
<td>5 000</td>
<td>6 000</td>
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Draw the plot \( U_R = f(U_c) \).

Find the regression coefficient \( a \) and the uncertainty \( \sigma_a \) for these data.

Use the one parameter linear regression (b=0)

Compare the slopes:

\[
U_R = a \cdot U_c \\
U_R = \sigma \frac{R_wd}{l} U_c \\
a = \sigma \frac{R_wd}{l} \\
\sigma = \frac{al}{R_wd}
\]

Find the unit of \( \sigma \).

Calculate the uncertainty of \( \Delta \sigma/\sigma \) using the method of logarithmic derivative:

Build-up such sub-tables for each values of \( I_{\text{em}} \) = 1, 1.5, 2, 2.5, 3.0, 3.5, 4.0 A and respective \( B_z \). Calculate \( R_H \) and \( \Delta R_H/R_H \) in each case and collect the results in one final table.
4. \( n = \frac{1}{R_n e} \)

Remember: \( e = 1.6 \times 10^{-19} \) C.

Determine the unit of \( n \). Calculate the uncertainty \( \Delta n \) using the total differential method:

\[
\Delta n = \left| \frac{\delta n}{\delta R_n} \right| \cdot |\Delta R_n|
\]

Collect the results in the final table:

<table>
<thead>
<tr>
<th>( B_z ) [Wb/m²]</th>
<th>( n ) [........]</th>
<th>( \Delta n ) [........]</th>
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*Updated: 15.02.2009, 11.03.2010 by Barbara Dziurdzia*